Anisotropies of solar wind turbulence through out the inertial range

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Solar Wind turbulence

A number of properties are found/explained by DNS of homogeneous turbulence

- Global anisotropy : more power in $k_\perp$ than in $k_\parallel$
- Spectral scaling w.r.t. large-scale mean field $\mathbf{B_0}$:

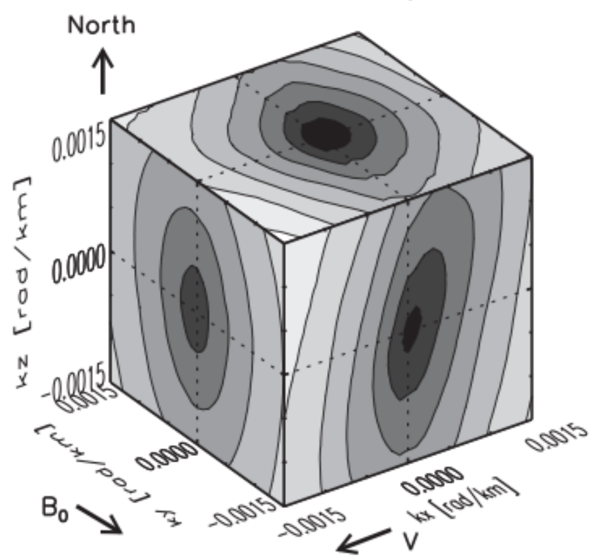
  -5/3 at all θ_{B0V}
- Structure Function scaling w.r.t. scale-dependent mean field $\mathbf{B_\ell}$:

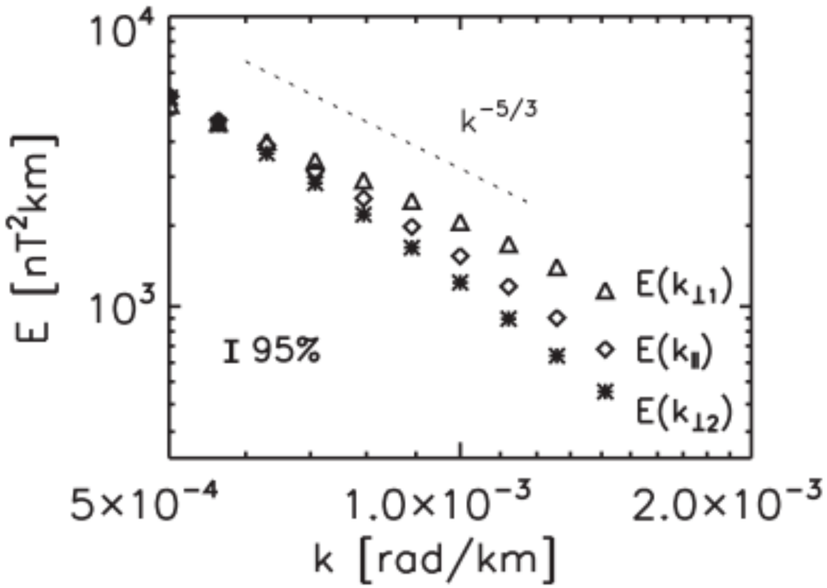
  -5/3 → -2 when θ_{BV}=0 → 90º
- The III-order structure function scales linearly with increments
- The cascade rate is consistent with the heating rate required to sustain the solar wind

All measurements assume axi-symmetry around B_0, but **solar wind turbulence** is **not axi-symmetric.**

Global Anisotropy

2D spectrum **CLUSTER** **1D spectrum**

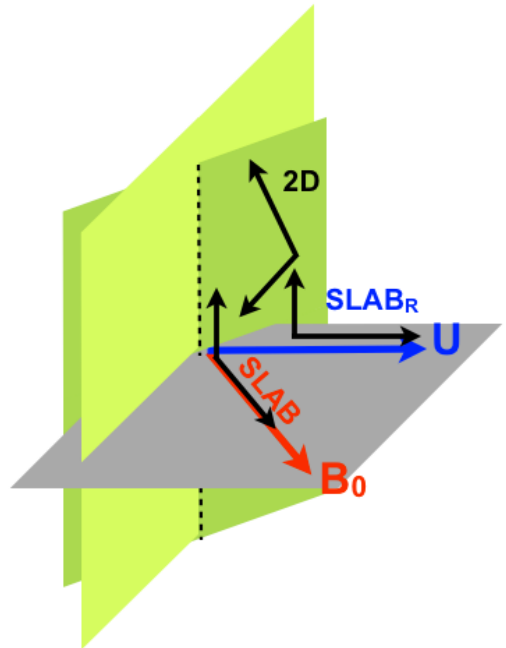




Narita et al 2010

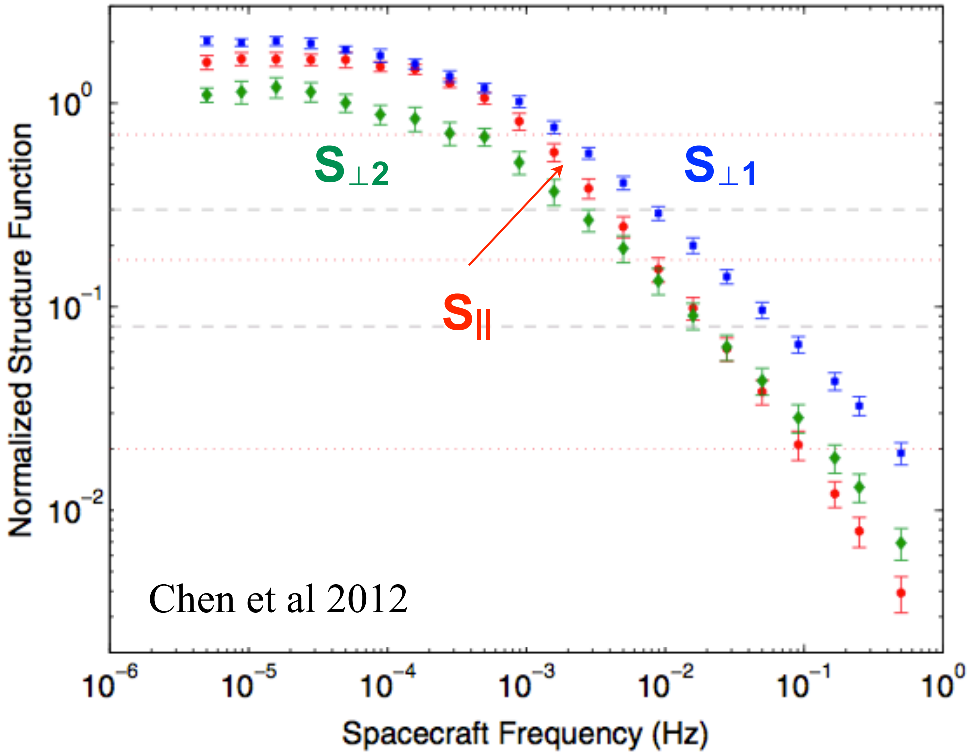
additional axis of symmetry?

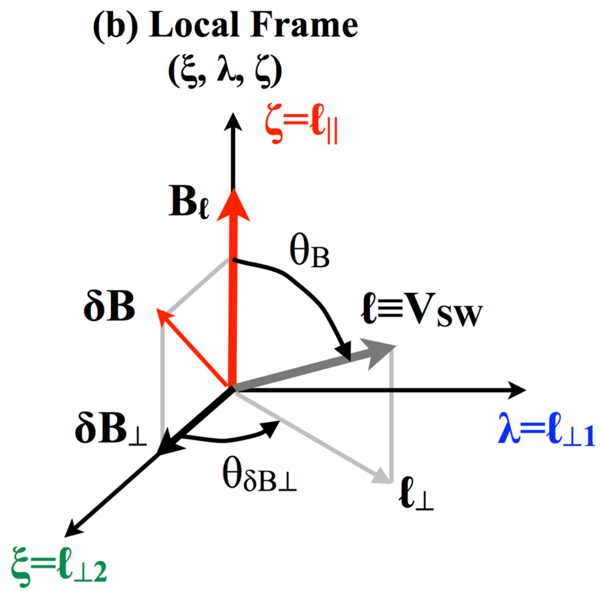
early hints from Saur & Bieber 1996
SLAB$_R$ + 2D decomposition fits better than
SLAB + 2D decomposition



Local Anisotropy

Anisotropy w.r.t B_ℓ (local)





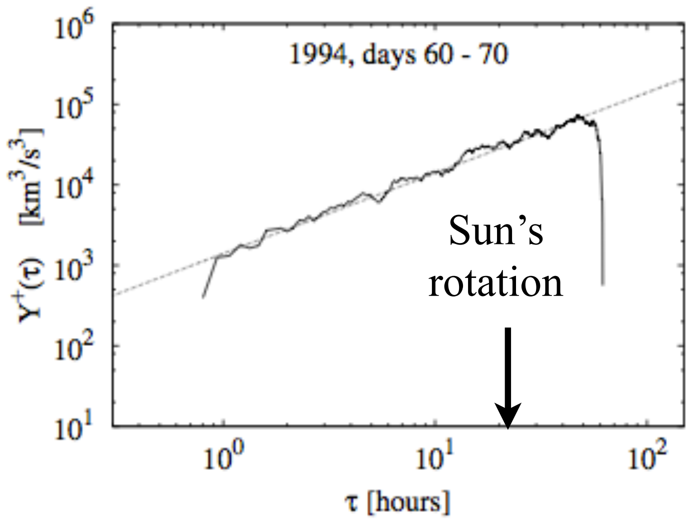
- small scales $S_{\perp 1} > S_{\perp 2} > S_{||}$ (~Boldyrev 2005, b-v alignment)
- large scales $S_{\perp 1} \sim S_{||} > S_{\perp 2}$ why ?

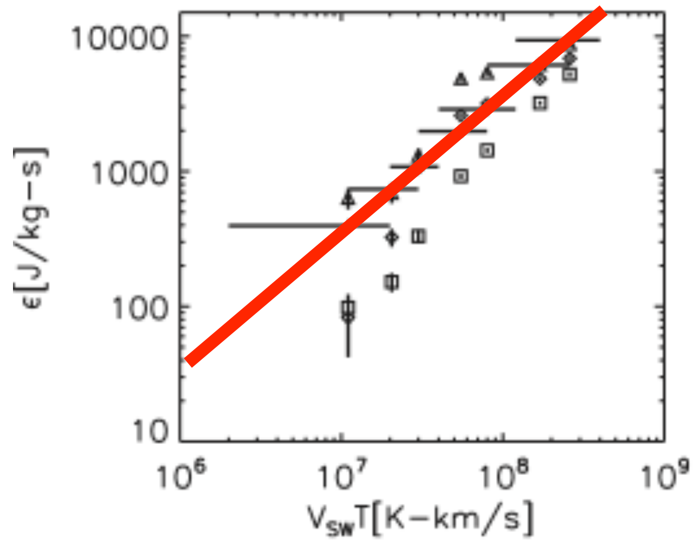
additional axis of symmetry3D?

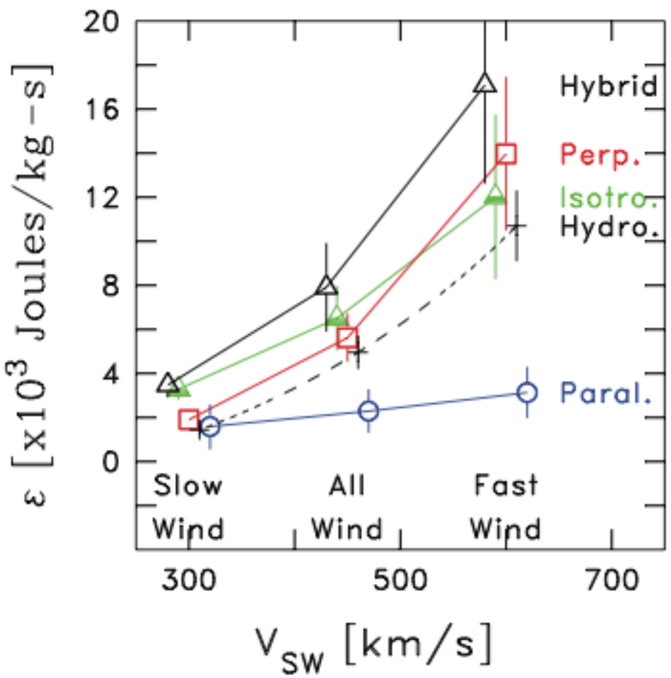
Cascade ≈ Heating

$$Y\cdot\boldsymbol{\ell}/|\boldsymbol{\ell}| \propto -\varepsilon\ell$$

$$\epsilon_{heat} = 3.6 \times 10^{-5} T_{pr} V_{SW}\ [\mathrm{J/(kg\,s)}],$$





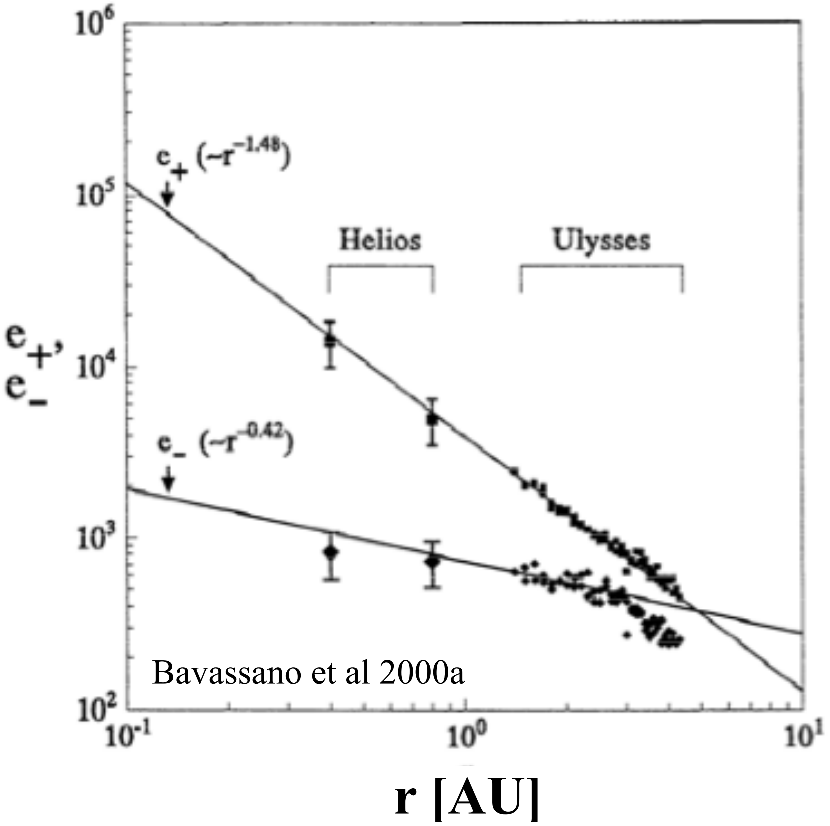


The cascade rate ε is consistent with the required heating at 1AU

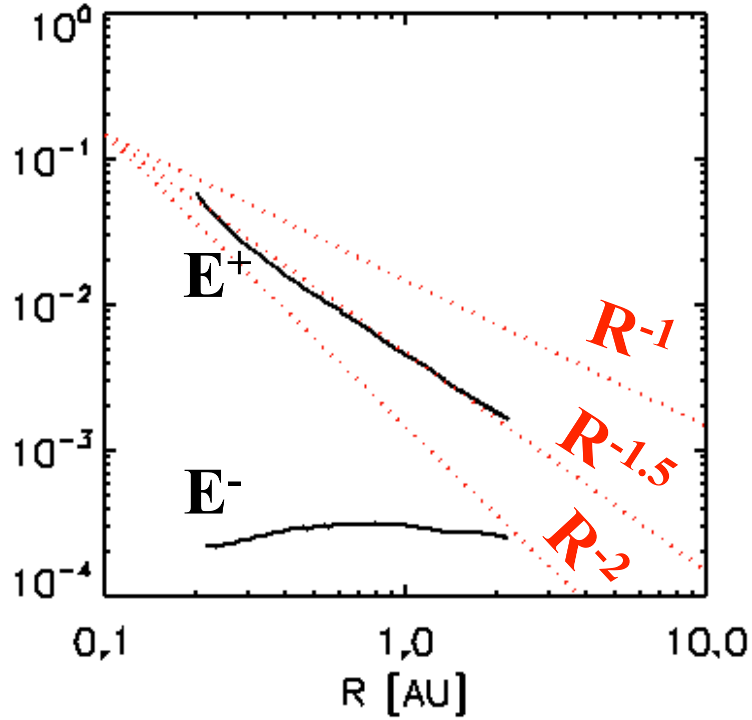
But the value of ε is strongly model dependent (**Y** is unknown)

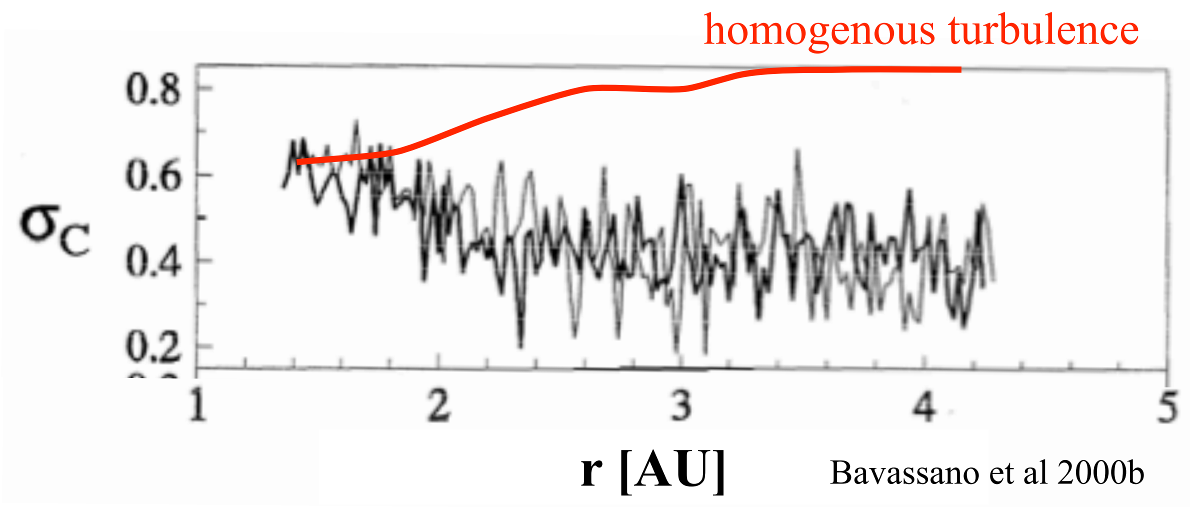
Alfvénic Turbulence

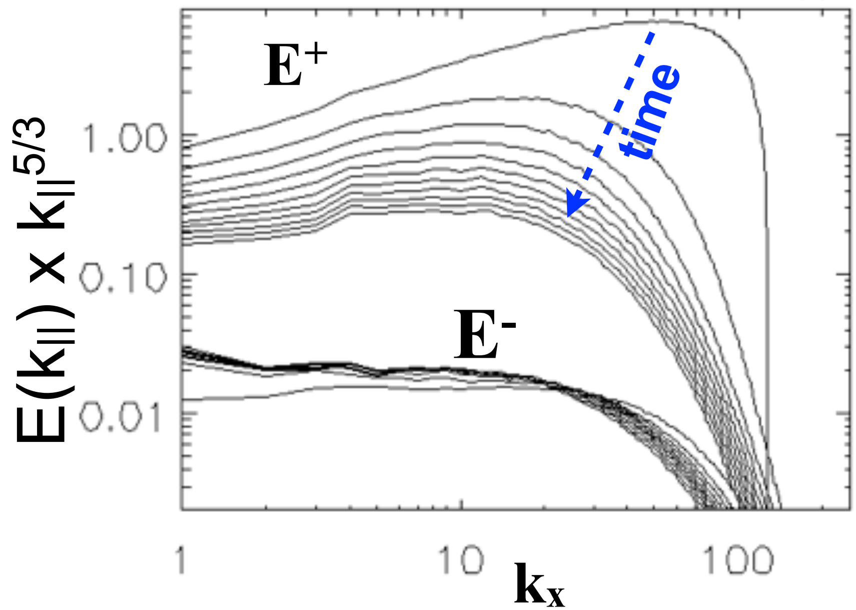
Elsasser energies



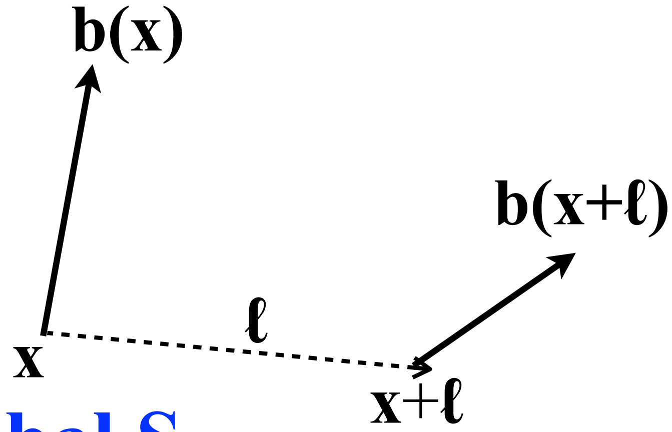
Elsässer energies (k_x=10~1hour)







II-order Structure Functions



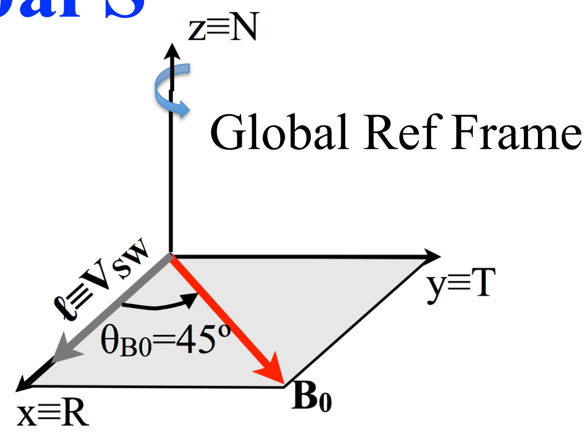
II-order SF $S(\ell) = \langle |\Delta b|^2 \rangle_x$

two variants:

- define ℓ w.r.t. the global field $\mathbf{B_0}$ => **Global S**

II order statistics, anisotropy

Related to Fourier Spectra



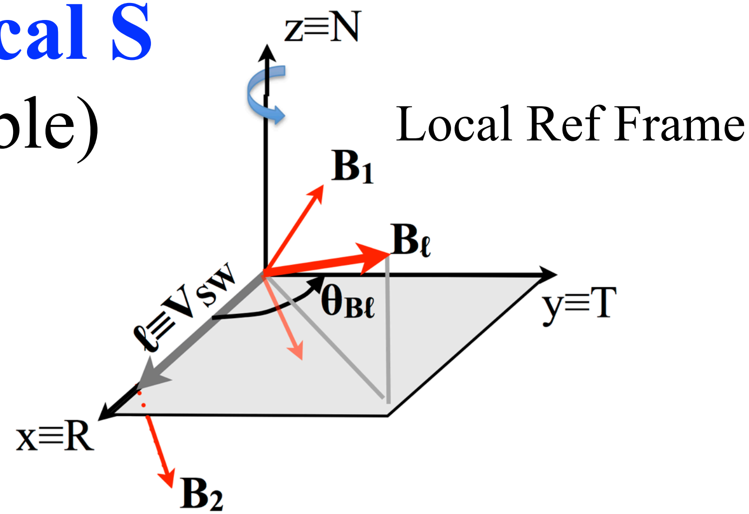
- define ℓ w.r.t. the local field $\mathbf{B_\ell}$ => **Local S**

higher order statistics (B_ℓ is a random variable)

anisotropy with respect to B_ℓ

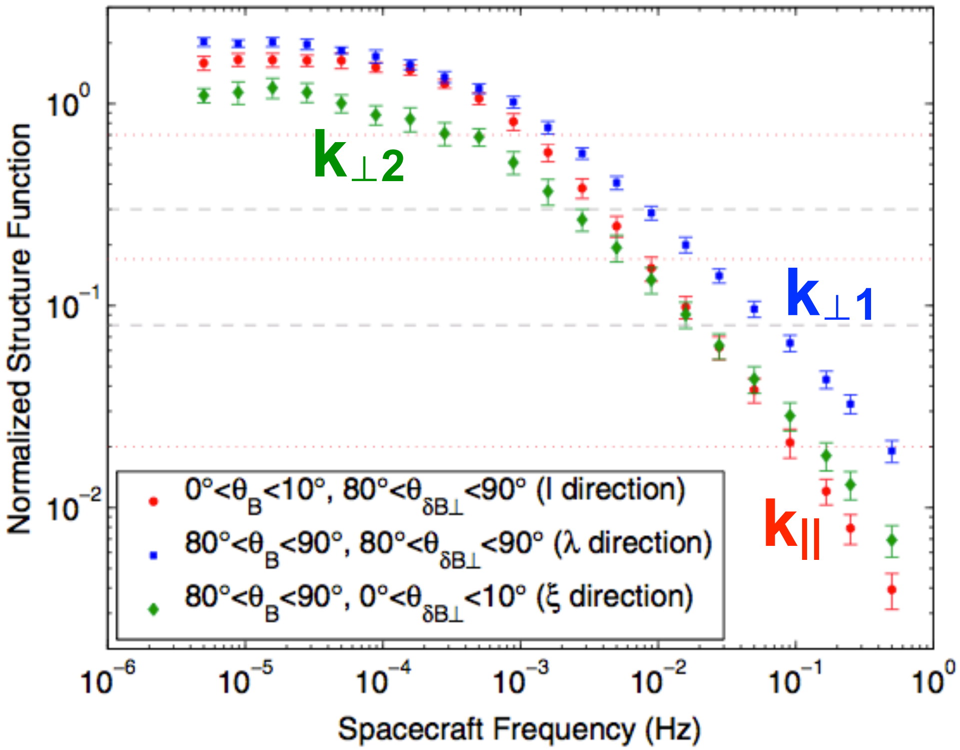
highlight local dynamics

(e.g. critical balance)

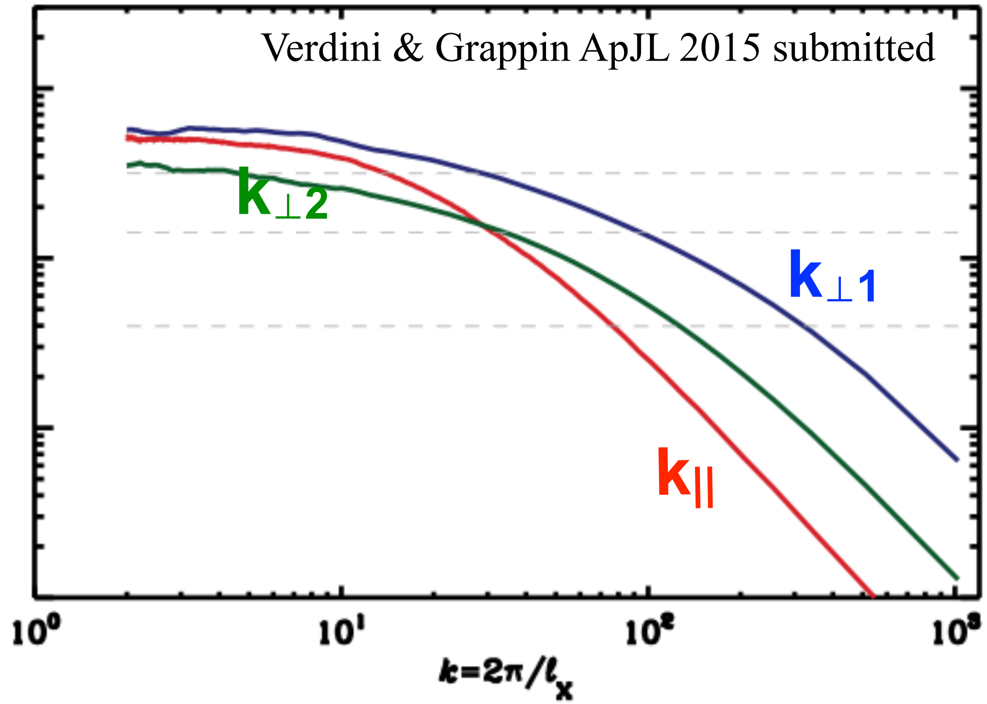


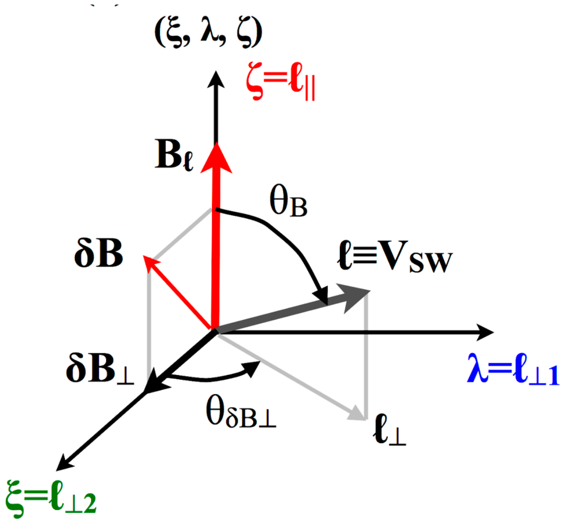
Local Anisotropy

Observations Chen et al 2012



Expanding DNS

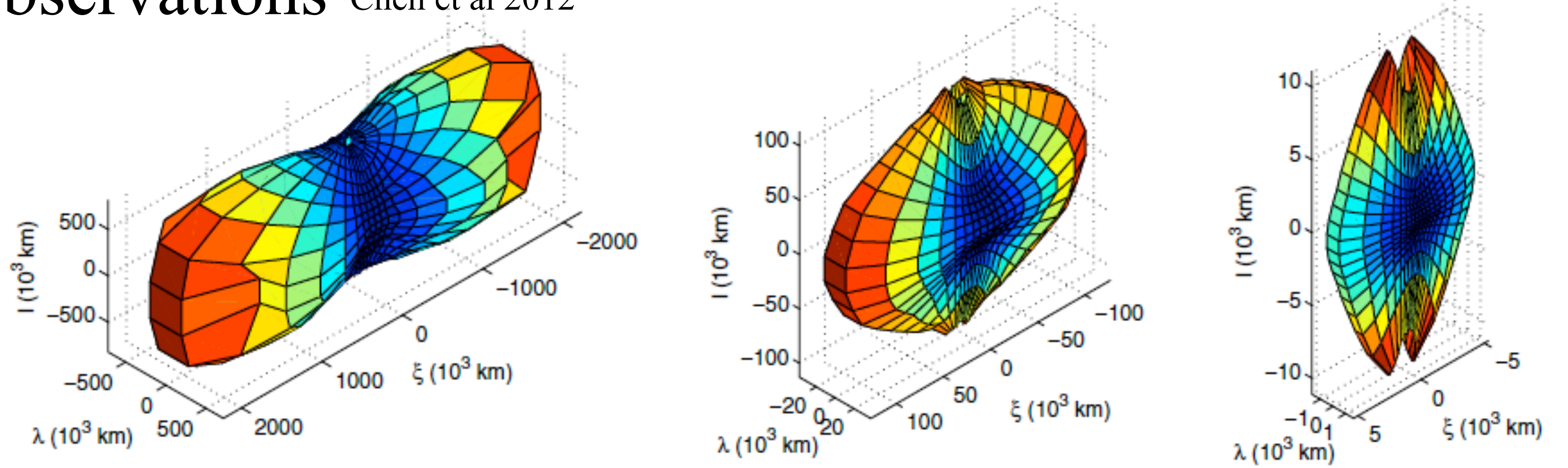




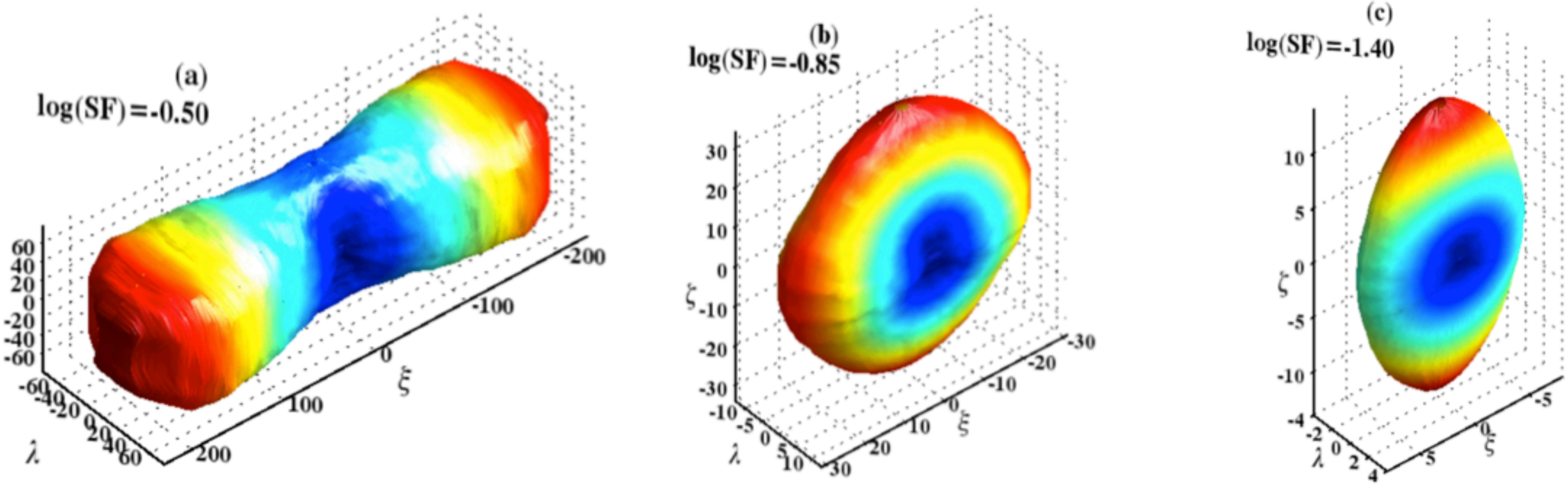
- large scales $S_{\perp 1} \sim S_{\|} > S_{\perp 2}$
- small scales $S_{\perp 1} > S_{\perp 2} > S_{\|}$

Local Anisotropy: eddy shape

Observations Chen et al 2012



Simulation Verdini & Grappin ApJL 2015 submitted

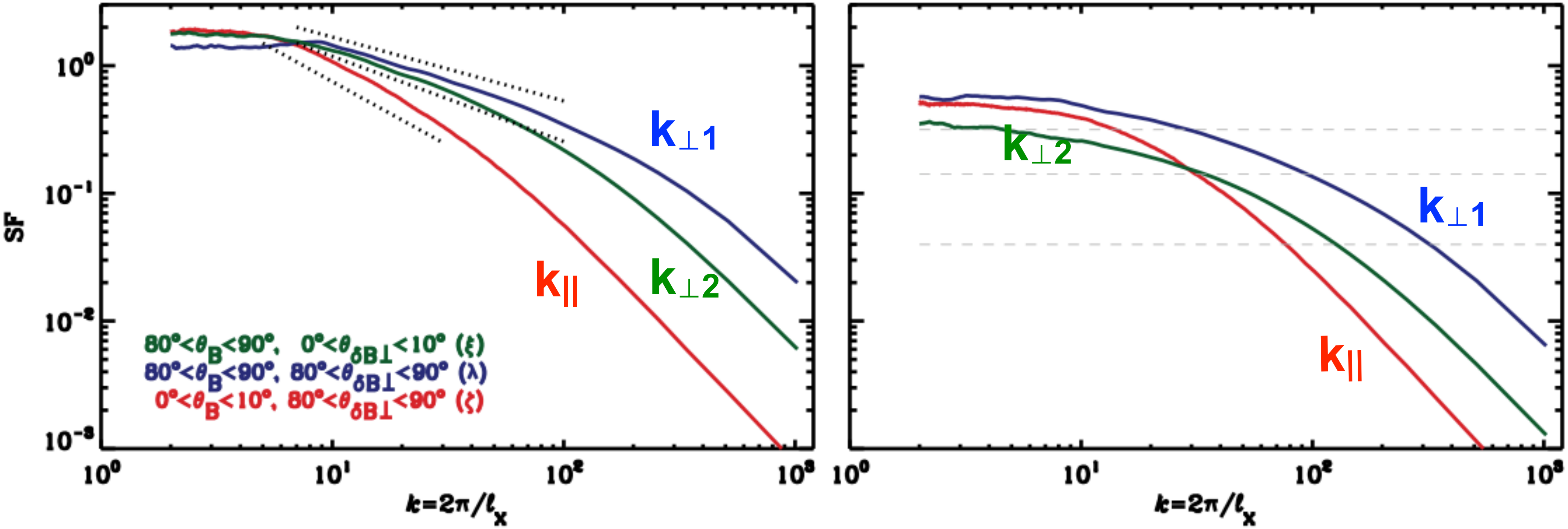


Local Anisotropy

Verdini & Grappin ApJL 2015 submitted

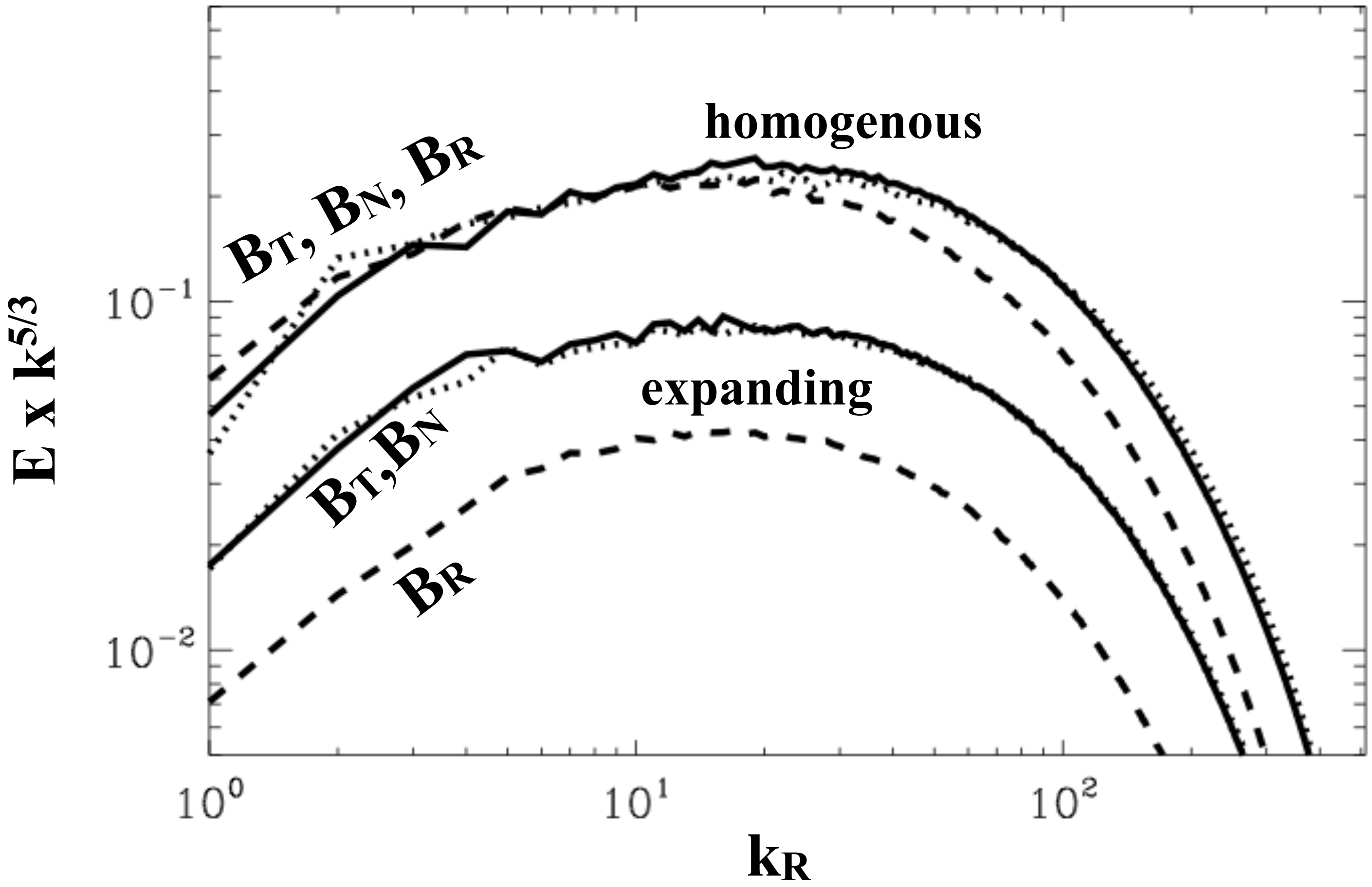
Homogenous DNS

Expanding DNS



Component Anisotropy

Dong, Verdini, Grappin ApJ 2014
Verdini & Grappin ApJL 2015 submitted



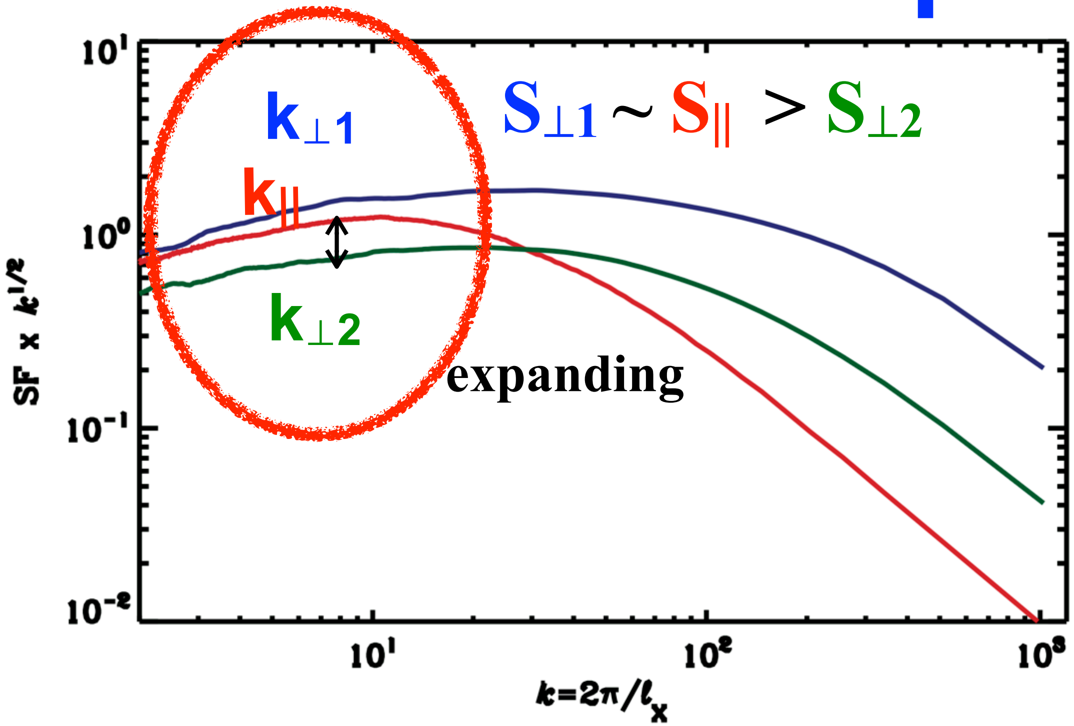
With expansion: fluctuations are confined in the (T,N) plane

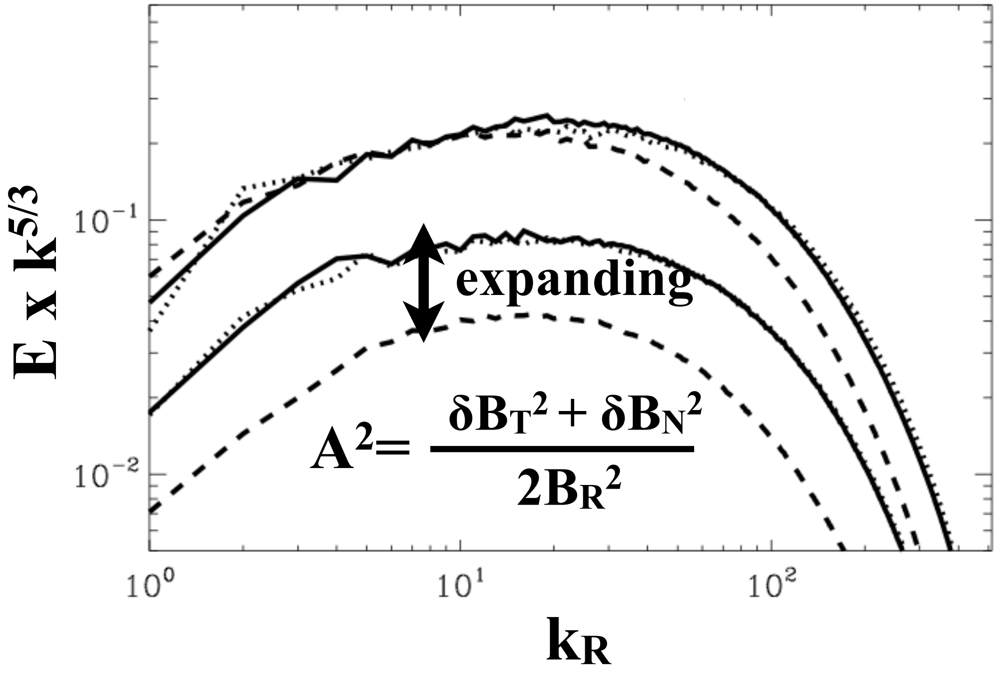
$$\mathbf{B_{\perp 1}, B_{\perp 2} \sim O(1) >> B_{||1}, B_{||2} \sim O(2)}$$

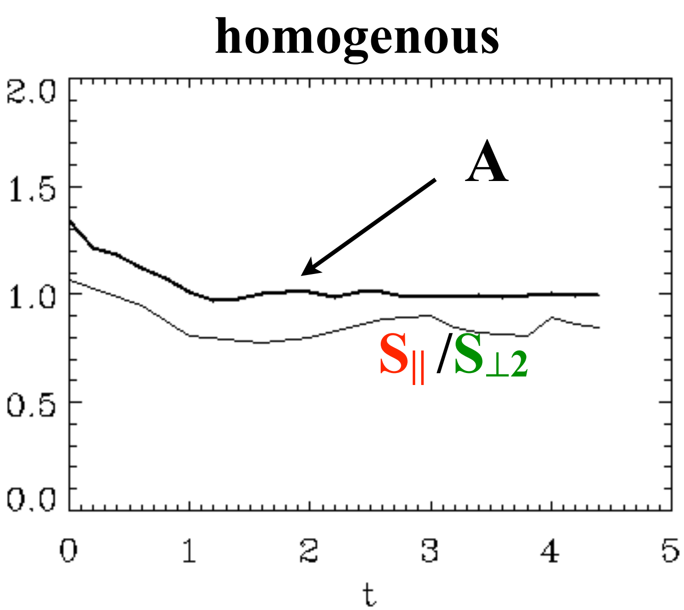
=> affect local field $B_\ell = B_1 + B_2$

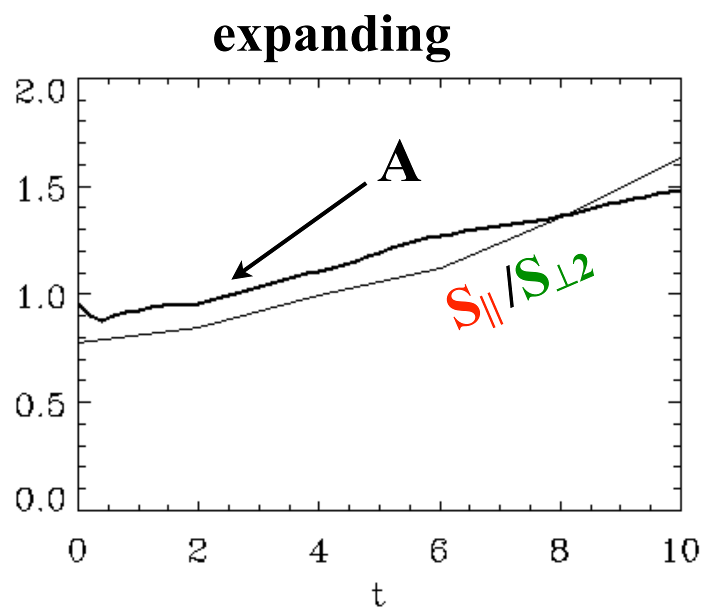
=> affect power in SF=$|\delta B|^2 = |B_1 - B_2|^2$

Local & Component Anisotropy





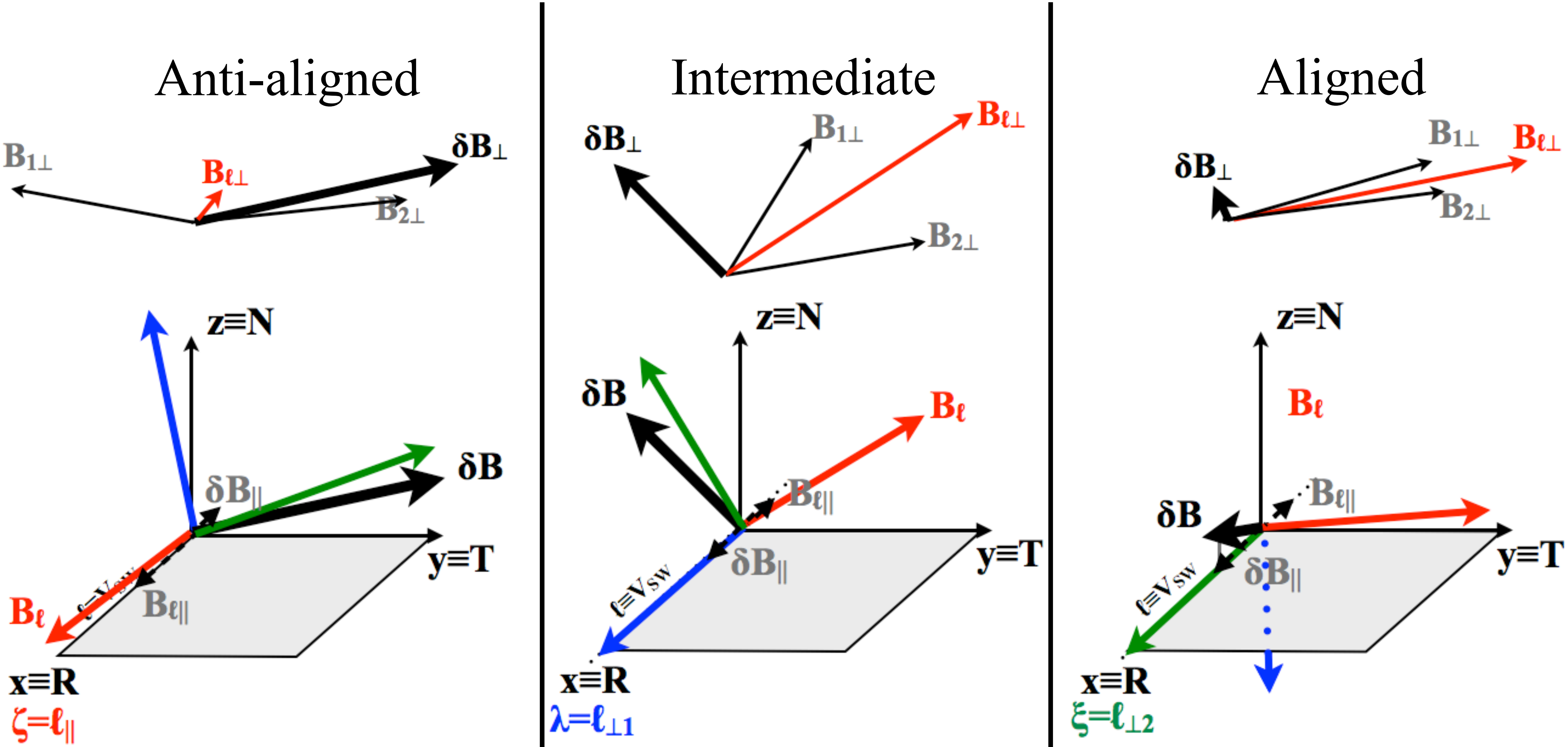




Local & Component Anisotropy

$S_{\perp 1} \sim S_{\parallel} > S_{\perp 2}$ assume $B_{\perp 1}, B_{\perp 2} \sim O(1) >> B_{\parallel 1}, B_{\parallel 2} \sim O(2)$

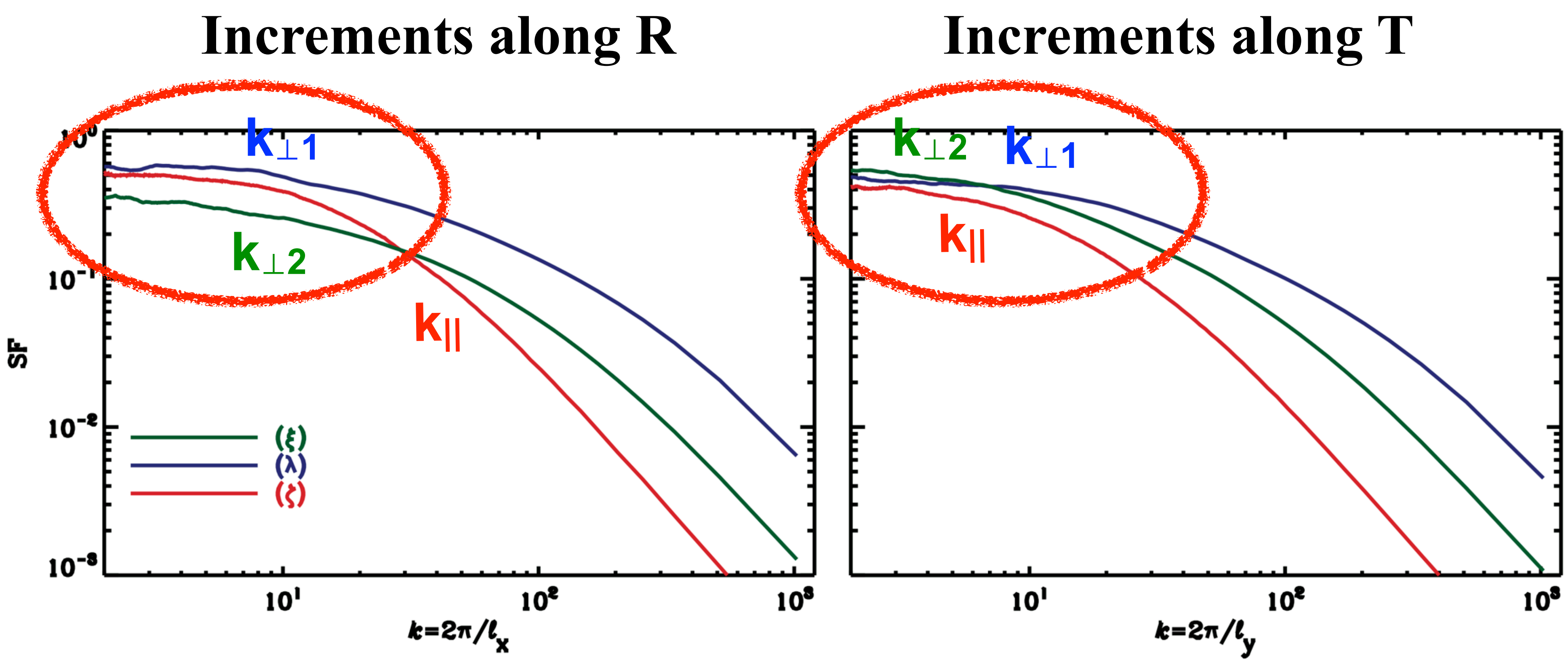
the orientation & SF power depend on the alignment of $B_{\perp 1}, B_{\perp 2}$



$SF(\zeta) = |\delta B|^2 \sim O(1)$ $SF(k_{\perp 1}) = |\delta B|^2 \sim O(1)$ $SF(k_{\perp 1}) = |\delta B|^2 \sim O(2)$

Local anisotropy depend on the direction of increments

EXPANDING CASE

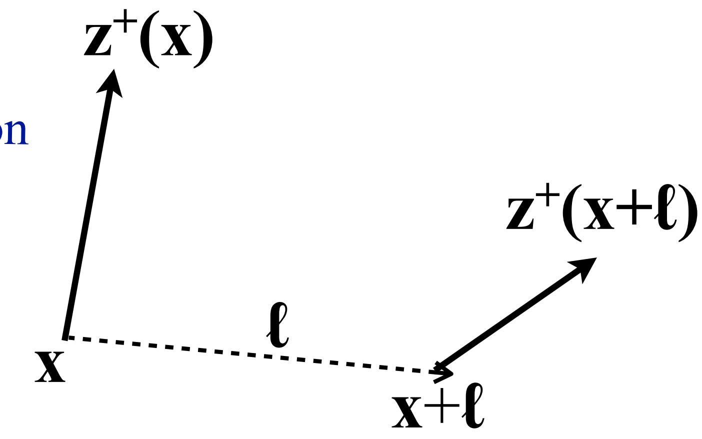


III-order Structure Function

$$\mathbf{z}^{\pm} = \delta\mathbf{u} \mp \delta\mathbf{b}/\sqrt{4\pi\rho}$$
$$\Delta\mathbf{z}^{\pm}(\mathbf{x}, \boldsymbol{\ell}) = \mathbf{z}^{\pm}(\mathbf{x} + \boldsymbol{\ell}) - \mathbf{z}^{\pm}(\mathbf{x})$$

$$S = 1/2\left[\langle|\Delta\mathbf{z}^{-}|^2\rangle + \langle|\Delta\mathbf{z}^{+}|^2\rangle\right]$$ II-order Structure Function

$$\epsilon = \nu/2\left[\langle\Sigma_i(\partial_{\mathbf{x}} z_i^{+})^2\rangle + \langle\Sigma_i(\partial_{\mathbf{x}} z_i^{-})^2\rangle\right]$$ Dissipation rate



III-order Structure Function

$$\mathbf{Y} = 1/2\left[\left\langle\Delta\mathbf{z}^{-}|\Delta\mathbf{z}^{+}|^2\right\rangle + \left\langle\Delta\mathbf{z}^{+}|\Delta\mathbf{z}^{-}|^2\right\rangle\right]$$

Connected to global II-order SF by a dynamical equation

$$\partial_t S + \boldsymbol{\nabla}_{\ell} \cdot \mathbf{Y} = -4\epsilon + 2\nu\boldsymbol{\nabla}_{\ell}^2 S$$

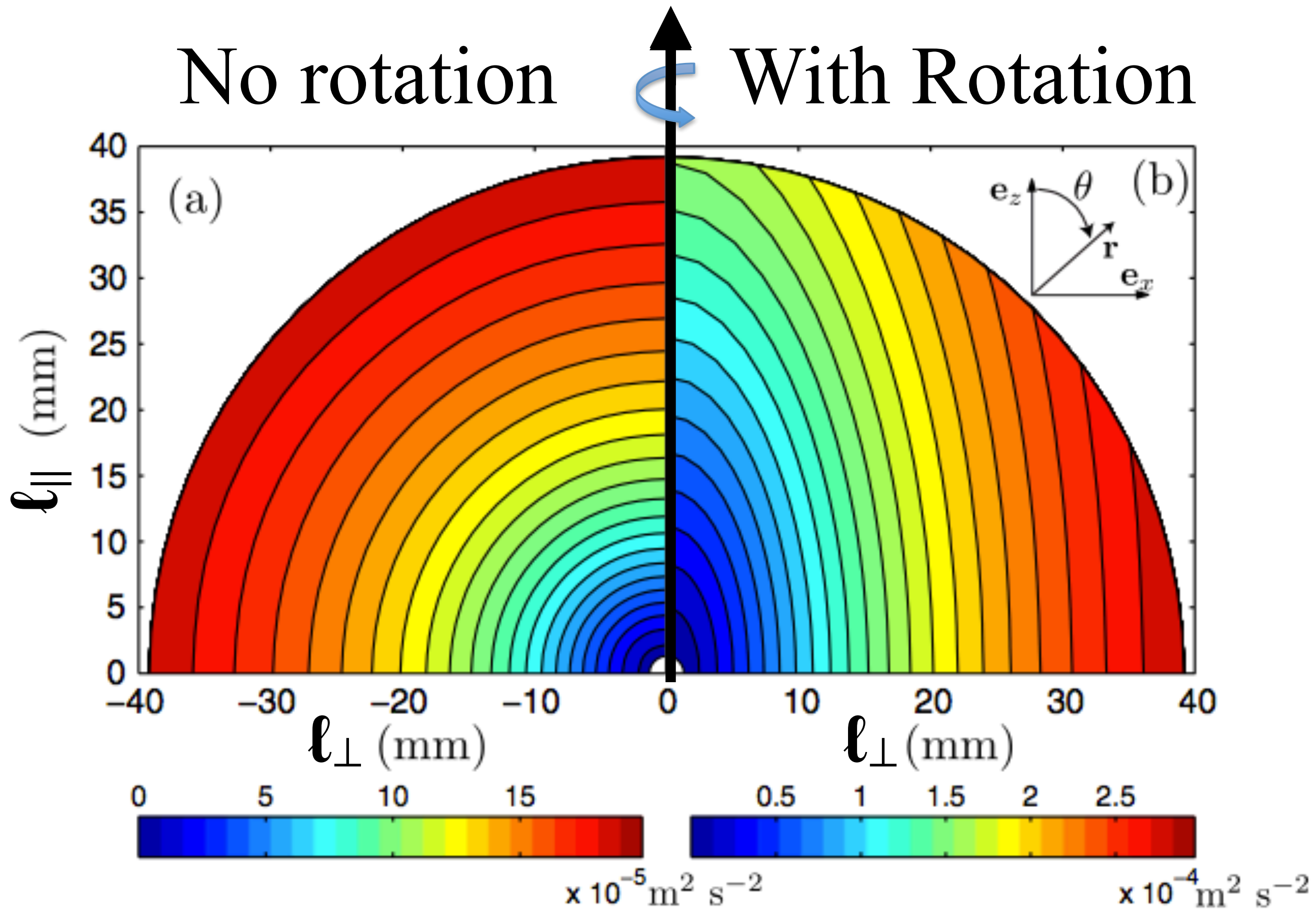
Returns the **cascade rate (ε)** assuming **stationary** and **isotropic** turbulence and vanishing dissipation

$$Y_{\ell} = -\frac{4}{3}\epsilon\ell$$

Politano & Pouquet 1998

$$Y_{\ell} = \mathbf{Y} \cdot \hat{\ell}$$

Fluid turbulence: anisotropy of II-order SF



Lamriben et al 2011, PRL

Fluid turbulence: anisotropy of III-order SF

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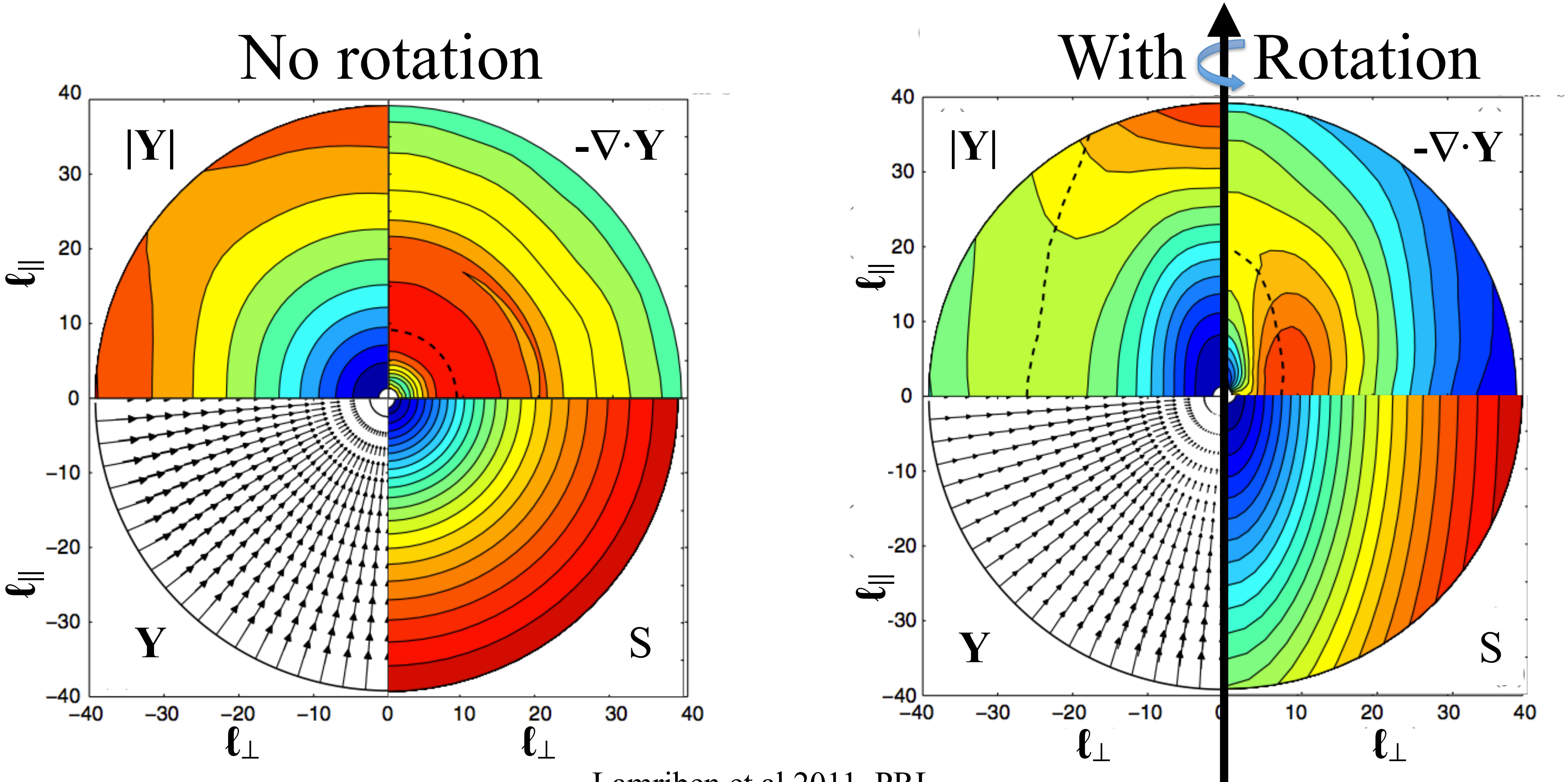
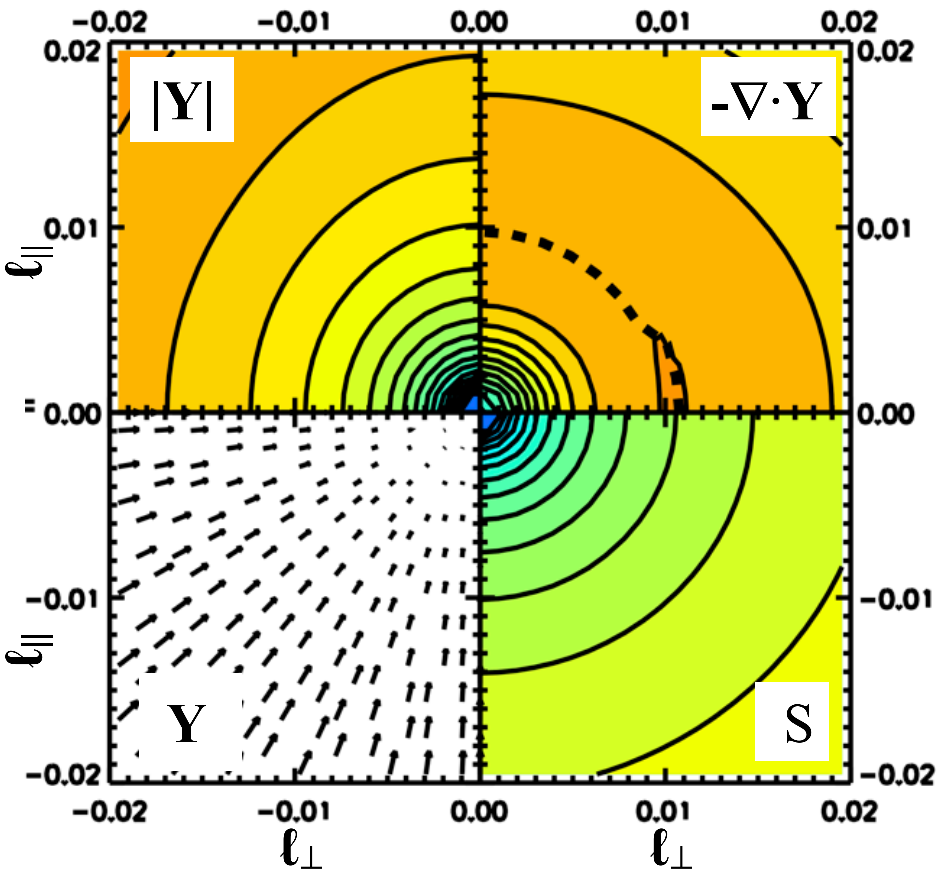


Lamriben et al 2011, PRL

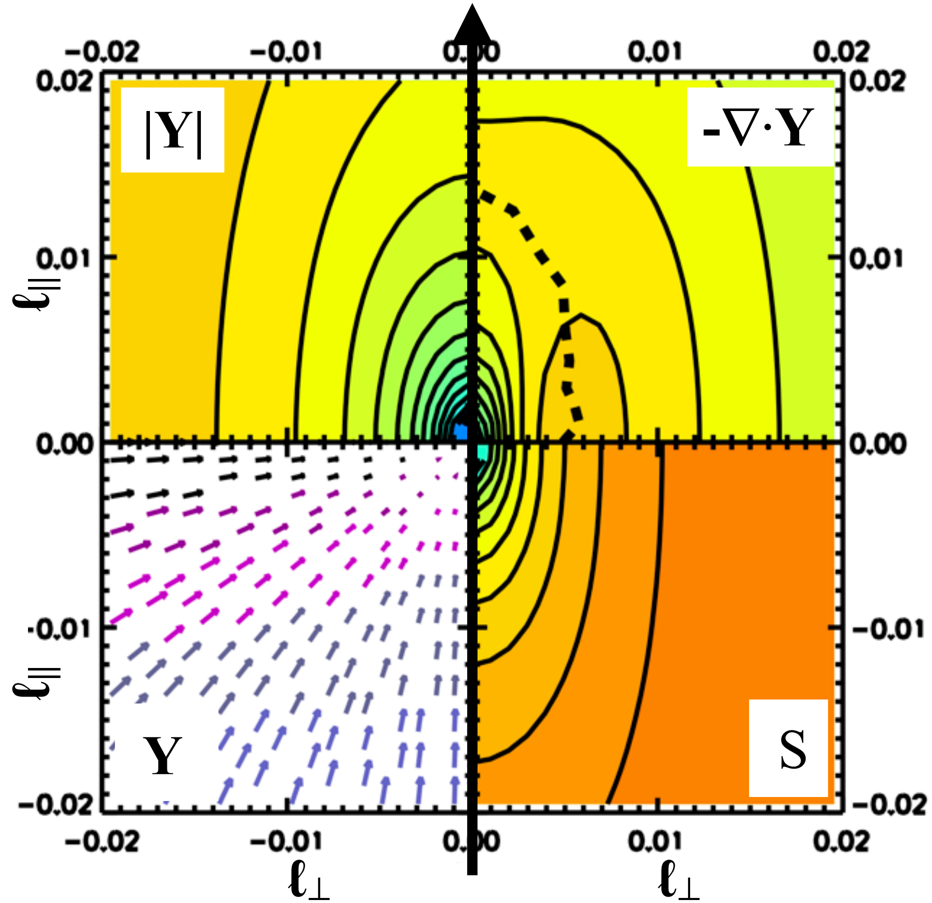
MHD turbulence
anisotropy of III-order SF

$$\partial_t S + \boldsymbol{\nabla}_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \boldsymbol{\nabla}_\ell^2 S$$

B_0=0



B_0=1

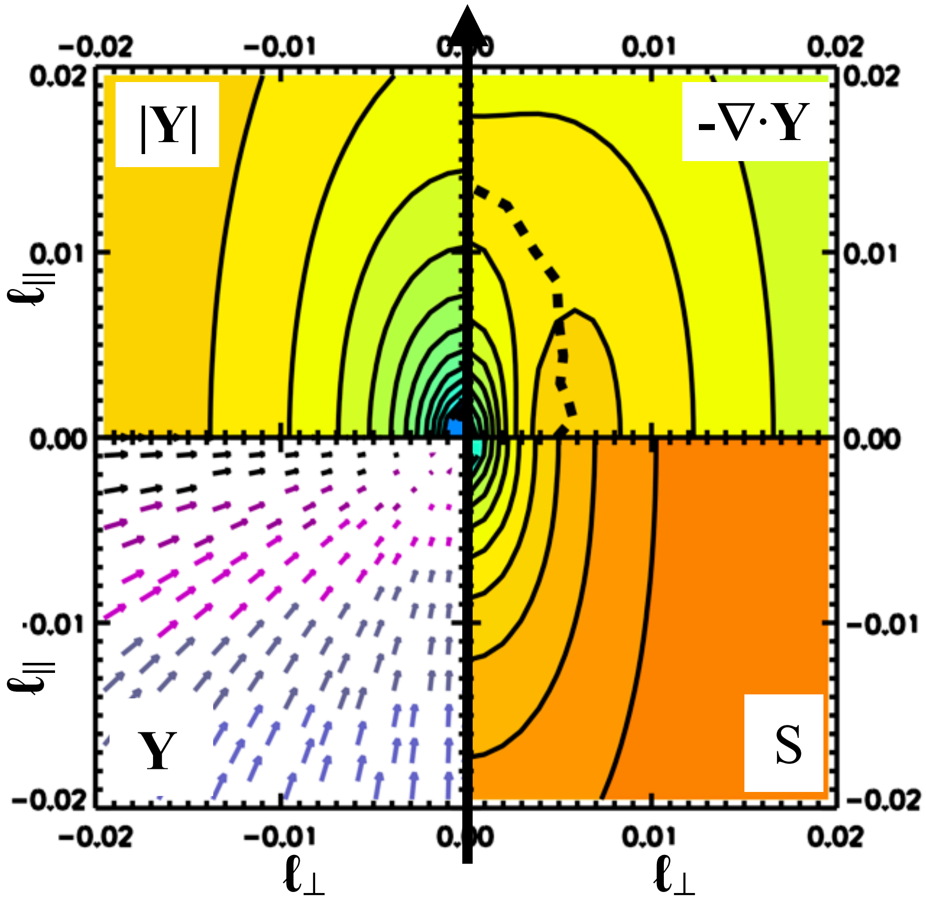


Verdini et al. 2015 ApJ

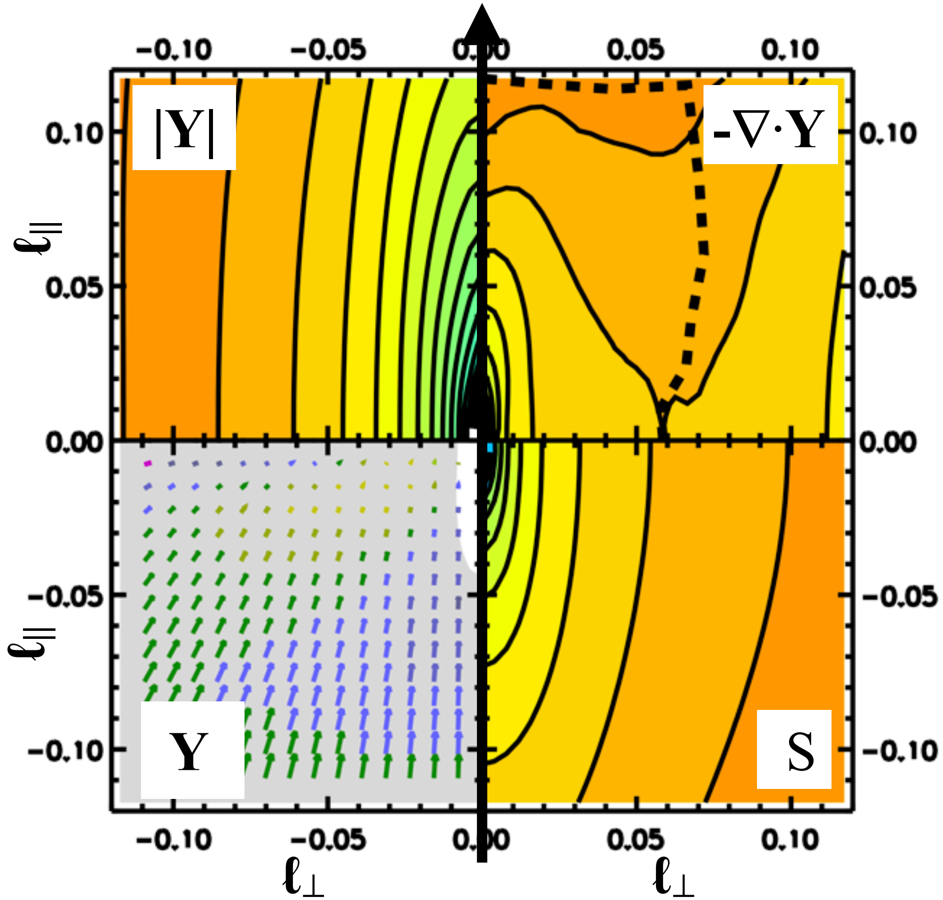
MHD: several anisotropies?

$$\partial_t S + \boldsymbol{\nabla}_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \boldsymbol{\nabla}_\ell^2 S$$

Weak B_0=1, decaying



Strong B_0=5, Forced



Verdini et al. 2015 ApJ

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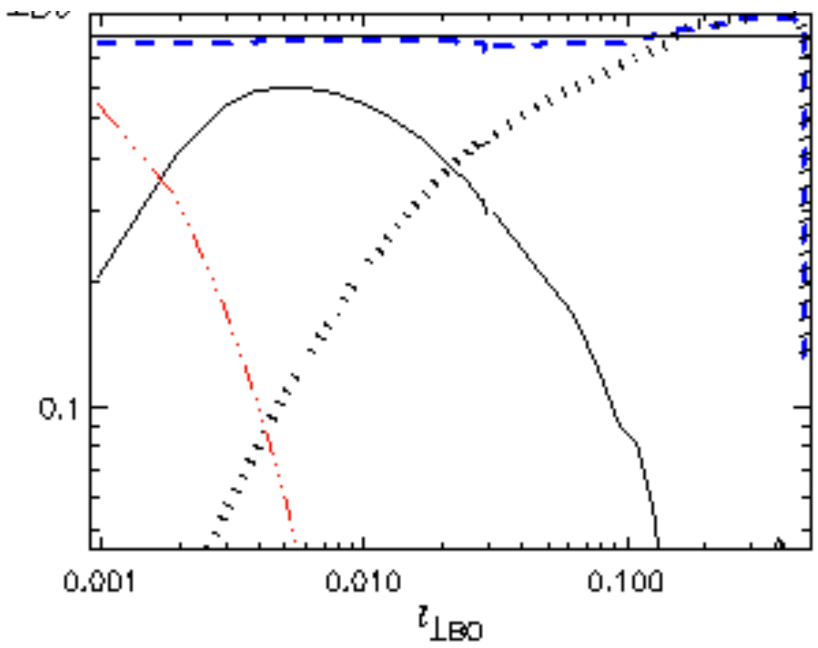

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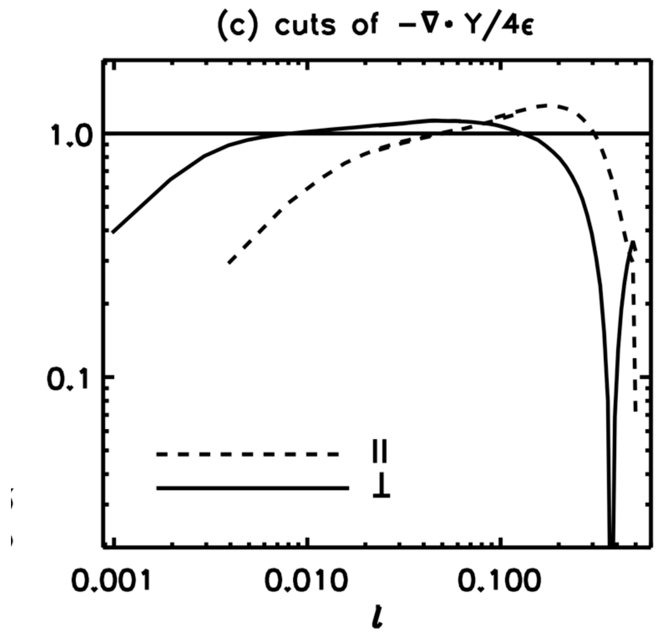


Verdini et al. 2015 ApJ

III order SF and Expansion

In a frame co-moving with the solar wind

Hellinger al 2013

$$\partial_t S + \nabla_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \nabla_\ell^2 S - \frac{U_0}{R}\left[S + \Delta\mathbf{z}^+ \cdot \Delta\mathbf{z}^- - 2\Delta z_R^+ \Delta z_R^-\right]$$

homogenous expansion

- Expansion introduces additional decay/source terms
- $R=R_0+Ut$ => $\partial_t S$ includes radial evolution

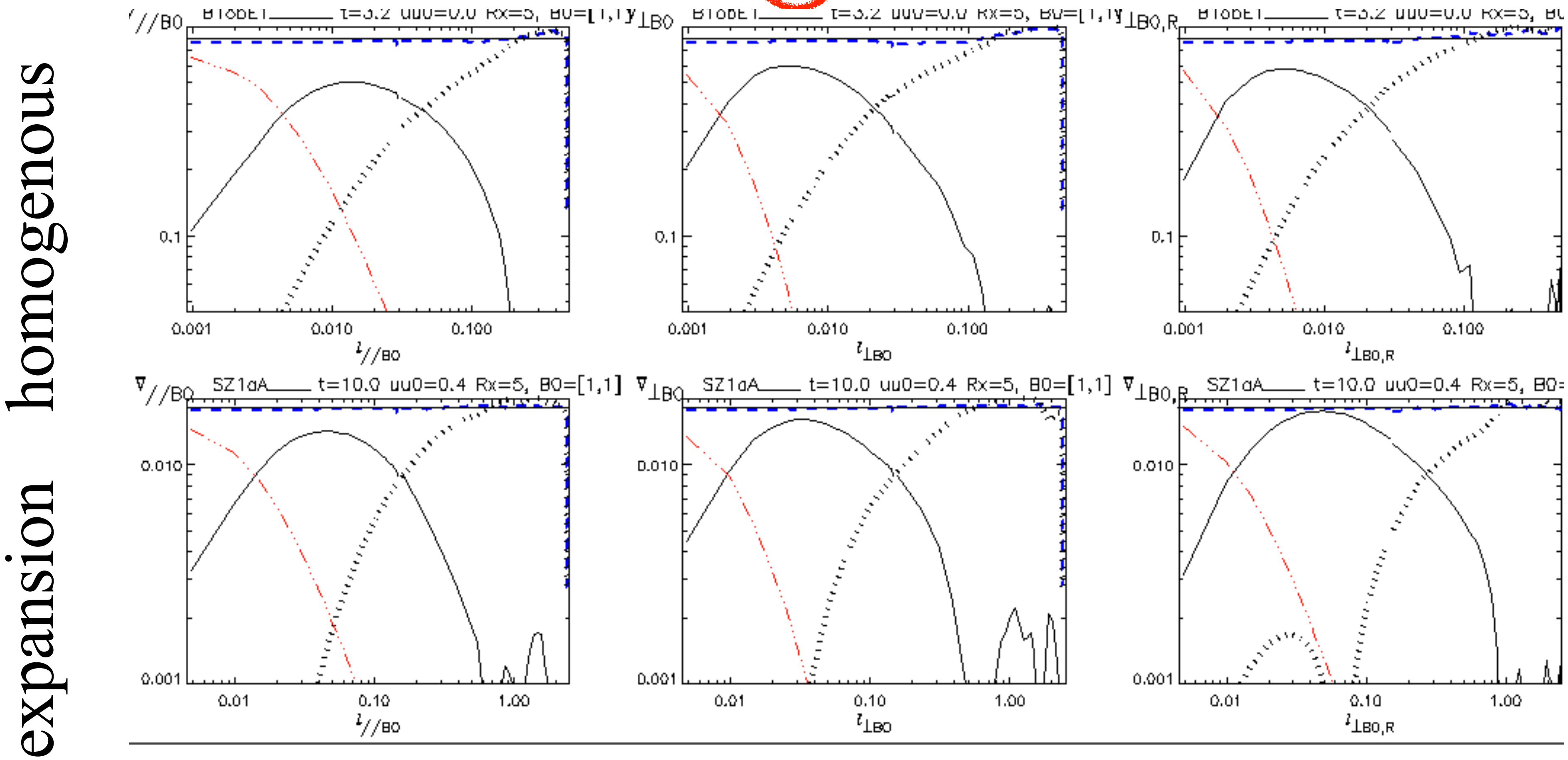
=> The cascade (Y) contribute still to the spectral anisotropy (S) but it is not the unique source of anisotropy

III order SF and Expansion

In a frame co-moving with the solar wind

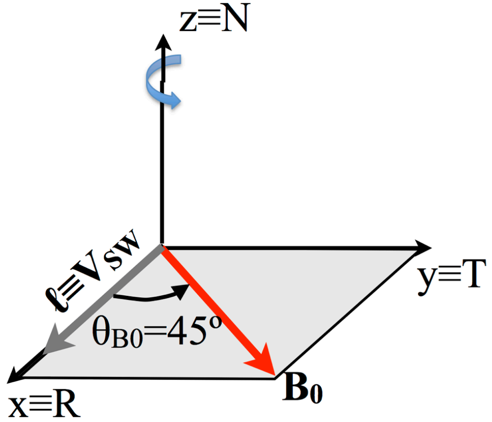
Hellinger al 2013

$$\partial_t S + \nabla_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \nabla_\ell^2 S - \frac{U_0}{R}\left[S + \Delta \mathbf{z}^+ \cdot \Delta \mathbf{z}^- - 2\Delta z_R^+ \Delta z_R^-\right]$$



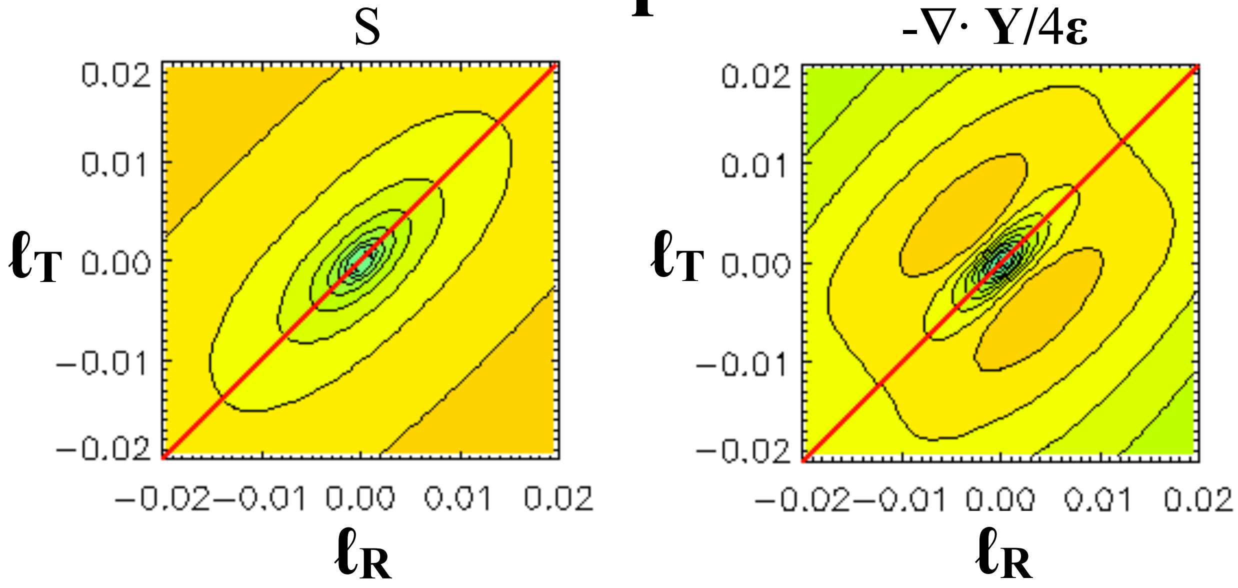
Expansion : Non Axisymmetry

S and ∇·Y have the
same anisotropy

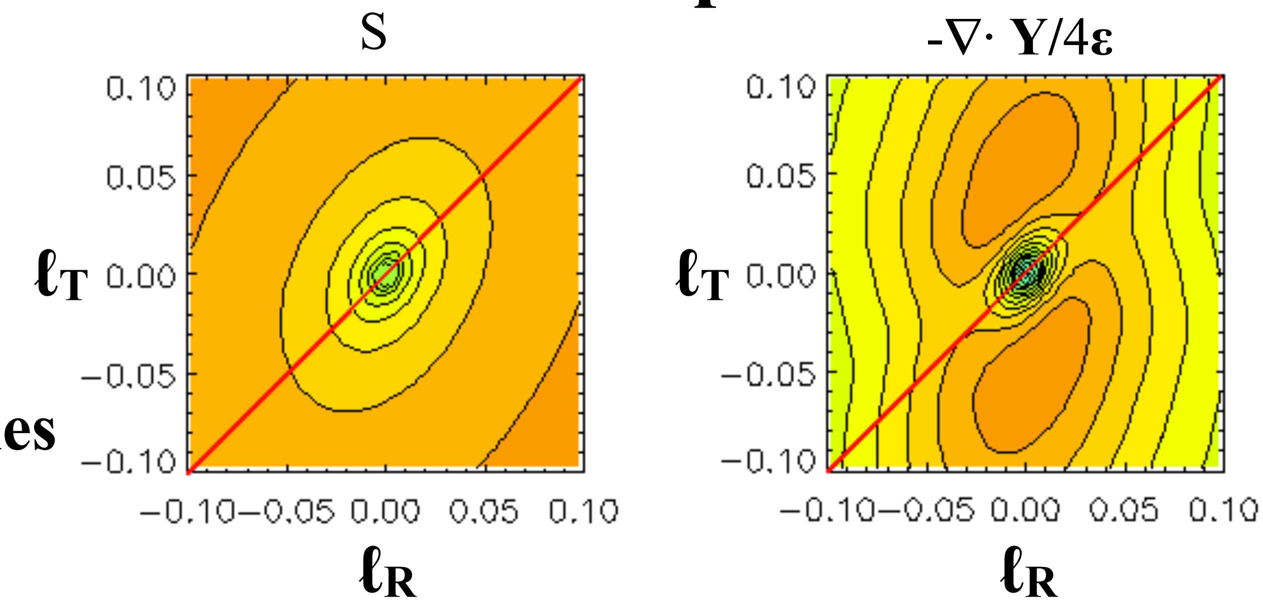


S and ∇·Y have
different anisotropies

No Expansion

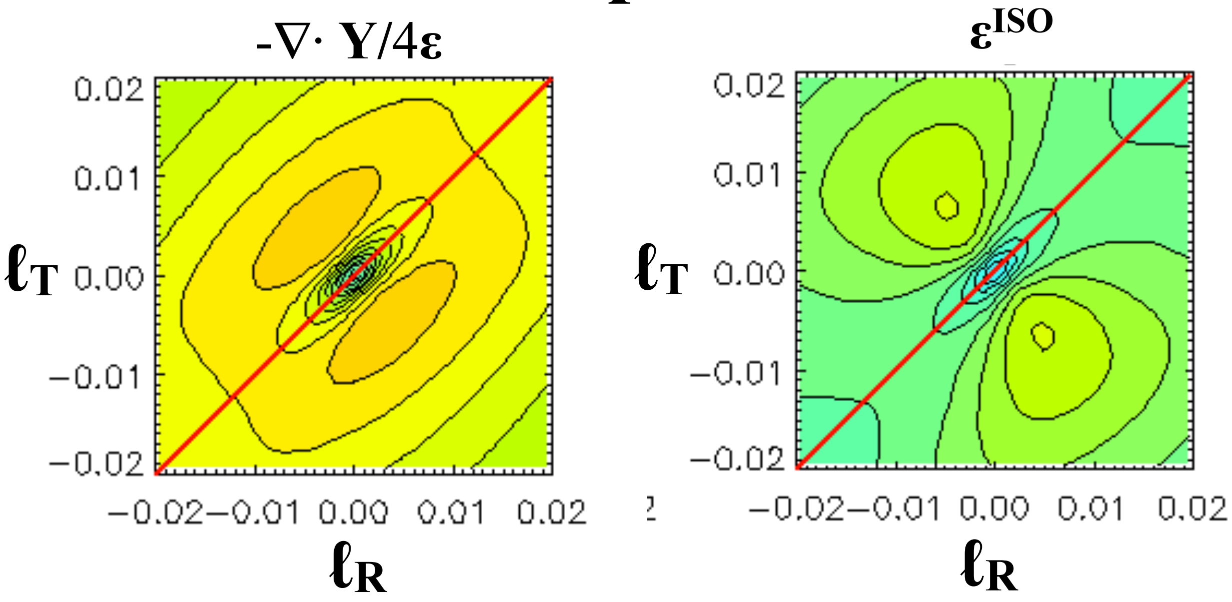


With Expansion

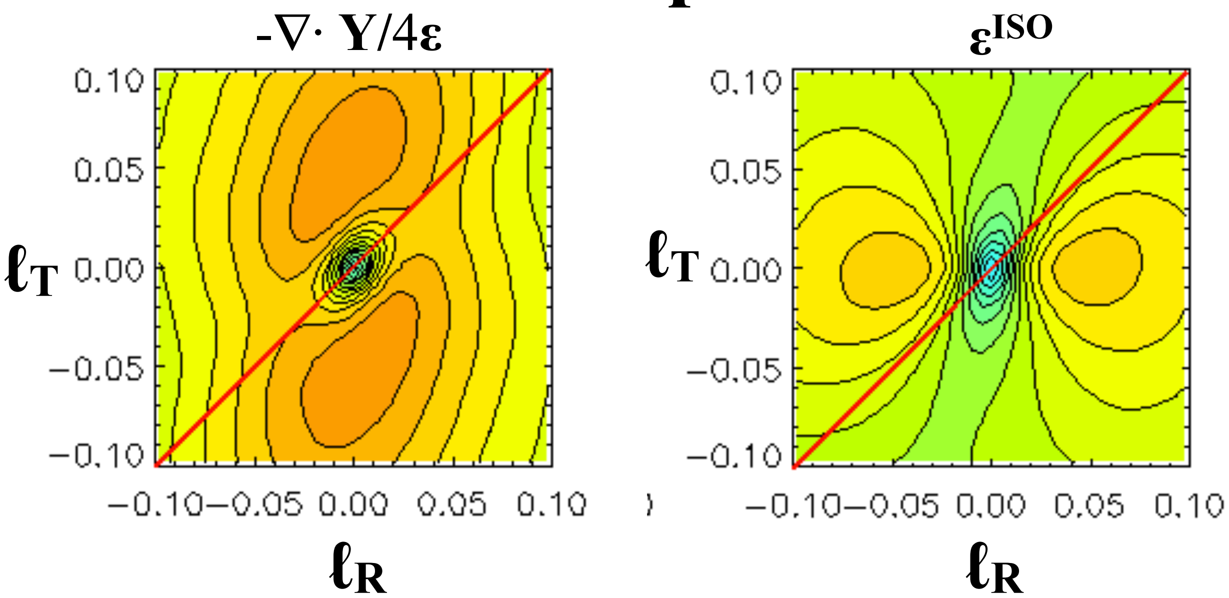


III-order SF & Isotropic model

No Expansion



With Expansion



$$\epsilon^{ISO} = -\frac{3}{4}\frac{\mathbf{Y}\cdot\hat{\boldsymbol{\ell}}}{\ell}$$

The isotropic prescription can return a false impression of where the cascade is at work

Conclusions

The solar wind turbulence has **3D anisotropies**

Expansion affects at large and small scales

- spectral anisotropy, B_0
- component anisotropy, $B_{T,N}>B_R$
- local anisotropy, B_ℓ

Dong, Verdini, Grappin ApJ 2014

Verdini & Grappin ApJL 2015 submitted

The **cascade (III-order SF)** anisotropy is less constrained:

- MHD with B_0 : varying anisotropy (perp>par)
- MHD with expansion: reduced anisotropy (perp~par)

Measurements depend on the sampling direction

- local anisotropy Verdini et al. ApJ 2015
- cascade rate Verdini et al. in prep