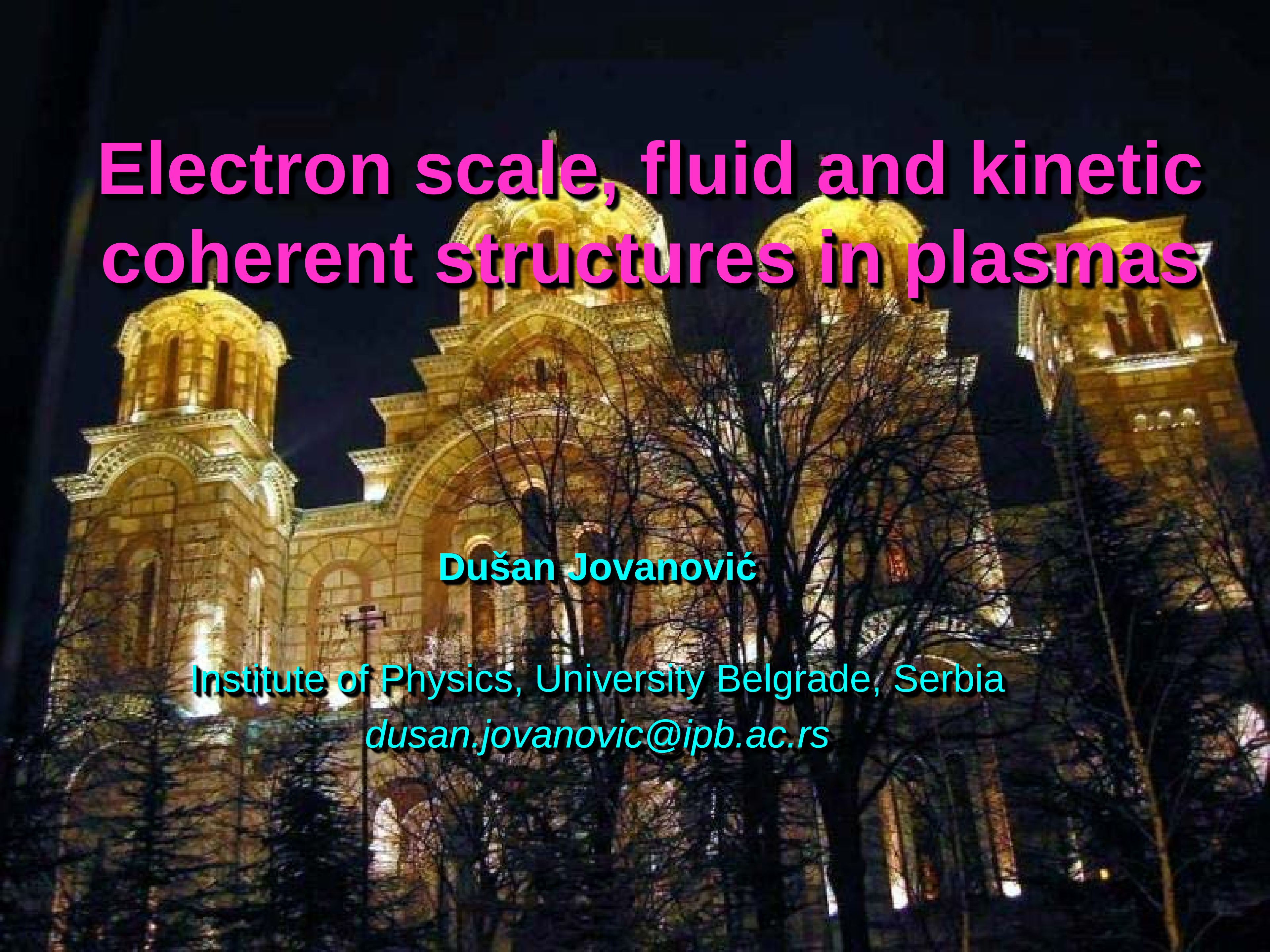

Electron scale, fluid and kinetic coherent structures in plasmas

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Introduction

- Solar wind & magnetosheath downstream of a quasi-perpendicular bow shock
- Instabilities (linear Vlasov theory, homogeneous plasma $T_{\perp} > T_{\parallel}$).
- For $\beta = 2p/c^2\varepsilon_0 B^2 \leq 1$ - instability of Alfven ion cyclotron waves
- For $\beta > 5$ - mirror mode instability
- Energy transfer to time-growing waves reduces anisotropy $T_{\perp}/T_{\parallel}$
- Turbulent spectrum established by energy distribution among unstable and other linear modes, as well as with plasma particles, by wave-wave and wave-particle interactions.
- Creation of (meta)stable coherent structures in the plasma, which may scatter and/or trap (turbulent) radiation - affects the spectrum
- Spectral shapes over broad range established (Alexandrova *et al*)
- Two distinct power laws separated by a "knee" observed, somewhat different from typical Kolmogorov turbulence of 3-D incompressible

- Kolmogorov spectrum emerges on the intermediate spatial scale (inertial range), if the energy injection scale is sufficiently far from the dissipation scale. Spectrum is independent on both the energy injection and the energy dissipation mechanisms.
- At the edges of the inertial range, turbulent spectrum influenced by intermittency. The latter results from the presence of coherent structures, in the form of filaments of vorticity.

- The observed two-power-laws' spectrum of magnetic fluctuations in the magnetosheath features a "knee", close to Ω_i
- Above Ω_i the spectrum has a steeper power law k^{-s} with $2 < s < 4$
- Spectral steepening due to dissipation?
- Or another turbulent cascade? (in compressible Hall MHD model)
- In this range, intermittency increases toward small scales, similarly to what happens in the Kolmogorov's inertial range.
- The observed intermittency was related with coherent structures.

- Alfven current filaments were documented in the magnetosheath, downstream of quasi-perpendicular bow shock
- They were interpreted as a sort of torsional Alfv´en vortices.
- When the ratio of the plasma thermodynamic and magnetic pressures is not very large, $\beta < 1$, Alfvenic vortices larger than the ion length are described by the standard Strauss's equations of reduced MHD (magnetohydrodynamic)
- It was proposed (O. Alexandrova *et al*) that these were created in the filamentation of nonlinear slab-like structures created in the saturation of unstable Alfven ion cyclotron waves.
- The location of the dissipation range of the solar wind and the magnetosheath turbulence still remains elusive.
- The particle collisions can not provide the dissipations - the mean free path comparable with the Sun-Earth distance.
- In contrast to neutral fluids, in magnetized plasma there is a number of characteristic (microscopic) space and temporal scales, related with the electron dynamics.

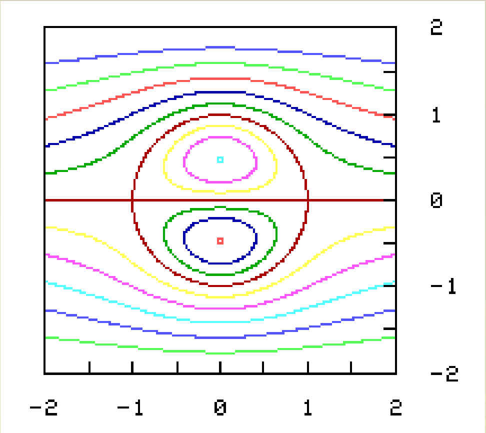
- The question is: what happens to turbulent magnetic fluctuations observed at spatial scales around the ion inertial length c/ω_{pi} and at time scales around the ion cyclotron frequency Ω_i, when their energy has cascaded towards electron scales?
- The satellite observations on the electron scales (electron inertia length d_e and the Debay length λ_{De}) crucial to answer this question.
- We are not aware of any observations of the solar wind and magnetosheath turbulent spectra on such small scales.
- No direct observational evidence, either, for the dissipation of the turbulent fluctuations; the association of the high frequency range with the dissipation range remains just a hypothesis.

Vortices

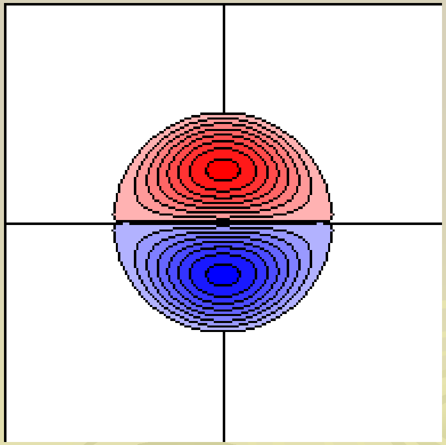
- Vortices are a major component of turbulent flow, most notably of incompressible flows.
- In the absence of external forces, viscous friction and/or turbulent dissipation within the fluid tends to organize the flow into a collection of vortices, possibly superimposed to larger-scale flows, including larger-scale vortices.
- Dual cascade in turbulence
- Energy flows toward large wavenumbers, enstrophy toward small – tendency towards self-organization?

Lamb dipole

- dipolar vortex, continuous vorticity distribution in a circular region
- moves along a straight line, constant velocity, no change of form.
- stationary solution of 2-d vorticity equation (Navier-Stokes equation) when there are no viscous effects and the domain is infinite.
- When viscous effects play a role, the vorticity is spread over a larger area and the dipole decreases in strength, so that the velocity also decreases in the course of time; the motion remains along a straight line though. In case the domain is finite, the velocity of the dipole is less than in an infinite domain.



Streamlines in the moving frame



vorticity

More about vortices

- Rotating shallow fluid (Rossby), Charney-Hasegawa-Mima equation
- Drift waves in plasma, CHM equation
- Shear-Alfven (low β plasma) two coupled equations
- Reduced MHD (Strauss' equation)
- Electron-MHD
- Flute mode, blobs in tokamak edge, ….
- Interact with high-frequency waves (upper hybrid, lower hybrid, ..)
- Interact with particles (Landau damping, particle trapping)
- For all the vortex equations, in the stationary case it is possible to find a sufficient number of integrals of motion.

Self-organization of Lamb vortices

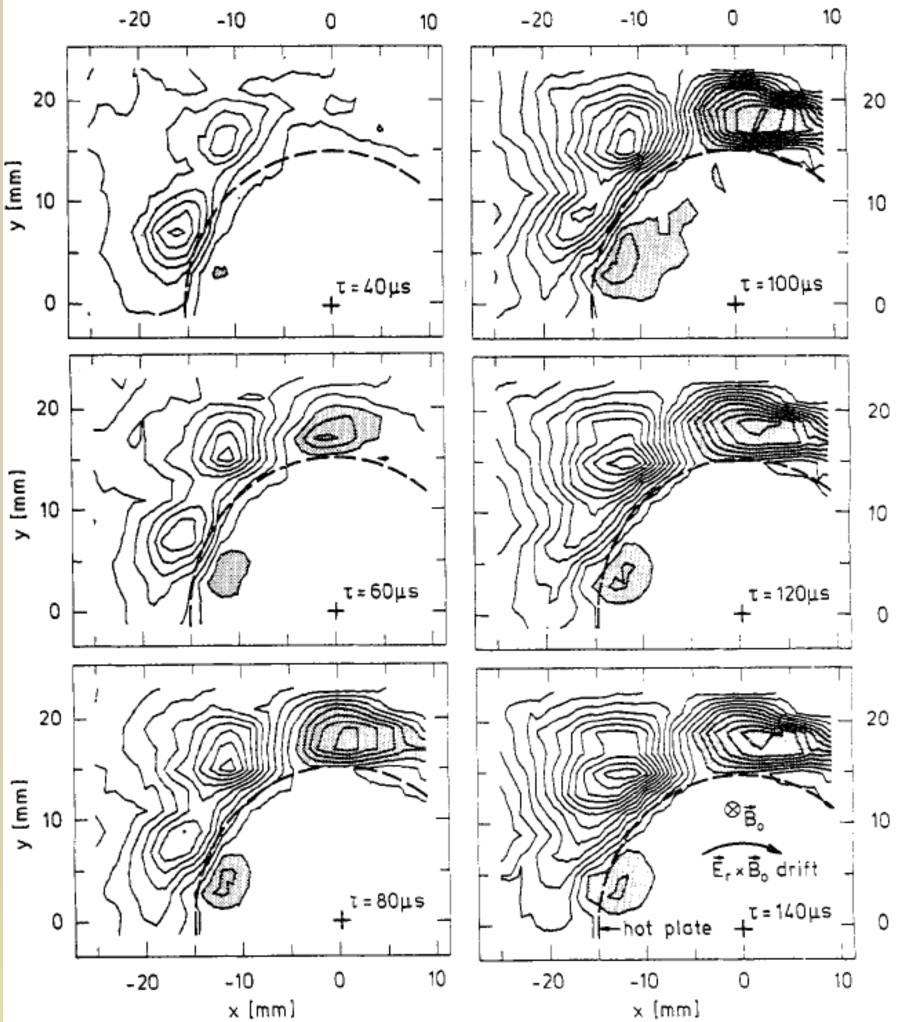
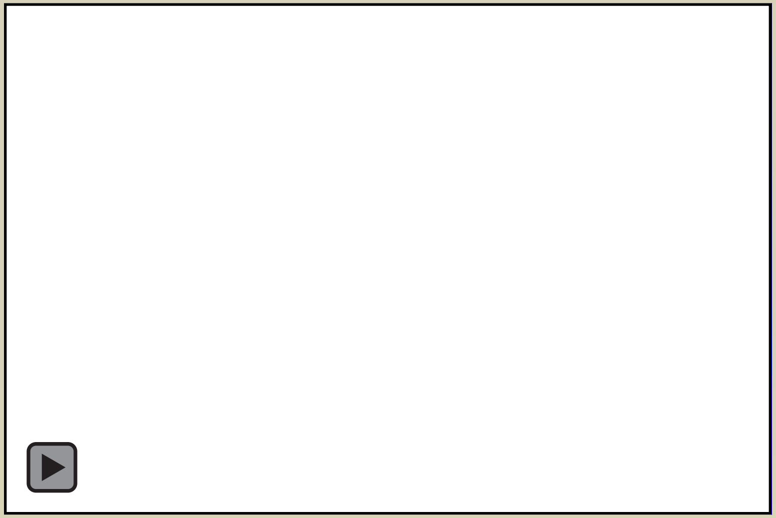


FIG. 3.—Temporal evolution of two interacting cells measured at different times τ after the turn-on of the exciters. The pulse durations were 15 μs, amplitudes 1 V. The potential difference between two adjacent contours is 4 mV. The first positive contour is at 2 mV. Negative potential regions are denoted by shading. The dashed circle shows the projection of the hot plate.

H. L. PECSELI, J. JUUL RASMUSSEN and K. THOMSEN
Plasma Physics and Controlled Fusion, 27, 837-846 (1985)

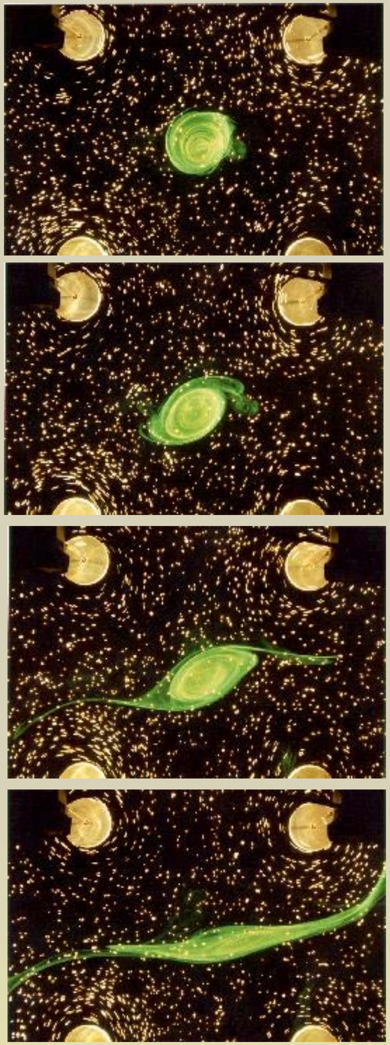
Merging of two externally injected monopoles and the emergence of a third vortex with opposite polarity

Does the nature prefer monopoles or dipoles?
Lamb Vortex creation by a jet flow

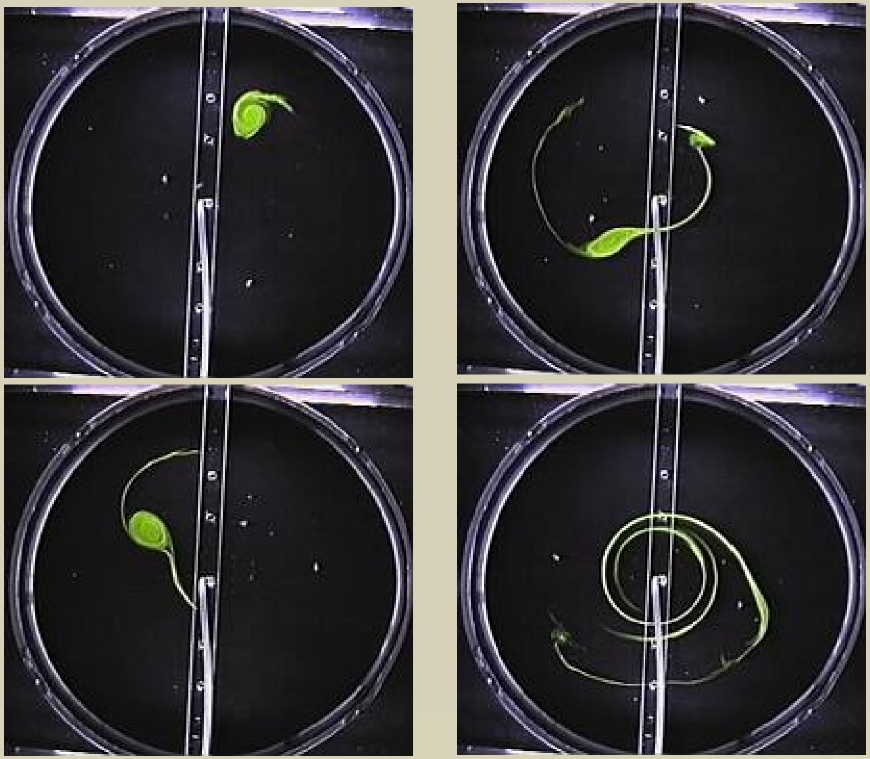


http://bugman123.com/FluidMotion/LeapFrog2.m1v

Stability of Rossby monopolar vortices



vortex in a strain flow

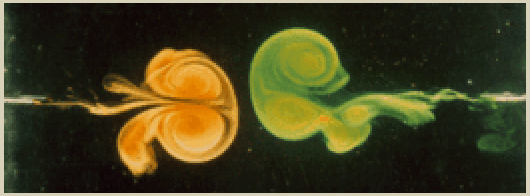
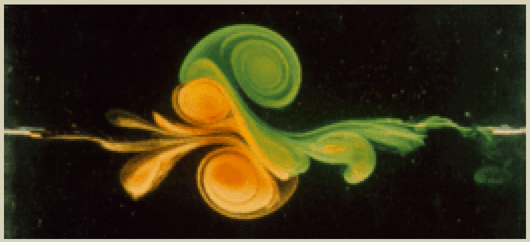
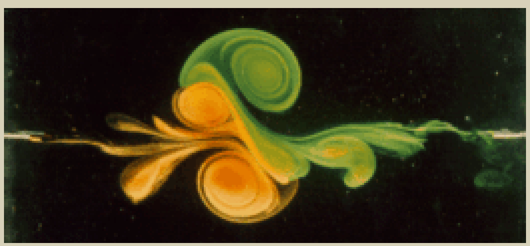


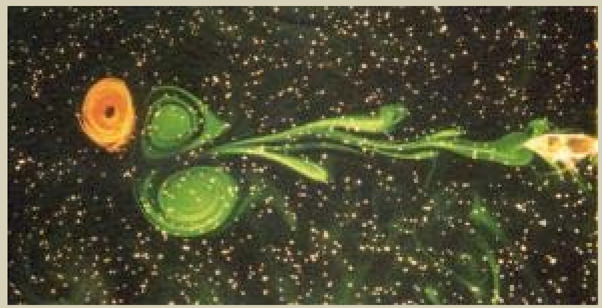
vortex in an irrotational annular shear flow

Gert Jan van Heijst, Ruben Trieling and Theo Schep,
Eindhowen University of Technology

http://web.phys.tue.nl/nl/de_faculteit/capaciteitsgroepen/transportfysica/fluid_dynamics_lab/research/vortex_dynamics/

Stability of Rossby dipolar vortices

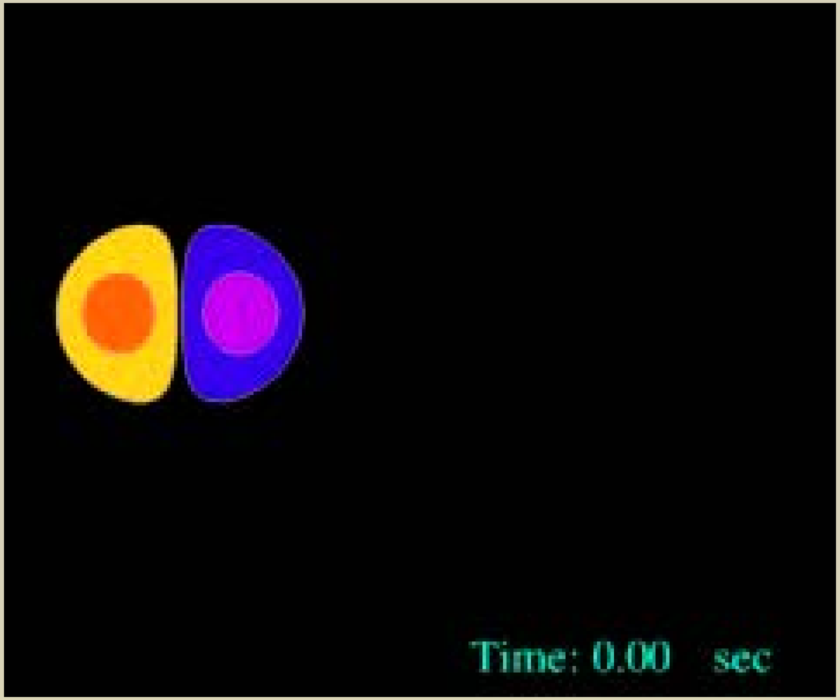




GertJan van Heijst, Ruben Trieling and Theo Schep,Eindhowen University of Technology

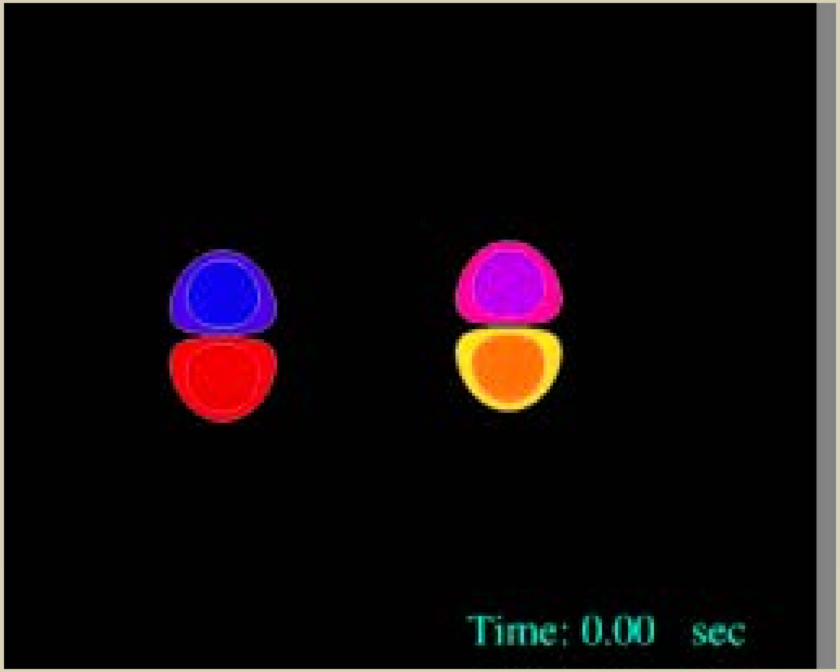
http://web.phys.tue.nl/nl/de_faculteit/capaciteitsgroepen/transportfysica/fluid_dynamics_lab/research/vortex_dynamics/

Rossby dipole + KdV scalar nonlinearity

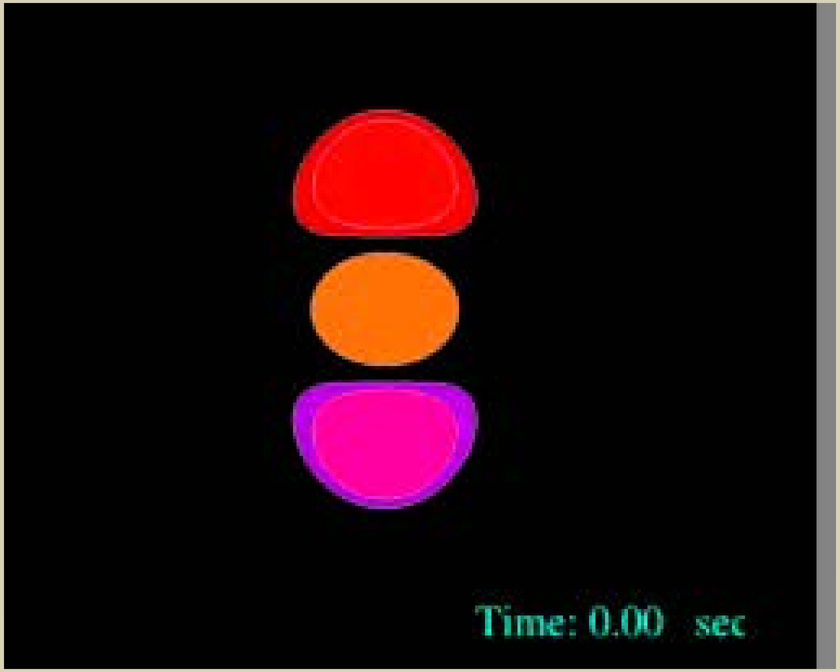
GertJan van Heijst, Ruben Trieling and Theo Schep,
Eindhowen University of Technology



Dipoles survive collisions (Rossby)



Tripolar vortices (rotating or imbedded in shear flow)



Electron-scale vortices

- Shear-Alfven dipolar vortices: MHD temporal scale, $d/dt \ll \Omega_i$, electron inertial spatial scale, $(c/\omega_{p,e})\nabla\perp \sim 1$. (Jovanovic et al 1984)
- A pair of weakly charged filaments carrying counterstreaming currents and slightly tilted relative to the background magnetic field
- Localized torsion of the magnetic field and $\vec{E} \times \vec{B}$ plasma drift flow
- Exist only in low- β plasmas, $\beta \ll m_e/m_i$.
- Lots of work done, in the context of fast colisionless reconnection.
- In a large-β plasma of the solar wind and magnetosheath, the nonlinear self organization and the resulting intermittency on the electron scale comes mostly from the short-wavelength nonlinear processes with a higher-than-MHD-frequency.
- It is necessary to study the self-organization in the form of vortices on a faster temporal scale $\Omega_e \gg \frac{d}{dt} \gg \Omega_i$

Electron magneto-hydrodynamic, EMHD

- Regime with immobile ions and drifting electrons:

$$\max(\Omega_i, \omega_{pi}) \ll \frac{d}{dt} \ll \min(\Omega_e \omega_{pe}), \qquad \nabla_\perp \gg \omega_{pi}/c$$

- In the original derivation of EMHD equations, electrons were assumed to have a zero temperature and the electron fluid was regarded as essentially incompressible.
- This permitted a simple mathematical description, obliquely propagating whistlers are the only linear eigenmode.
- Whistler waves and the associated electron dynamics critical to many aspects of astrophysical plasmas, such as the structure of collisionless quasi-perpendicular shocks, to our understanding of magnetic reconnection, dissipation, and heating processes in the solar wind, of the anomalous diffusion of field in plasma, and others.

Mathematical model for large $\beta \geq 1$

- Electron continuity + momentum eq. + eq. of state + Ampere's law
- Larmor radius terms via Braginski stress tensor
- Anisotropic electron temperature

$$\frac{1}{\Omega_e}\frac{\partial}{\partial t} \sim \frac{1}{\Omega_e}\vec{U}_e\cdot\nabla \sim \frac{\delta n_e}{n_e} \sim \frac{|\delta\vec{B}|}{|\vec{B}|} \sim \frac{\vec{b}\cdot\nabla}{\nabla_\perp} \sim \epsilon \ll 1$$

$$\left[\frac{\partial}{\partial t} + \left(\vec{e}_z\times\nabla_\perp d_e^2 B\right)\cdot\nabla\right](N-B) + \left[\frac{\partial}{\partial z} - \left(\vec{e}_z\times\nabla_\perp A_z\right)\cdot\nabla_\perp\right]\left(1-\frac{\beta_{e_\parallel}-\beta_{e_\perp}}{2}\right)\frac{\Omega_e^2}{\omega_{pe}^2}\frac{c^2k_{e_\parallel}^2}{\omega^2}\nabla_\perp^2 A_z +$$
$$\nabla_\perp\cdot\left[\left(\frac{\partial}{\partial t} + \left\{\vec{e}_z\times\nabla_\perp d_e^2\left[B+\frac{\beta_{e_\perp}}{2}(N-B)\right]\right\}\cdot\nabla_\perp\right)\nabla_\perp\, d_e^2 B\right] = 0,$$

$$\left(\frac{\partial}{\partial t} + \left\{\vec{e}_z\times\nabla_\perp d_e^2\left[B+\frac{\beta_{e_\perp}}{2}(N-B)\right]\right\}\cdot\nabla_\perp\right) d_e^2\nabla_\perp^2 A_z =$$
$$\frac{\partial A_z}{\partial t} + d_e^2\left[\frac{\partial}{\partial z} - \left(\vec{e}_z\times\nabla_\perp A_z\right)\cdot\nabla_\perp\right]\left[\left(1+\frac{\rho_{Le}^2\nabla_\perp^2}{2}\right)B - \left(\gamma_{e_\parallel}-1\right)N + \frac{\beta_{e_\parallel}+\beta_{e_\perp}}{2}(N-B)\right]$$

$$\left(1-\lambda'^2_{De}\nabla'^2_\perp\right)N' = \frac{2}{\beta_{e_\perp}}\left(1-\rho'^2_{Le}\nabla'^2_\perp\right)\lambda'^2_{De}\nabla'^2_\perp B'$$

Cold plasma limit, β<<1

$$\left[\frac{\partial}{\partial t}+(\mathbf{e}_z\times\nabla_\perp B_z)\cdot\nabla_\perp\right](1-\nabla_\perp^2)B_z-(\mathbf{e}_z\times\nabla_\perp A_z)\cdot\nabla(1-\nabla_\perp^2)A_z=0$$

$$\left[\frac{\partial}{\partial t}+(\mathbf{e}_z\times\nabla_\perp B_z)\cdot\nabla_\perp\right](1-\nabla_\perp^2)A_z=f(t)$$

- Moving 2-d solution, tilted to z-axes, $\partial/\partial t = -u_y\partial/\partial y = -u_z\frac{\partial}{\partial z}$

$$(1-\nabla_\perp^2)A_z=\mathcal{F}(B_z-ux),$$

$$(1-\nabla_\perp^2)B_z+A_z\frac{d\mathcal{F}(B_z-ux)}{d(B_z-ux)}=\mathcal{G}(B_z-ux)$$

$\mathcal{F}$ and $\mathcal{G}$ arbitrary functions.

Ansatz than produces dipoles (better than Larichev-Reznik's):

$\mathcal{F}$ and $\mathcal{G}$ part-by-part linear → equation that separates variables in cylindrical variables:

$$(\nabla_\perp^2+k_1^2)\left(\nabla_\perp^2+k_2^2\right)(B_z+v\,x)=0$$

k_1 , k_2 , and v have different values inside and outside vortex core

Dipole, tripole, vortex chain in cold EMHD

This may be the saturated state of magnetic reconnection !!

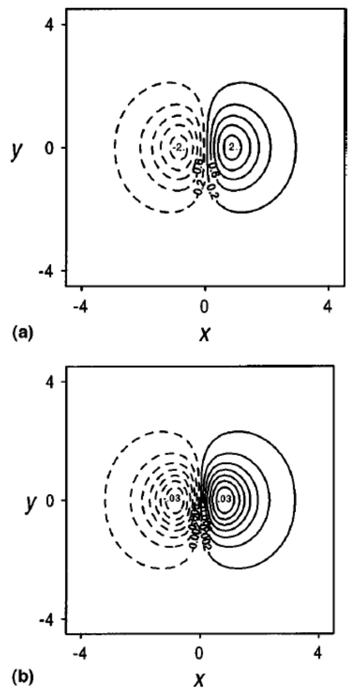


FIG. 1. Dipolar vortex. Contour plots of the parallel magnetic field perturbation δB_z (a) and the z-component of the vector potential A_z (b) are shown. Vortex parameters are $r_0=1.5$, $u=1$, $\kappa_2^2=-0.998$ $(\Rightarrow \kappa_1=2.703\ 59)$ and the scale length of the magnetic field is $L_z=10$.

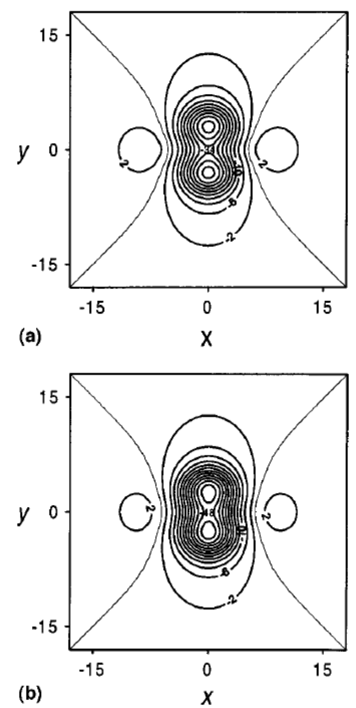


FIG. 3. Tripole. Perturbations of the parallel magnetic field δB_z (a) and the z-component of the vector potential δA_z (b) are plotted for the plasma parameters $L_y=-1$, $L_y=0.8$. The core radius is adopted to be $r_0=6$.

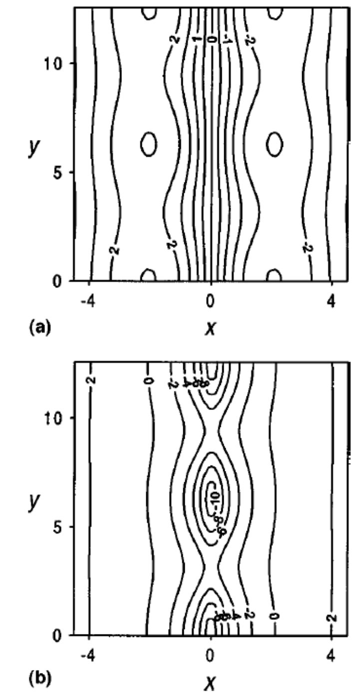


FIG. 2. Vortex chain. Perturbations of the parallel magnetic field $\delta B'$ (a) and of the z-component of the vector potential $\delta A'$ (b), defined in Eq. (24), are plotted. The chain wavelength is adopted to be 2π and the plasma parameter is $a=7$.

D. Jovanovic & F. Pegoraro, Phys. Plasmas, **7**, 889 (2000)

Dipoles in warm EMHD, β≥1

- Moving 2-d solution, tilted to z-axes, $\partial/\partial t = -u_y\,\partial/\partial y = -u_z\,\partial/\partial z$
- combine into a single equation (complex)

$$[\vec{e}_z \times \nabla_\perp (A_z - x)] \cdot \nabla_\perp \left\{ d_e^2 \nabla_\perp^2 A_z - iR \left[d_e^2 \left(\frac{\rho_{Le}^2}{2} \nabla_\perp^2 + 1 \right) B - x \right] \right\}$$
$$= \left\{ \vec{e}_z \times \nabla_\perp iR \left[d_e^2 \left(1 - \frac{\beta_{e_\perp}}{2} \right) B - x \right] \right\} \cdot \nabla_\perp \left\{ d_e^2 \nabla_\perp^2 A_z - iR \left[d_e^2 \left(\frac{\rho_{Le}^2}{2} \nabla_\perp^2 + 1 - \frac{\beta_{e_\perp}}{2} \right) B - x \right] \right\},$$

$$R = \pm \frac{2\omega}{k_{e_\parallel} v_{Te_\perp}} \left(2 - \beta_{e_\parallel} + \beta_{e_\perp} \right)^{-\frac{1}{2}}$$

- Secular terms give the linear dispersion relation $\omega = ck_y\,\Omega_e/\omega_{pe}$ where

$$\mathcal{D}(k_y) = k_y^2 \left[k_y^2 + \frac{R^2 \rho_{Le}^2 - 2d_e^2}{\rho_{Le}^2 d_e^2 (1 + R^2)} \right] = 0$$

- In the presence of temperature and density gradients, dispersion relation for whistler and electron gradient waves is modified to
- $\omega/\Omega_e = (ck_y/2\omega_{pe})(1 \pm \sqrt{1 - 4\beta\ L_n/L_p}\,)$

  which yields the ETG linear instability.

- For β≥1 cannot be integrated to obtain nonlinear characteristics.
- Seek a particular solution → introduce the following ansatz:

$$d_e^2 B - x - \mathcal{C}\left[d_e^2\left(1-\frac{\beta_{e\perp}}{2}\right)B - x\right] = \mathcal{F}\left(A_z - x\right)$$

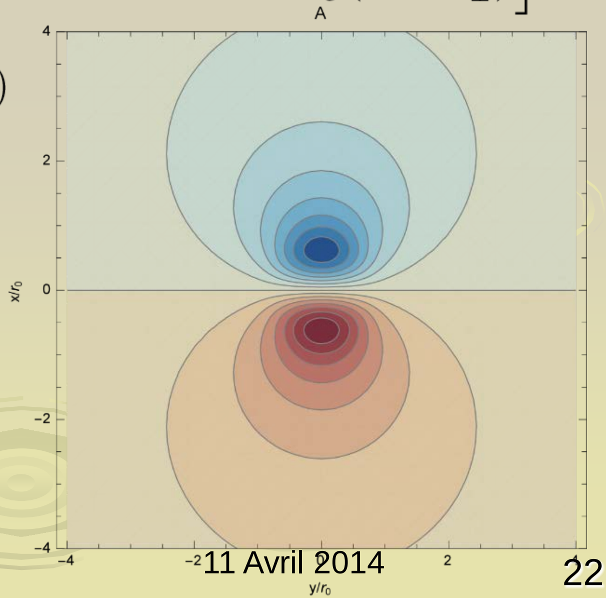
- now the nonlinear equation becomes integrable, yielding

$$d_e^2\nabla_\perp^2 A_z - iR\left\{d_e^2\,\frac{\rho_{Le}^2}{2}\,\nabla_\perp^2 B + \mathcal{C}\left[d_e^2\left(1-\frac{\beta_{e\perp}}{2}\right)B - x\right]\right\} = \mathcal{G}\left\{A_z - x - iR\left[d_e^2\left(1-\frac{\beta_{e\perp}}{2}\right)B - x\right]\right\},$$

- we obtain a standard dipole (strongly localized!)

$$A_z^{in}\left(r,\varphi\right) = \cos\varphi\;\left[\alpha^{in}J_1\left(k_1 r\right) - \left(a^{in}-1\right)r\right],\quad B^{in}\left(r,\varphi\right) = \cos\varphi\left[Q^{in}\alpha^{in}J_1\left(k_1 r\right) - \frac{2\left(b^{in}-1\right)r}{d_e^2\left(2-\beta_{e\perp}\right)}\right]$$

$$A_z^{out}\left(r,\varphi\right) = \cos\varphi\;\alpha^{out}K_1\left(\kappa r\right),\quad B^{out}\left(r,\varphi\right) = \cos\varphi\;Q^{out}\alpha^{out}K_1\left(\kappa r\right)$$



- Due to the use of the ansatz, this is not a general solution. It is obtained only for

$$\frac{\omega}{k_{e_\parallel}} = \frac{c\,|\Omega_e|}{2\,\omega_{pe}}\left[(2-\beta_{e_\perp})\left(2-\beta_{e_\parallel}+\beta_{e_\perp}\right)\right]^{\frac{1}{2}} \qquad \kappa^2 = \frac{R^2\rho_{Le}^2 - 2d_e^2}{\rho_{Le}^2 d_e^2\,(1+R^2)} \quad \Rightarrow \quad Q^{out} = \frac{2}{\rho_{Le}^2 R^2}$$

- The nonlinear dispersion relation of the dipole mode

$$a^{in} k_1 r_0 \frac{J_1'(k_1 r_0)}{J_1(k_1 r_0)} - a^{in} + 1 = \kappa r_0 \frac{K_1'(\kappa r_0)}{K_1(\kappa r_0)}$$

$$a^{in} = \frac{1}{k_1^2\, d_e^2}\,\frac{\beta_{e_\perp}}{2-\beta_{e_\perp}}\,\frac{G_1^{in}}{F^{in}}$$

$$Q^{out} = \frac{2}{d_e^2\,(2-\beta_{e_\perp})}.$$

- The dipole exists if: $\beta_{e\parallel}/\beta_{e\perp} < (1-\beta_{e\perp}/2) < 1$, for $\beta_{e\perp} < 2$ or
  $\beta_{e\parallel}/\beta_{e\perp} < (d_e^2/\varrho_{Le}^2)(2-\beta_{e\perp})$ for $\beta_{e\perp} > 2$.
- It corresponds to the second mode in the dispersion relation, with $k_y \neq 0$, which is not the whistler mode.
- Conversely, in cold plasma, the dipoles were constructed for the $k_y = 0$, which is the oblique whistler mode.

Gyrokinetic structures in EMHD plasmas

- Magnetized electron holes in collisionless plasmas, analyzed in connection with the satellite measurements

  Ergun et al, GRL **25** 2041 (1998); Mozer et al, Phys. Rev. Lett. **79** 1281 (1997).
- In laboratory plasmas, multi-dimensional PIC and Vlasov numerical simulations of the interaction of ultra-short ultra-intense laser pulse with underdense plasmas have shown the existence of coherent magnetic vortices and relativistic solitary waves

  Bulanov et al Phys. Rev. Lett. (1996), (1997), (2001).
- 2-D PIC simulations showed that a bipolar magnetic field perpendicular to the simulation plane is created in the wake of the relativistically intense ultra short laser pulse.

Gyrokinetic description

- In a strictly 2-d regime $\frac{\partial}{\partial z} = \frac{\partial}{\partial v_z} = 0$, and using small parameters $(\frac{e\phi}{T})^2 \sim \left(\frac{v\nabla}{\Omega}\right)^2 \sim \log\frac{f}{f_M} \sim \log\frac{B}{B_0} \sim (\frac{d}{\Omega_e dt})^{2/3} \sim \varepsilon$
- Integrating the Vlasov equation for the gyroangle, we obtain the following gyrokinetic equation

$$\frac{d}{dt} f_0 + \frac{v}{2}\frac{\partial f_0}{\partial v}\frac{d}{dt}\left(\log\frac{B}{B_0} + \frac{\nabla^2\phi}{B\Omega_e}\right) = 0$$

- In the stationary case, we have the following conservation laws

$$W = \frac{m_e v^2}{2T} - \frac{e\bar{\phi}}{T} = \text{constant}, \qquad \mu = \frac{m_e v^2}{2euLB} = \text{constant}$$

Gyrokinetic cont'd

- The curl of the electron momentum equation + the electron continuity:

$$\left[\frac{1}{d_e^2 \Omega_{e,0}}\frac{\partial}{\partial t} + \left(\vec{e}_z \times \nabla \frac{B}{B_0}\right)\cdot\nabla\right]\left[\left(1 - d_e^2\nabla^2\right)\frac{B}{B_0} - \frac{n}{n_0}\right] = \frac{1}{d_e^2 \Omega_{e,0}^2}\vec{e}_z \cdot \left(\nabla \frac{n}{m_e n_0^2} \times \nabla p\right) \to 0$$

- Integrate in the stationary/travelling case:

$$d_e^2 \nabla^2 \frac{B}{B_0} + \frac{n}{n_0} - \frac{ux}{d_e^2 \Omega_{e,0}} = 1 + \mathcal{F}\left(\frac{B}{B_0} - \frac{ux}{d_e^2 \Omega_{e,0}}\right), \quad (14)$$

- where the electron density is: $\delta n = n - n_0$

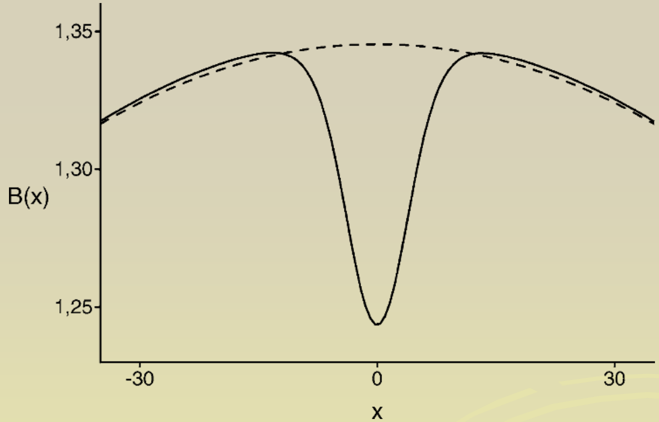
$$\frac{\delta n}{n_0} = -\frac{v_T^2 \nabla^2}{\Omega_{e,0}^2}\frac{e\phi}{T} + Y(lb_0)\left[\delta\varphi - l\delta b - a(\delta\varphi - l\delta b)^{\frac{3}{2}}\right]$$

- The last term is the density of trapped particles. $Y(l\, b_0)$ is the incomplete gamma function.

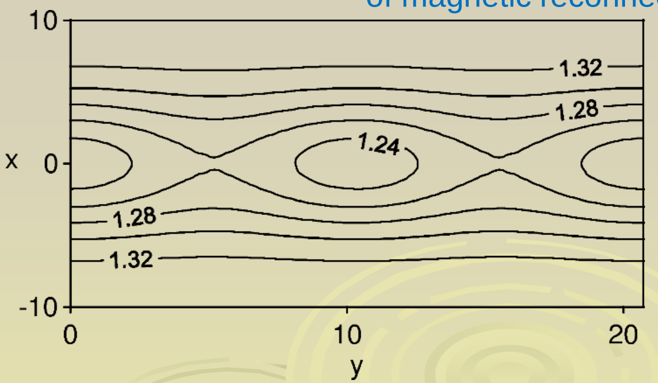
- Linear dispersion relation in the presence of a zero-order flow $v_0(x) = (c^2\epsilon_0/n_0 e)(d/dx)\, B_{z0}(x)$

$$\left[\nabla^2 - \kappa^2(x)\right]\nabla^2 \delta B = 0, \quad \kappa^2(x) \equiv \frac{d_e^2 v_0''(x) - u}{d_e^2[v_0(x) - u]} \tag{17}$$

- Assume a parabolic profile of $B_{z0}(x)$ ⇒ Kelvin-Helmholtz instability
- In the nonlinear slab case, large amplitude stationary magnetic field perturbation creates a "waveguide" inside which a new "(quasi)linear mode" can propagate, that has the form of a vortex chain.



This is NOT the saturated state of magnetic reconnection !!



D. Jovanovic, F. Califano, F. Pegoraro, Physics Letters A 303 (2002) 52–6

Conclusions

- We have derived nonlinear model equations that describe the plasma behavior and its turbulence at small length scales and relatively short time scales, i.e. on the electron spatial scale and with the characteristic frequency below the electron gyrofrequency.
- The corresponding linear mode is the obliquely propagating whistler.
- In the quasiperpendicular shock, the turbulence of the finite- β plasma is enhanced due to temperature anisotropy.
- While it has been known that the energy is cascading towards theelectron scales, the properties of the high- and moderate- β plasma turbulence at the electron scale and the physical nature of the energy sink at such scales have not yet been fully clarified.
- We have constructed a coherent, nonlinear dipole vortex, associated with obliquely propagating whistlers in the moderate β ($\beta \leq 1$) plasma of the magnetosheath, with an anisotropic temperature
- Electron trapping by mirror force provides a new mechanism for the self-organization in EMHD. Solutions known in laser-plasmas.
- Further work needed for interplanetary plasma. 3-d theory?

Thank you for your attention!!