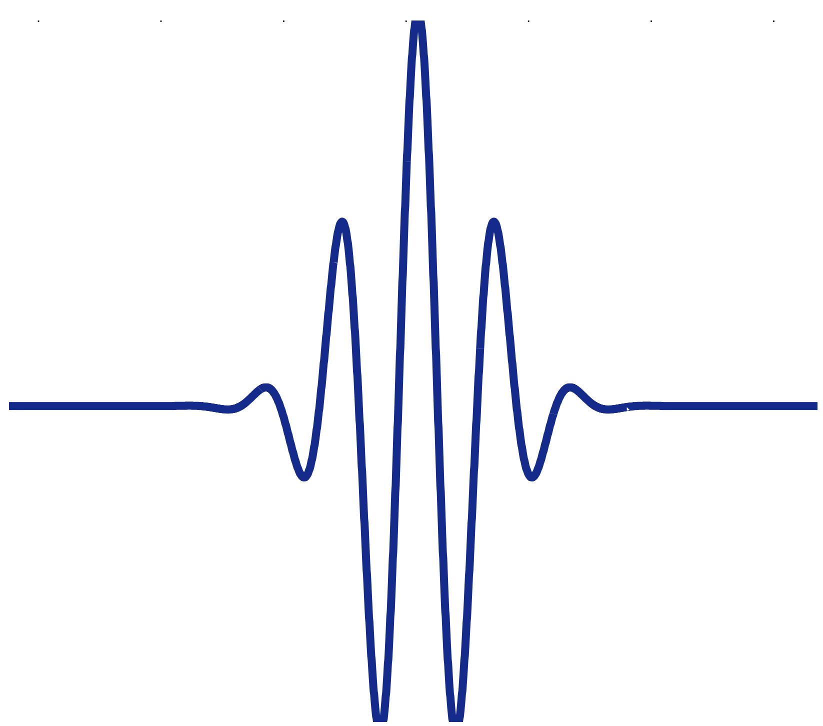
Using discrete wavelets to explore anisotropy in solar wind plasma turbulence

Khurom H. Kiyani



LPP

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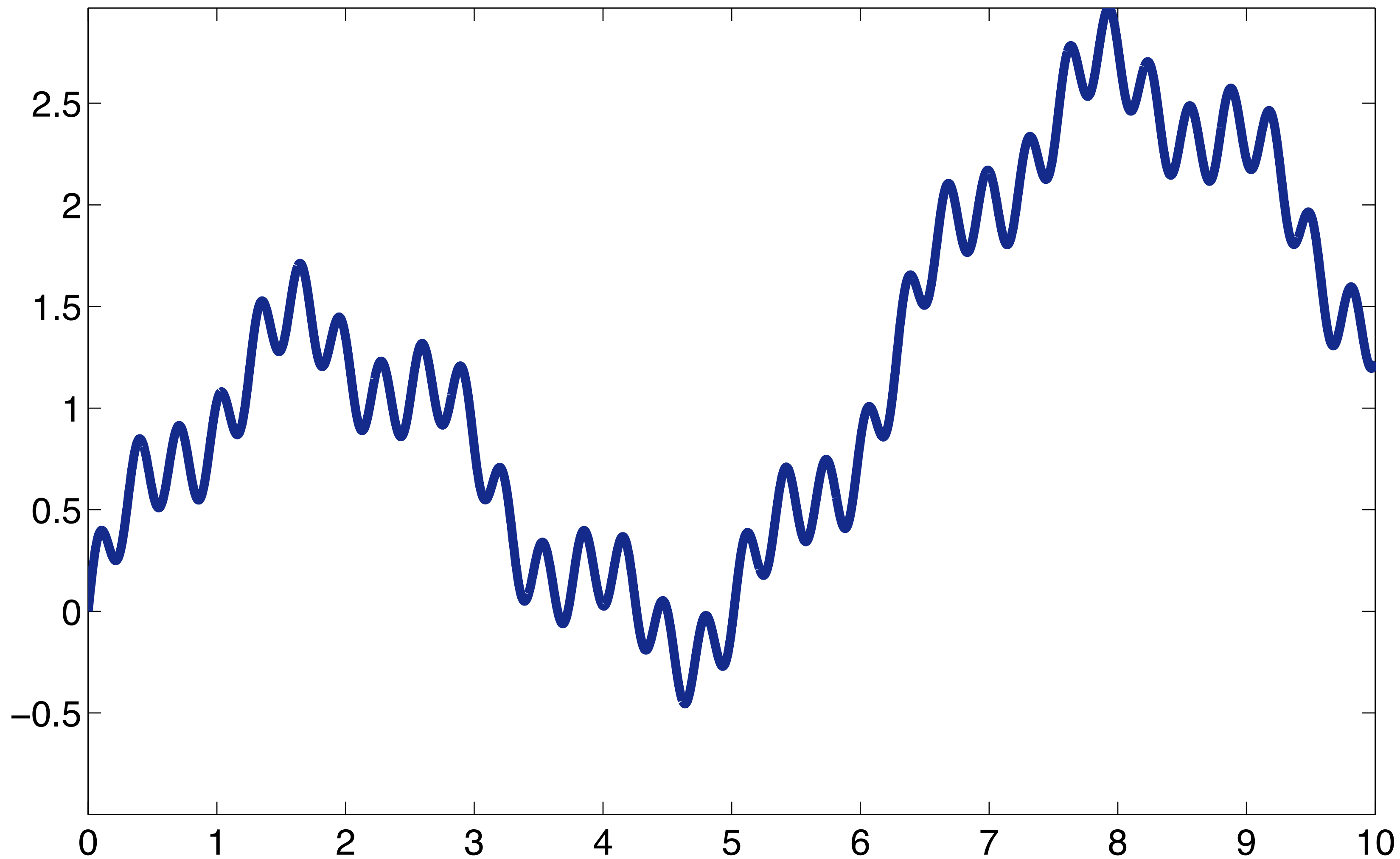
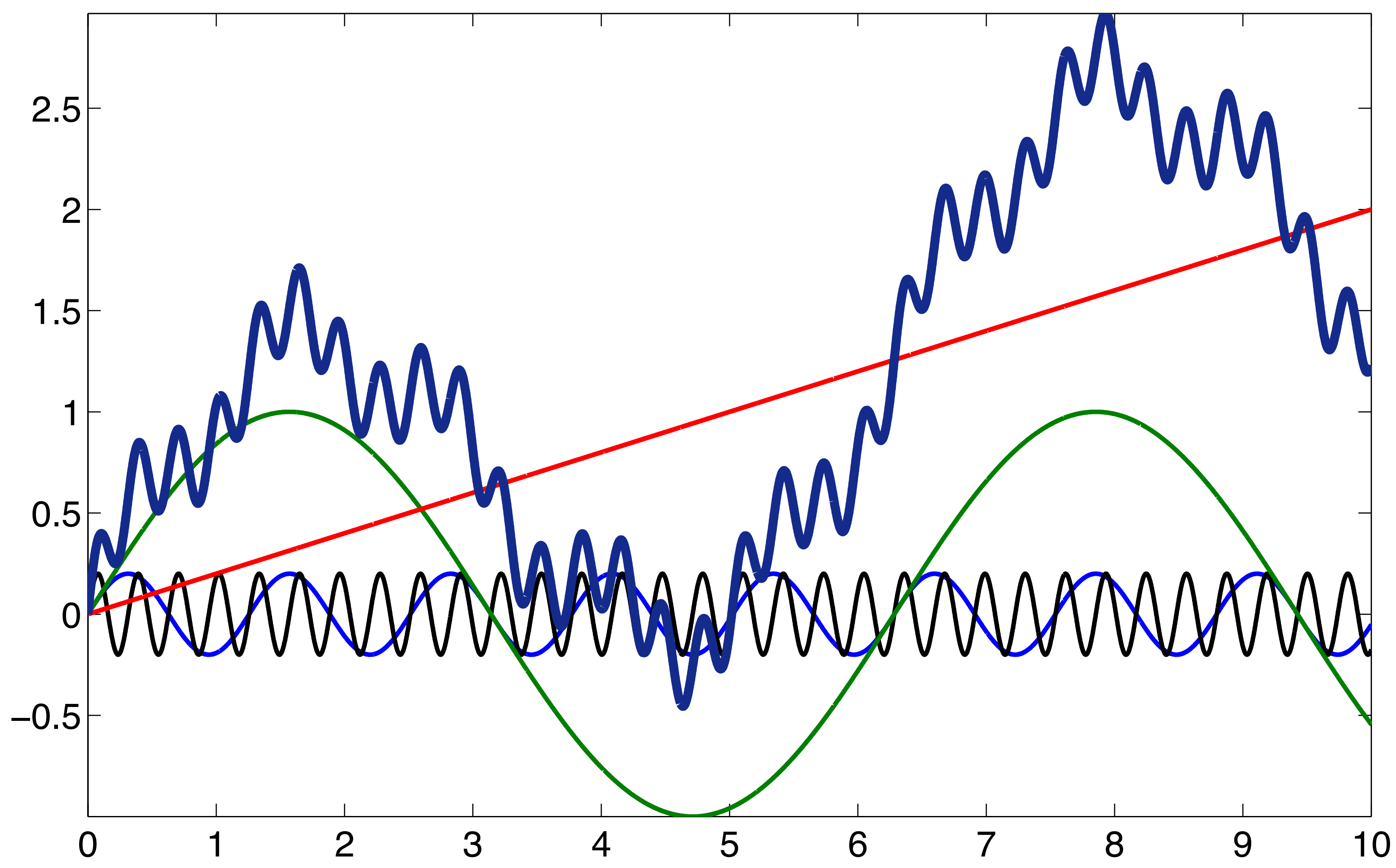
UNIVERSITÉ PARIS-SACLAY

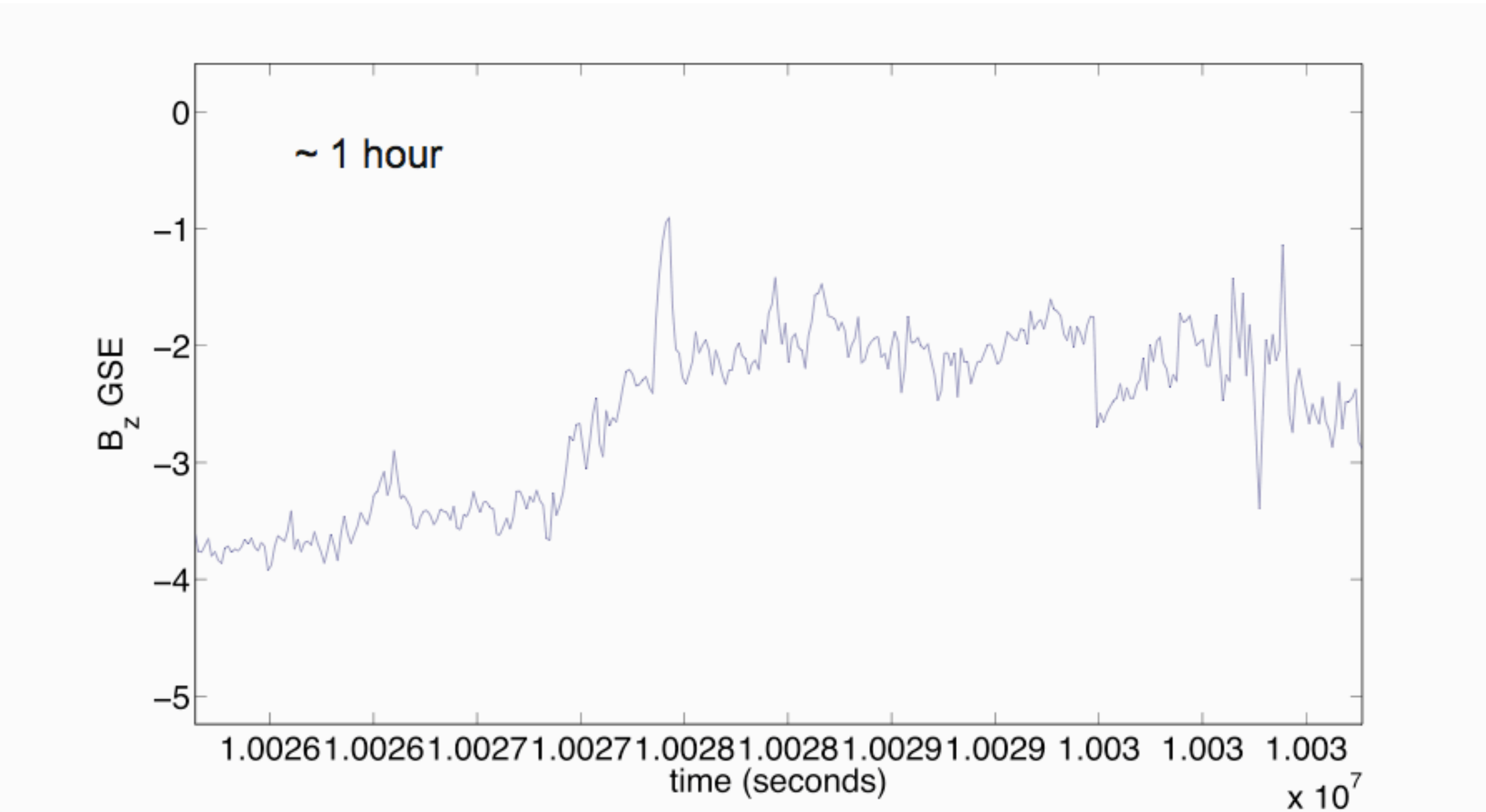
background guide field



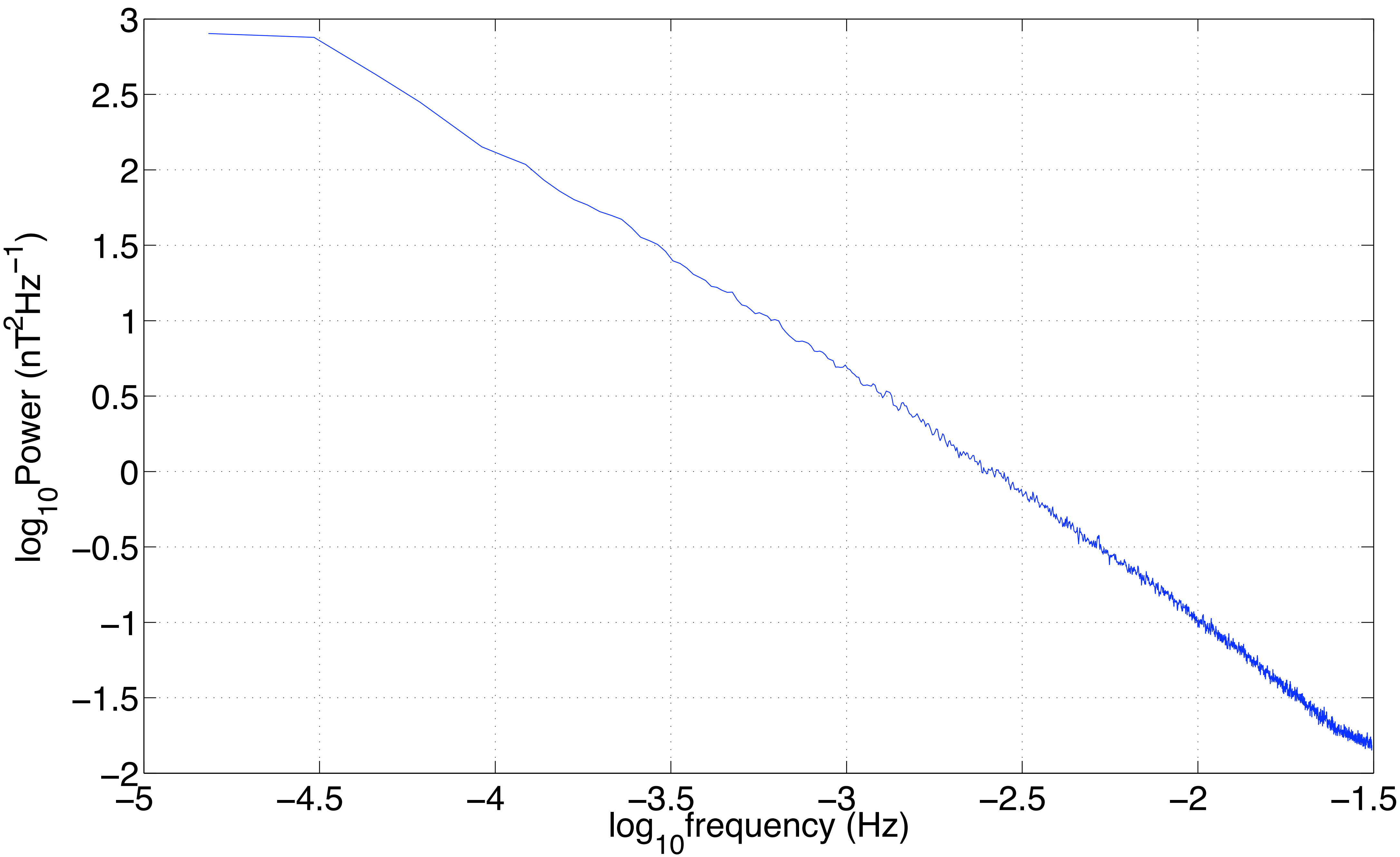
background guide field



background guide field

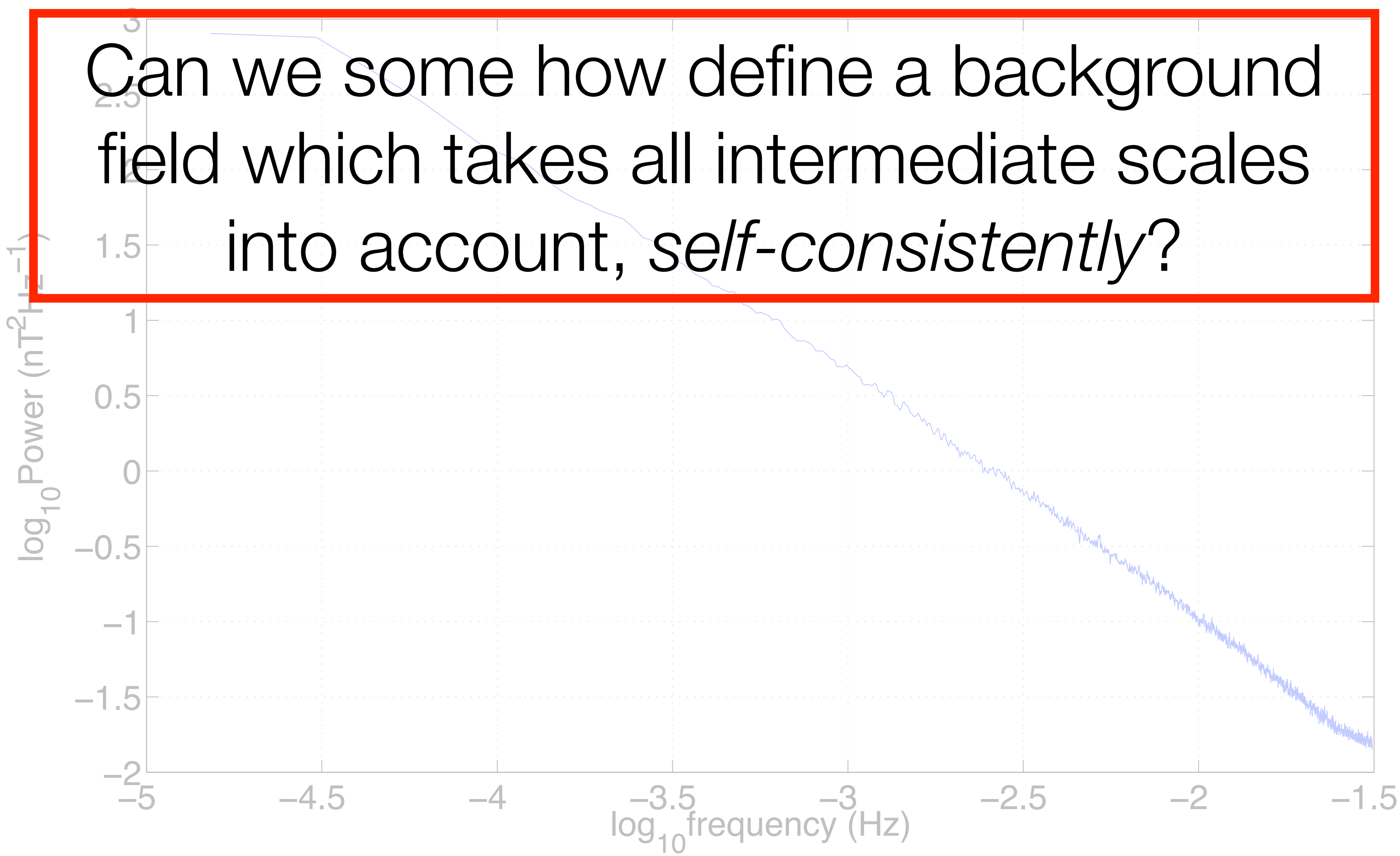


background guide field



question 1

Can we some how define a background field which takes all intermediate scales into account, *self-consistently*?



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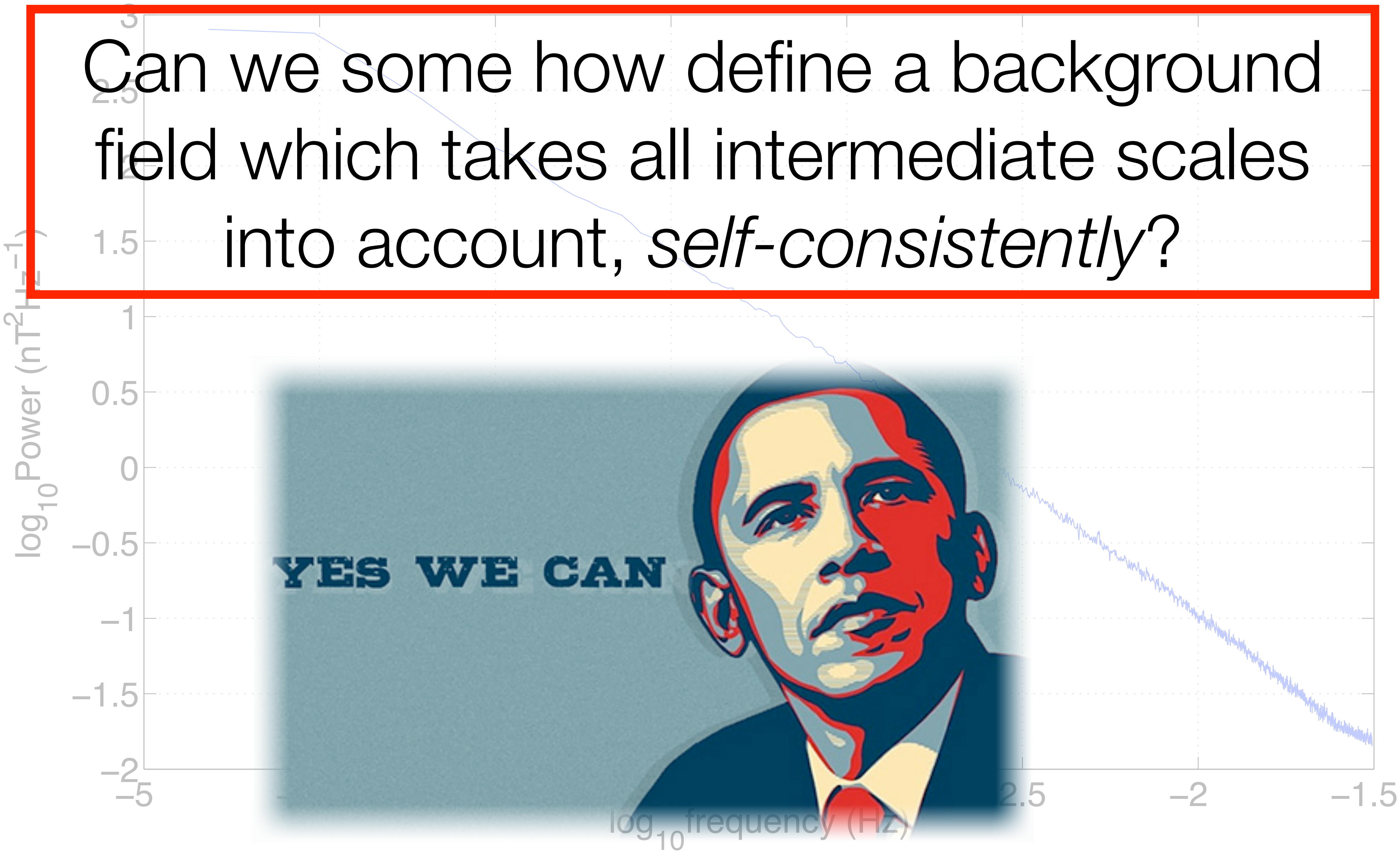
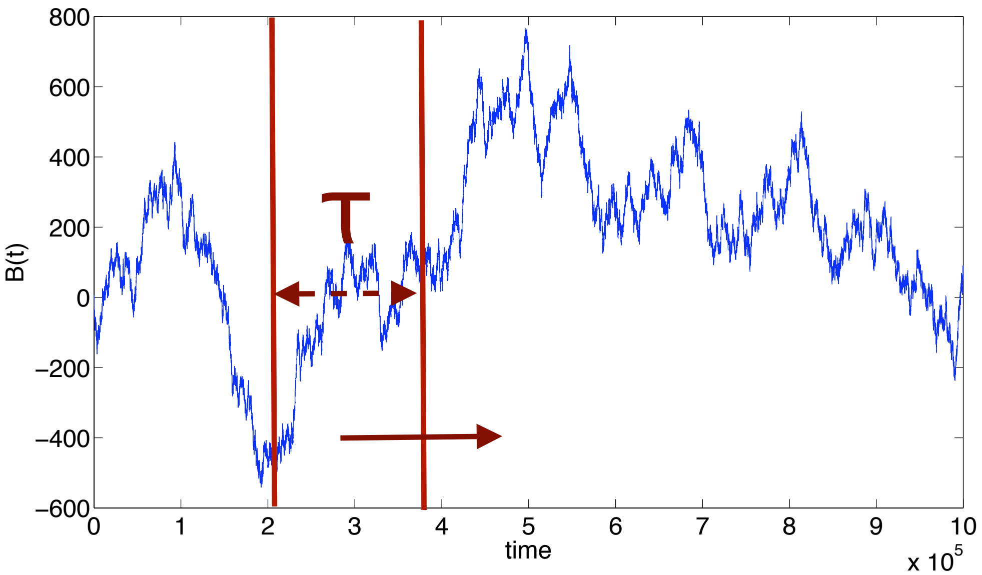
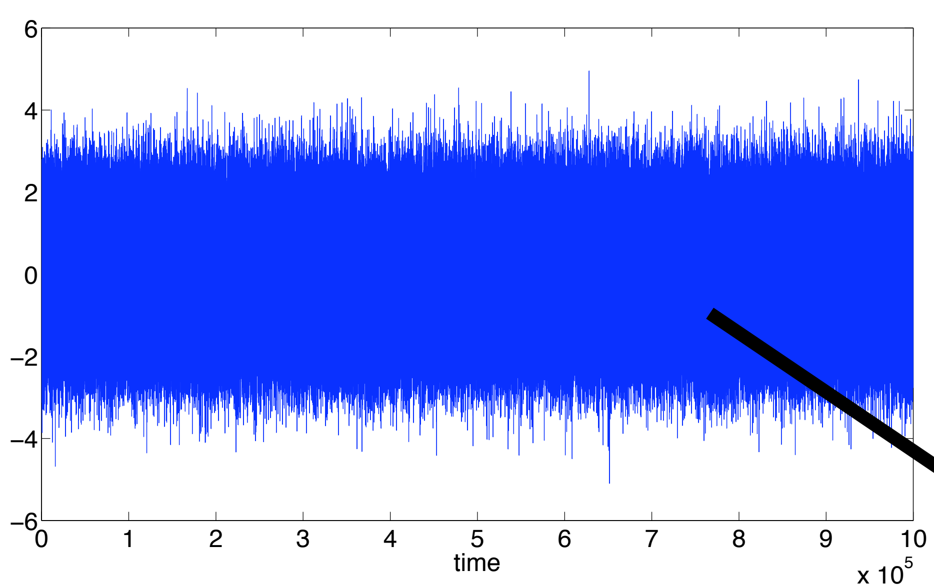


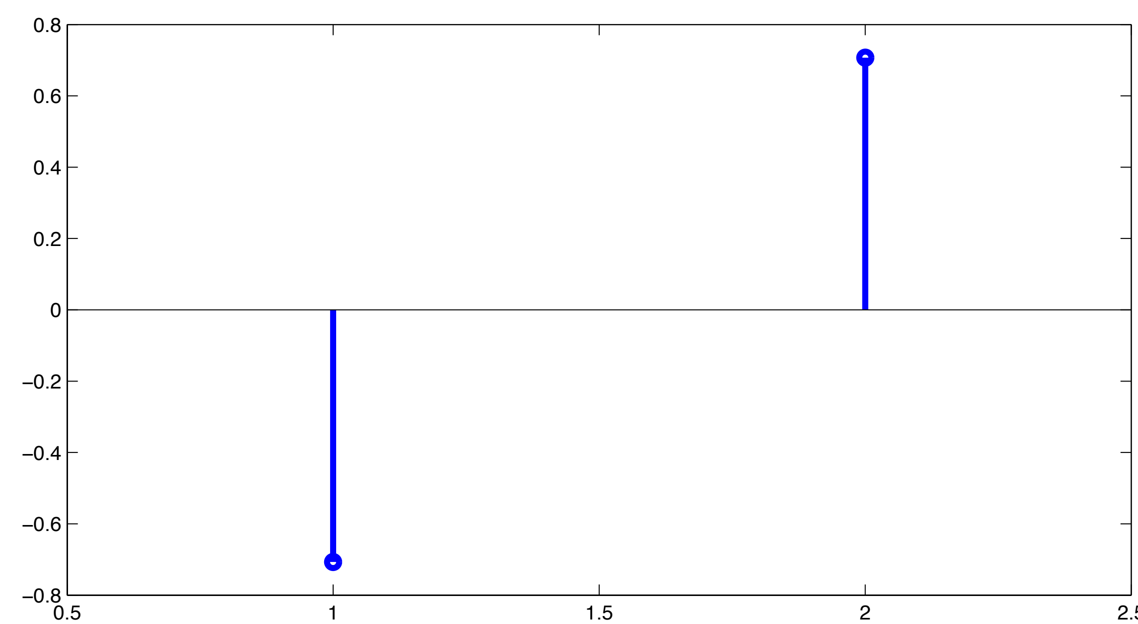
extreme time locality (difference filter)



increments

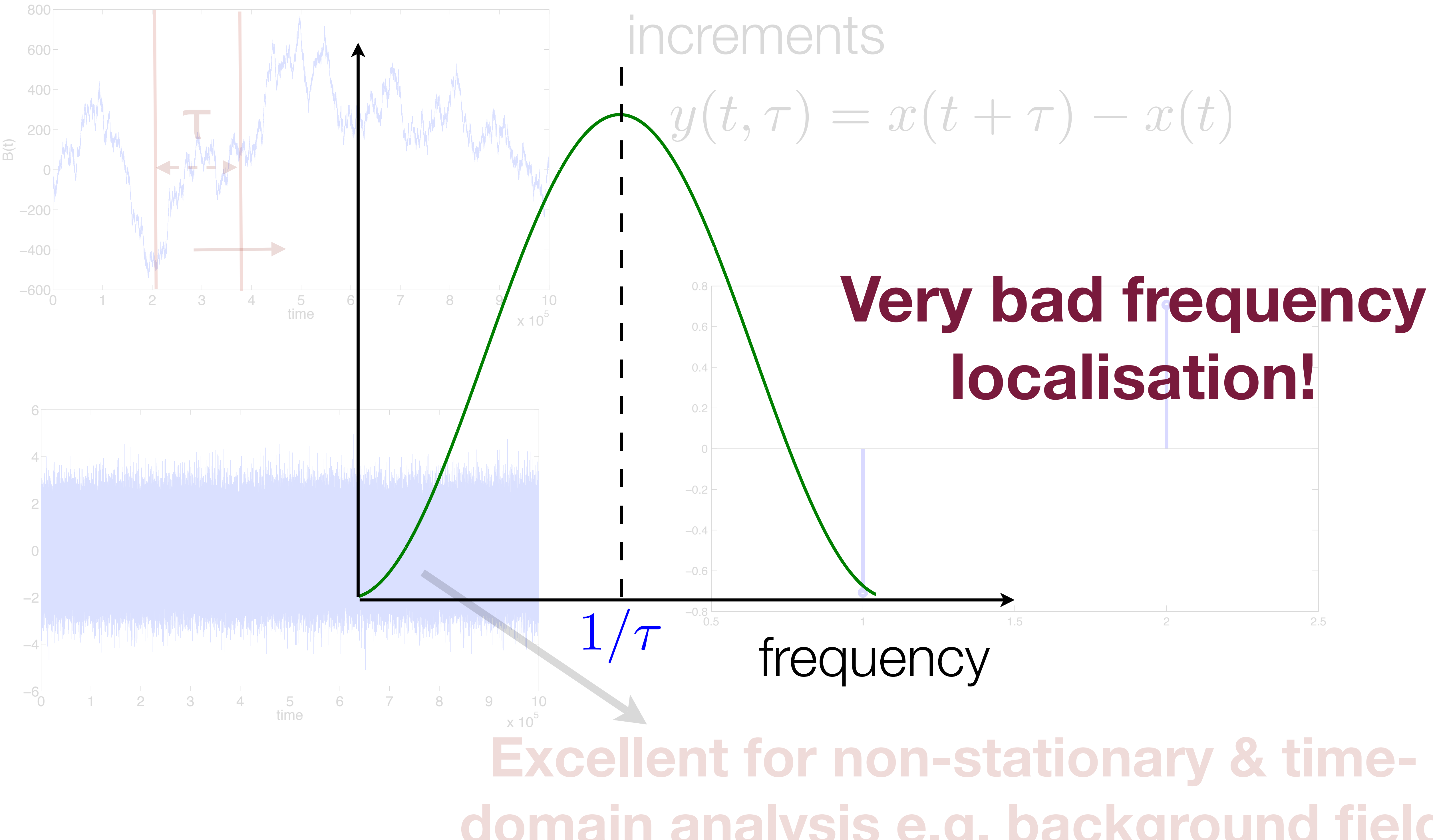
$$y(t, \tau) = x(t + \tau) - x(t)$$



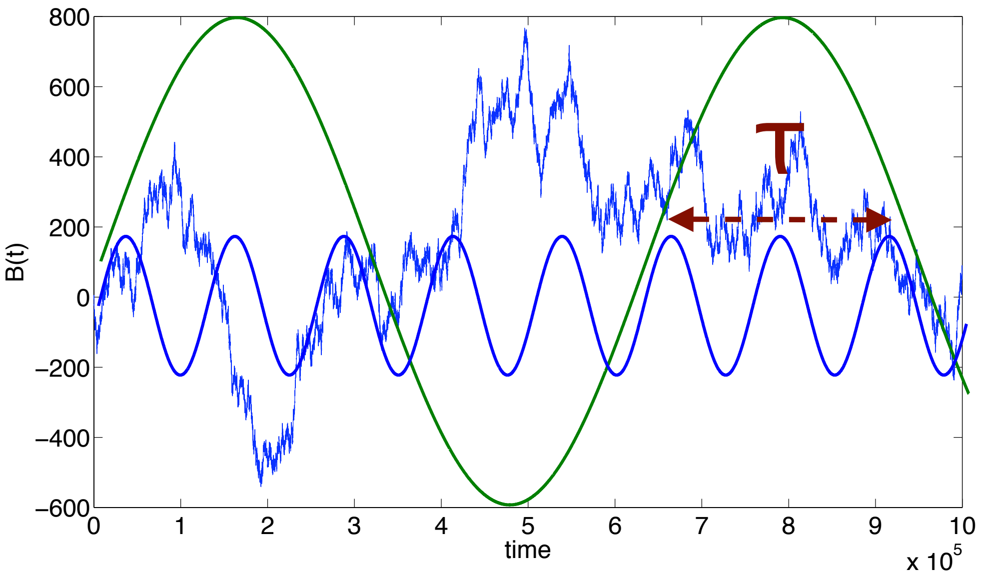


Excellent for non-stationary & time-domain analysis e.g. background field

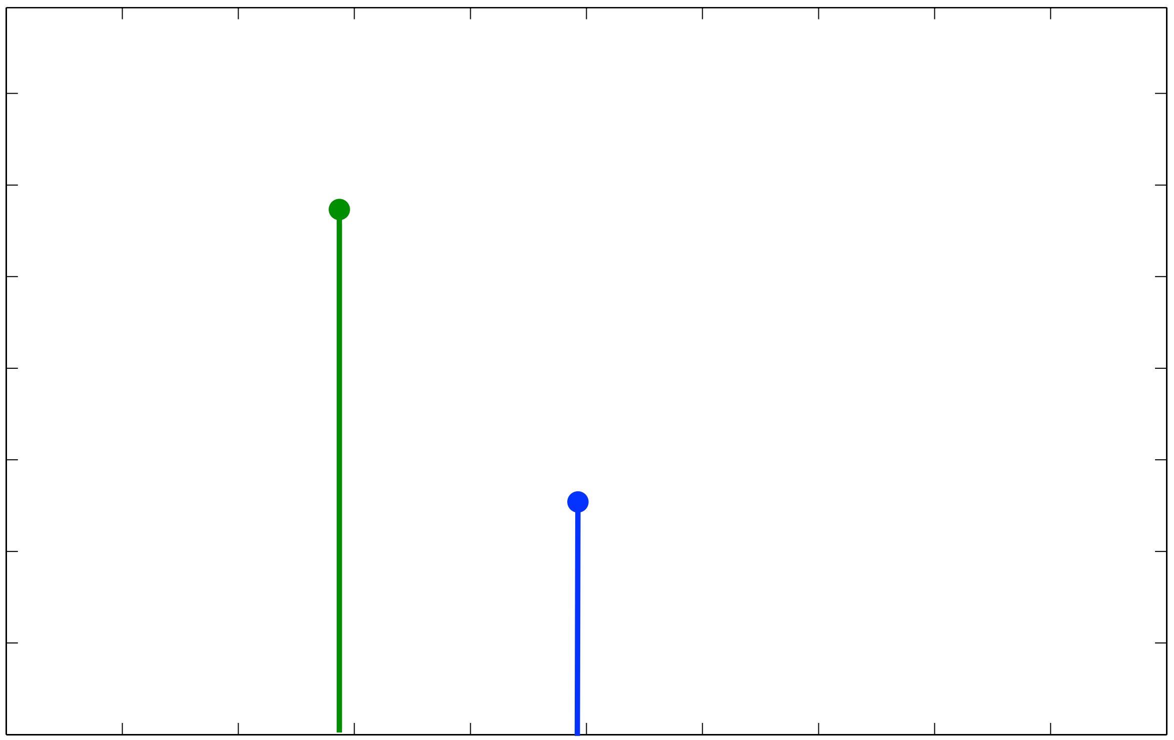
extreme time locality (difference filter)



extreme frequency locality (Fourier modes)

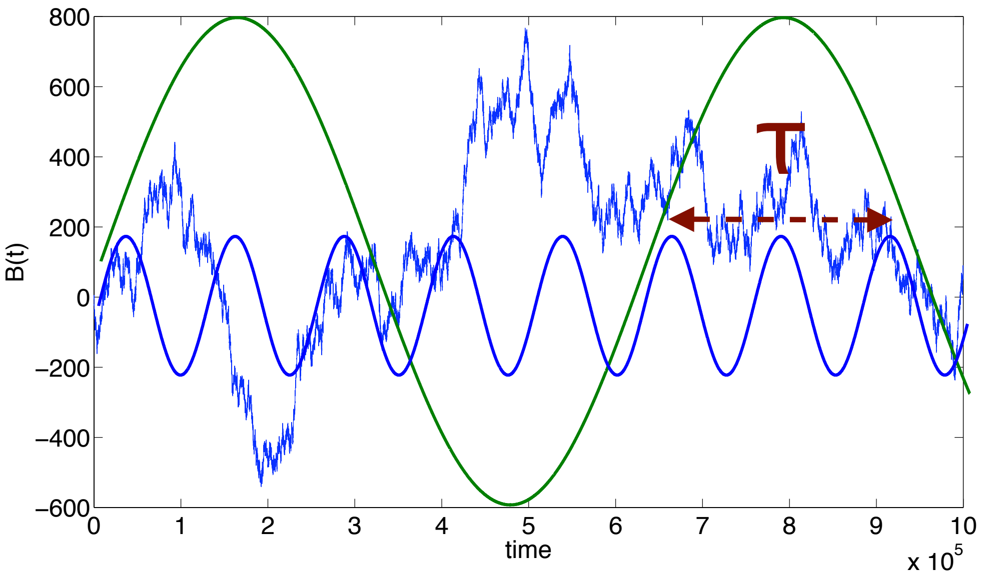


$$\frac{d}{dt}e^{i\omega t} = i\omega e^{i\omega t}$$

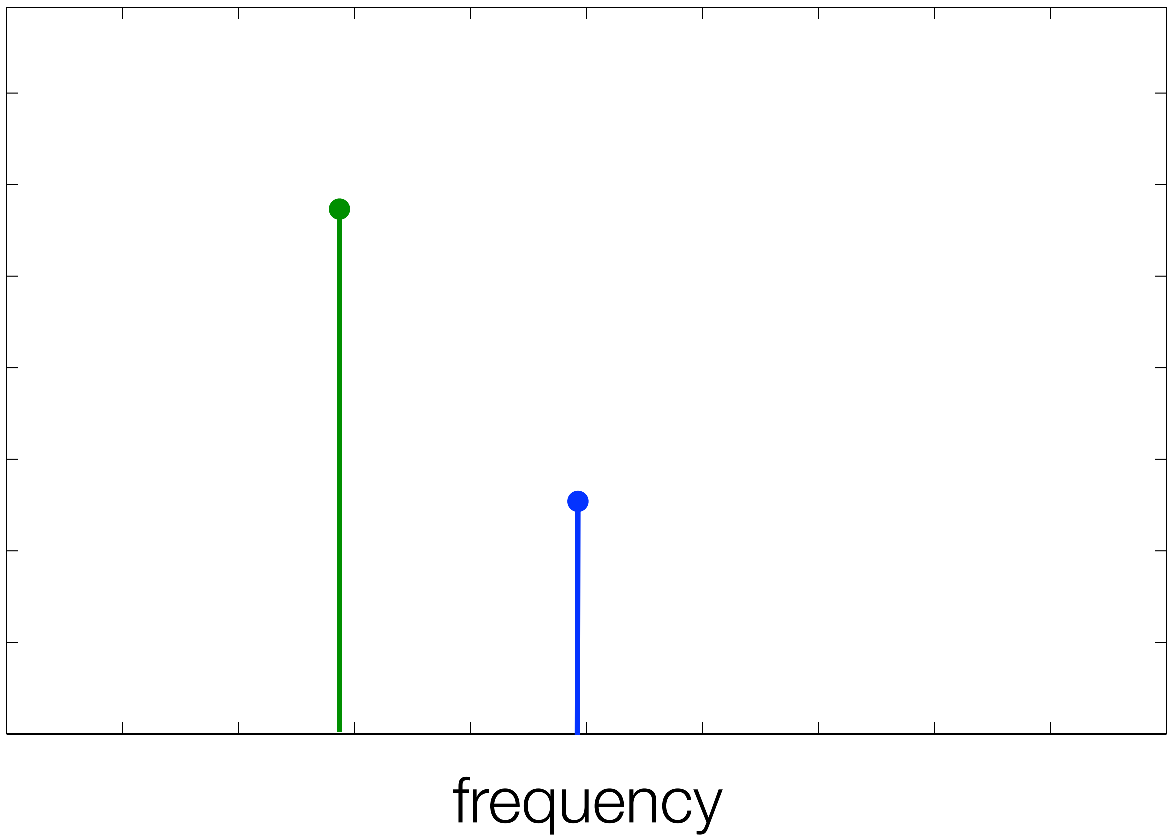


Good for stationary frequency-domain analysis; Good for theory - Fourier modes are eigenmodes for linear physics

extreme frequency locality (Fourier modes)



Extremely bad time-domain localisation!



$$\frac{d}{dt}e^{i\omega t} = i\omega e^{i\omega t}$$

Good for stationary frequency-domain analysis; Good for theory - Fourier modes are eigenmodes for linear physics

question 2

Can we obtain the best of both worlds and live ‘well’ in the time and frequency domains?

time (space) domain

better reference to detailed physics and structure

Fourier domain

much theory; normally coarse-grained picture

question 2

Can we obtain the best of both worlds and live ‘well’ in the time <u>and</u> frequency domains?

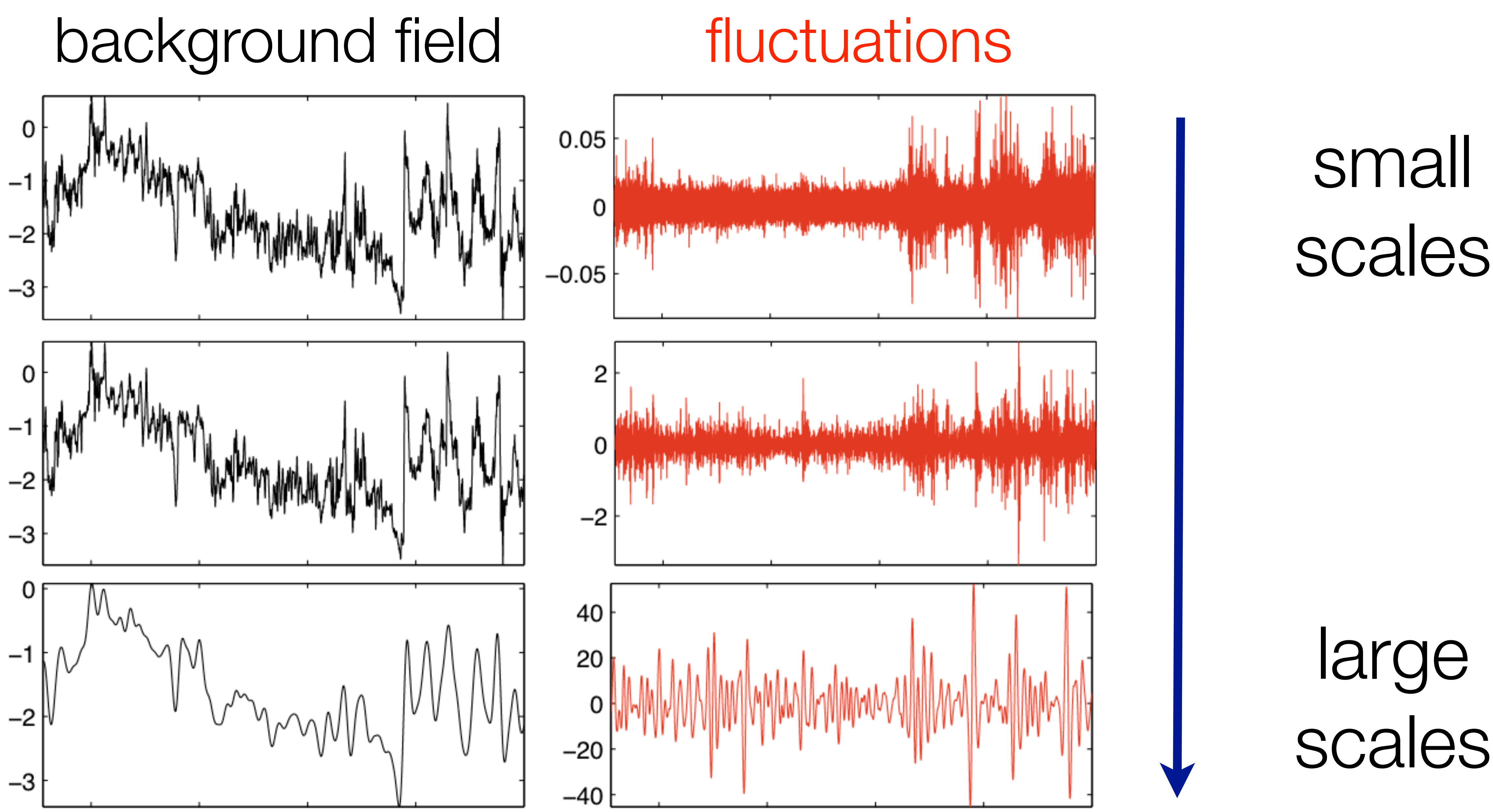
time domain

Fourier domain

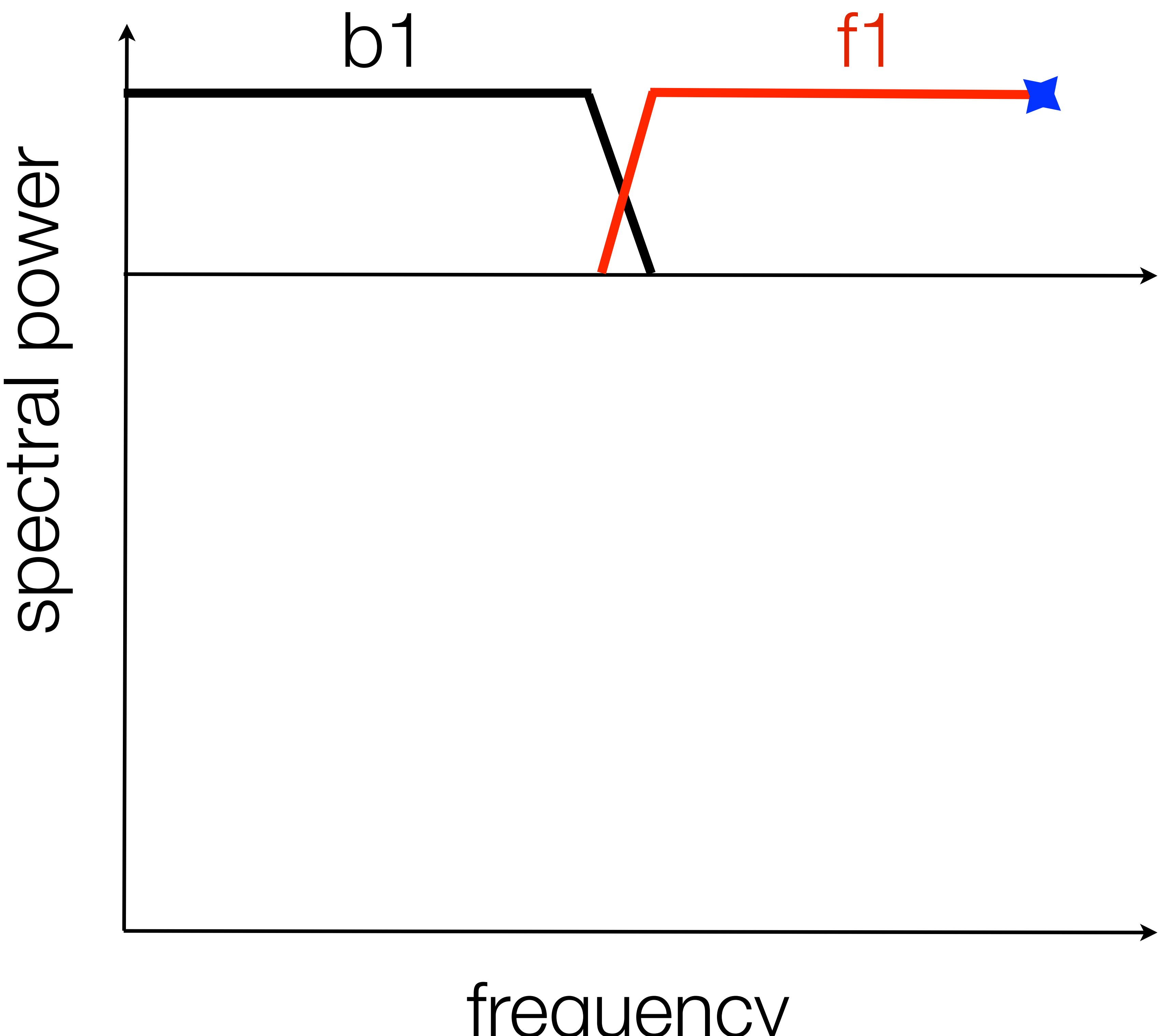
outline

- Discrete wavelets as band-pass digital filters
- Picking and designing an appropriate wavelet/transform
- Some results on separating anisotropic signatures in plasma turbulence in the solar wind

adaptive local scale-dependent fluctuations **and** background guide field



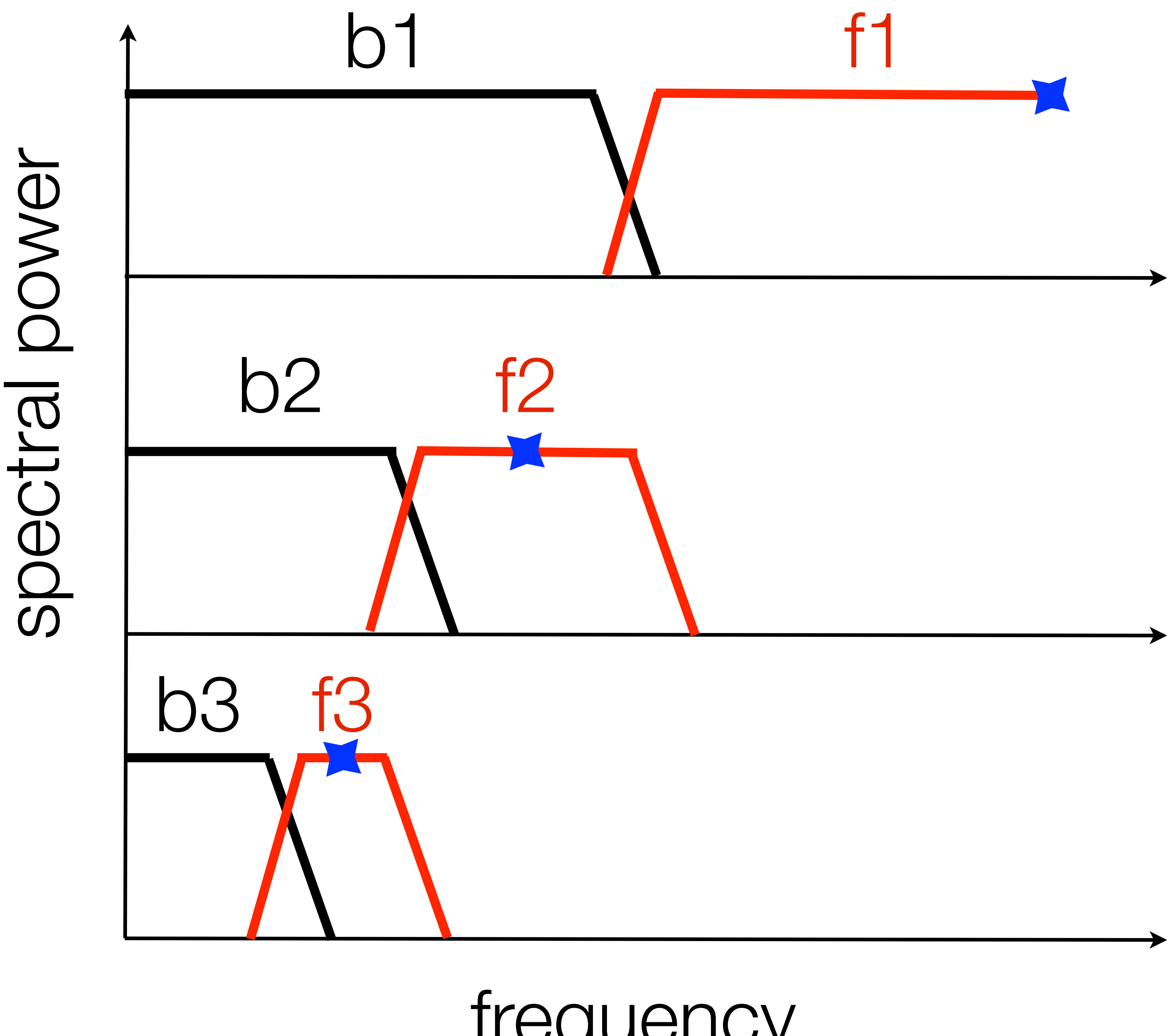
local scale-dependent fluctuations *and* background field using low/high pass filters



fluctuations

background field

local scale-dependent fluctuations *and* background field using low/high pass filters

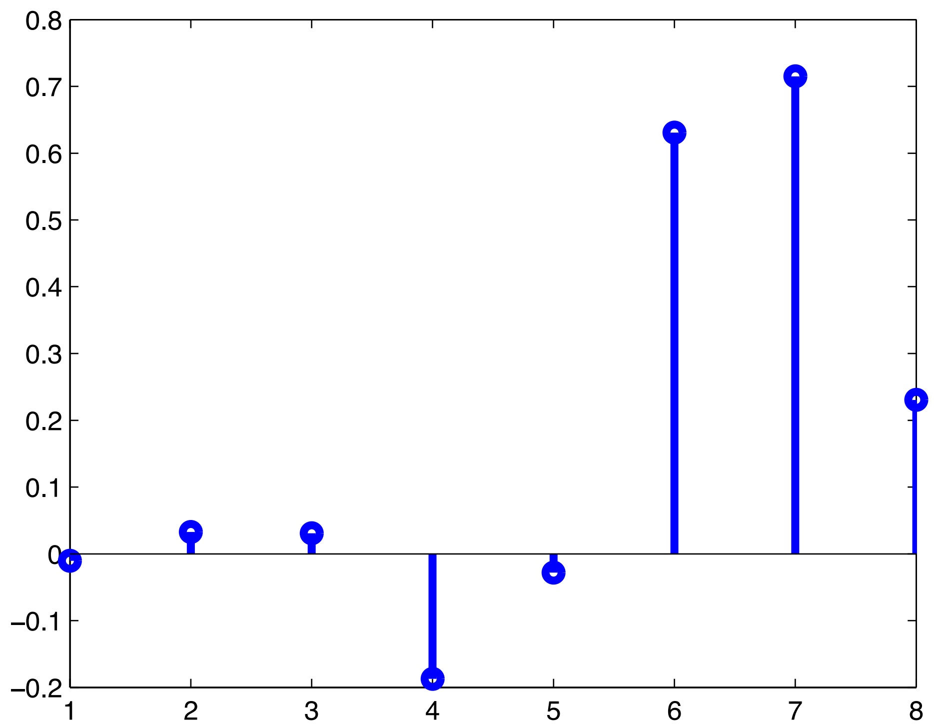


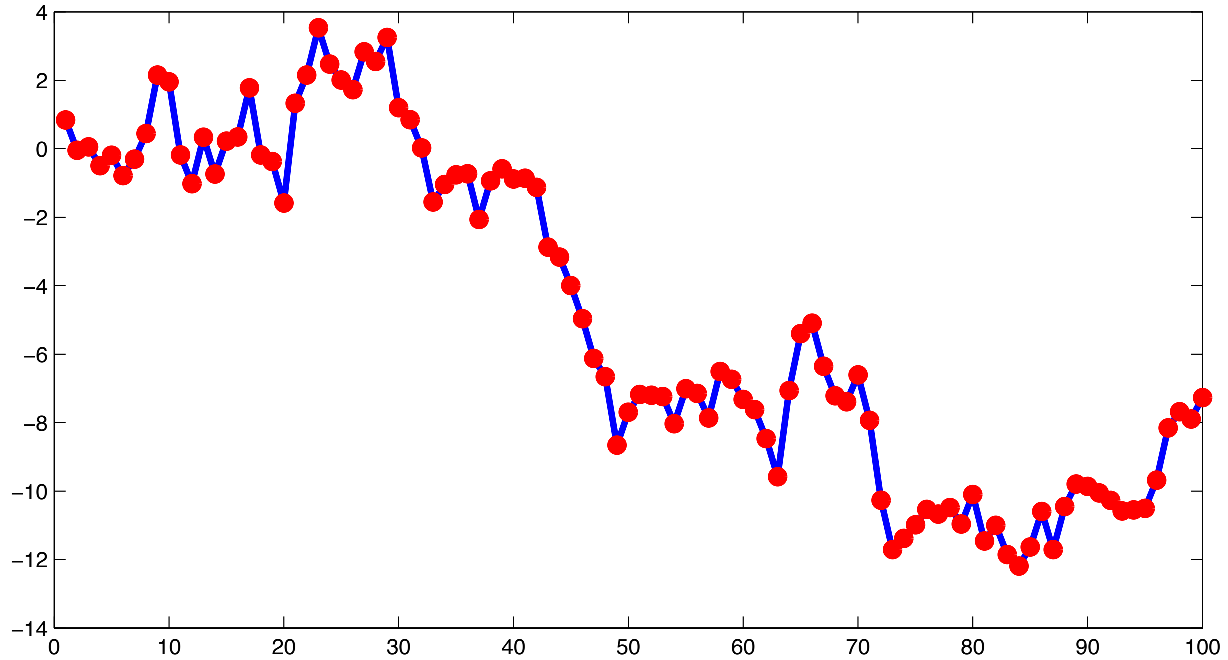
fluctuations

background field

filter bank implementation using quadrature mirror filters

discrete wavelet transform (dwt)





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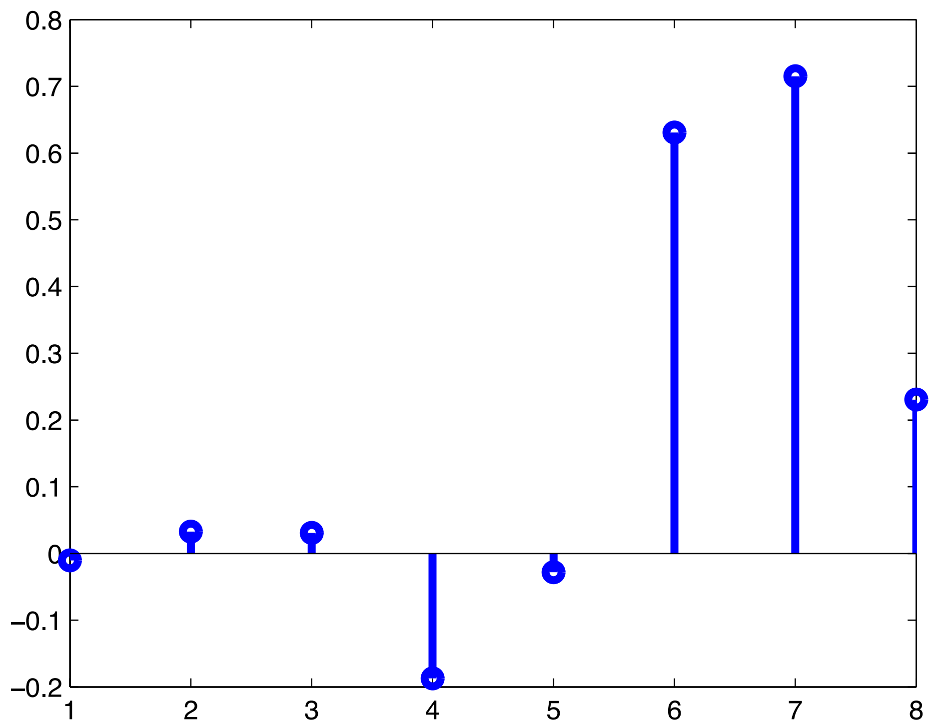
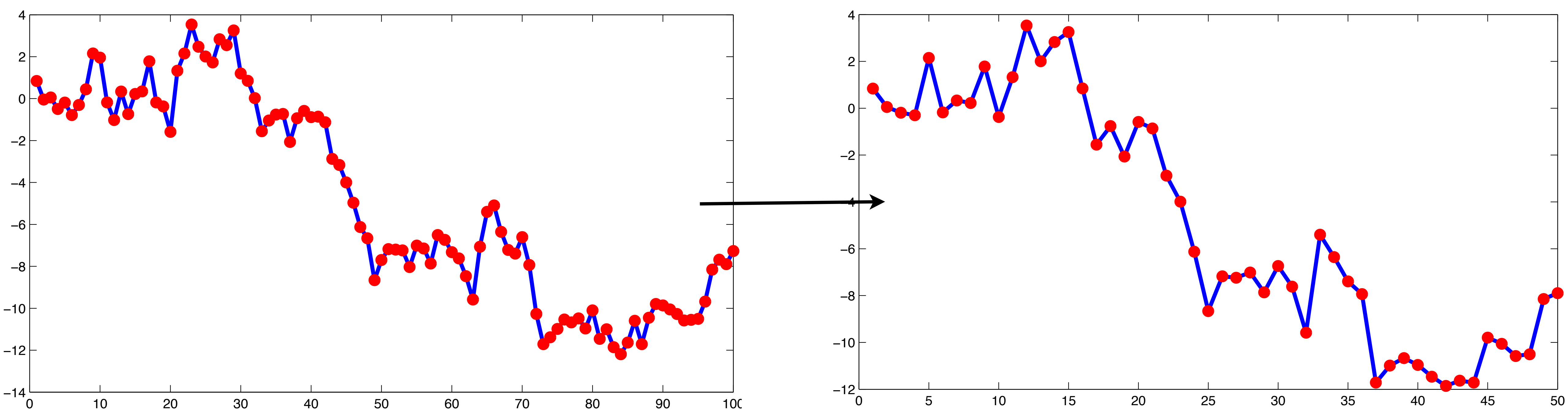


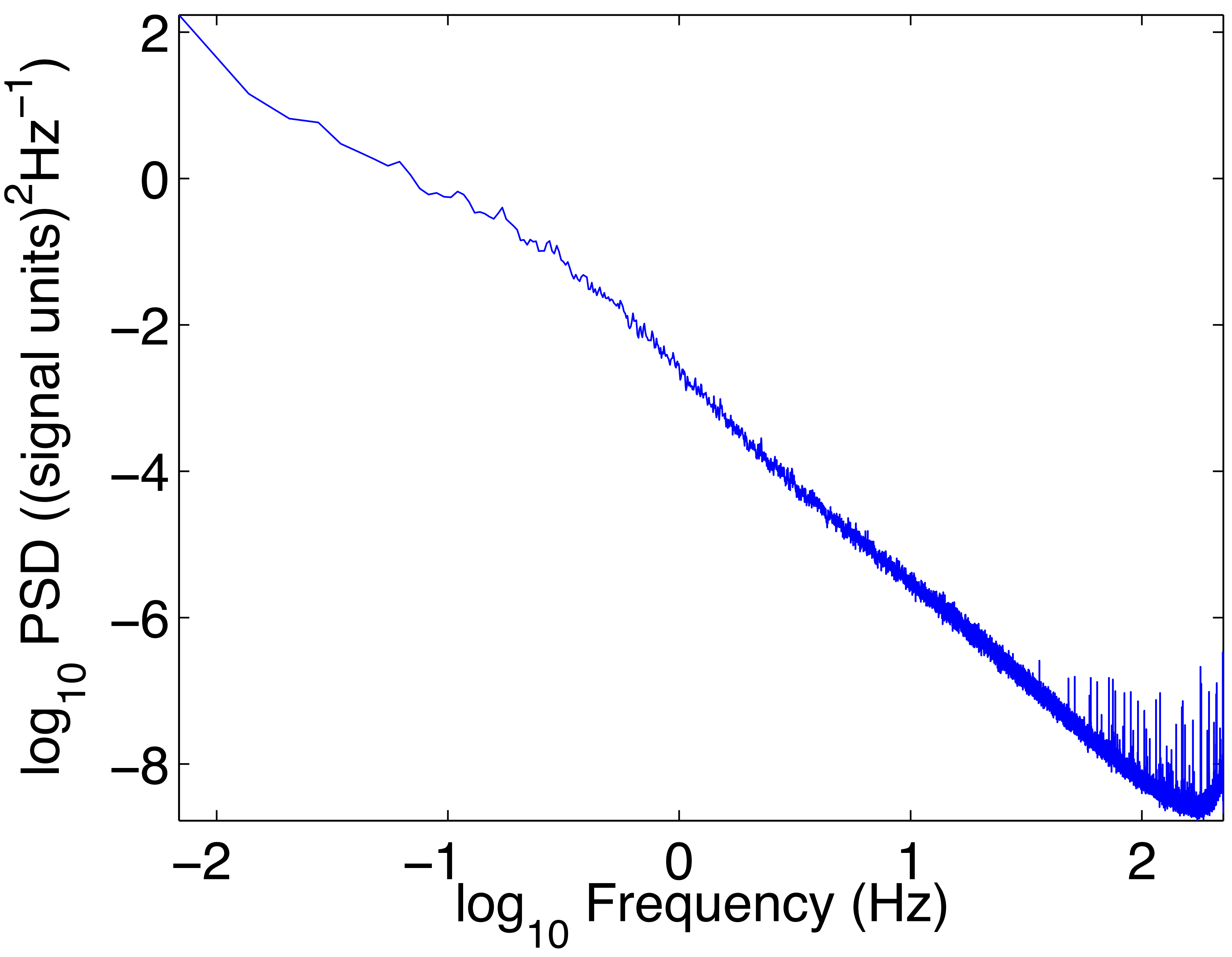
<u>DWT steps:</u>

Leave filter as it is

Decimate signal by 2



let’s see it working



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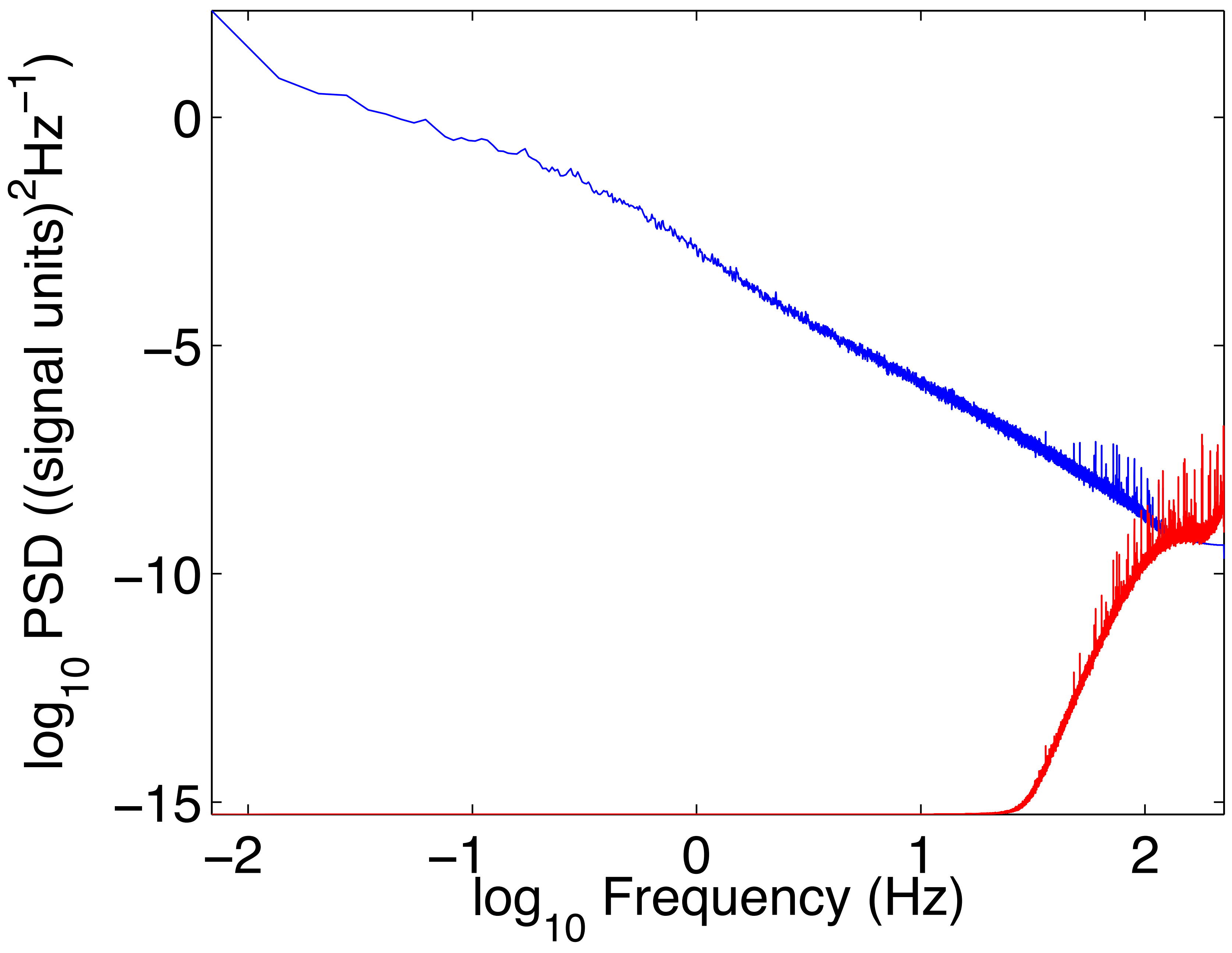

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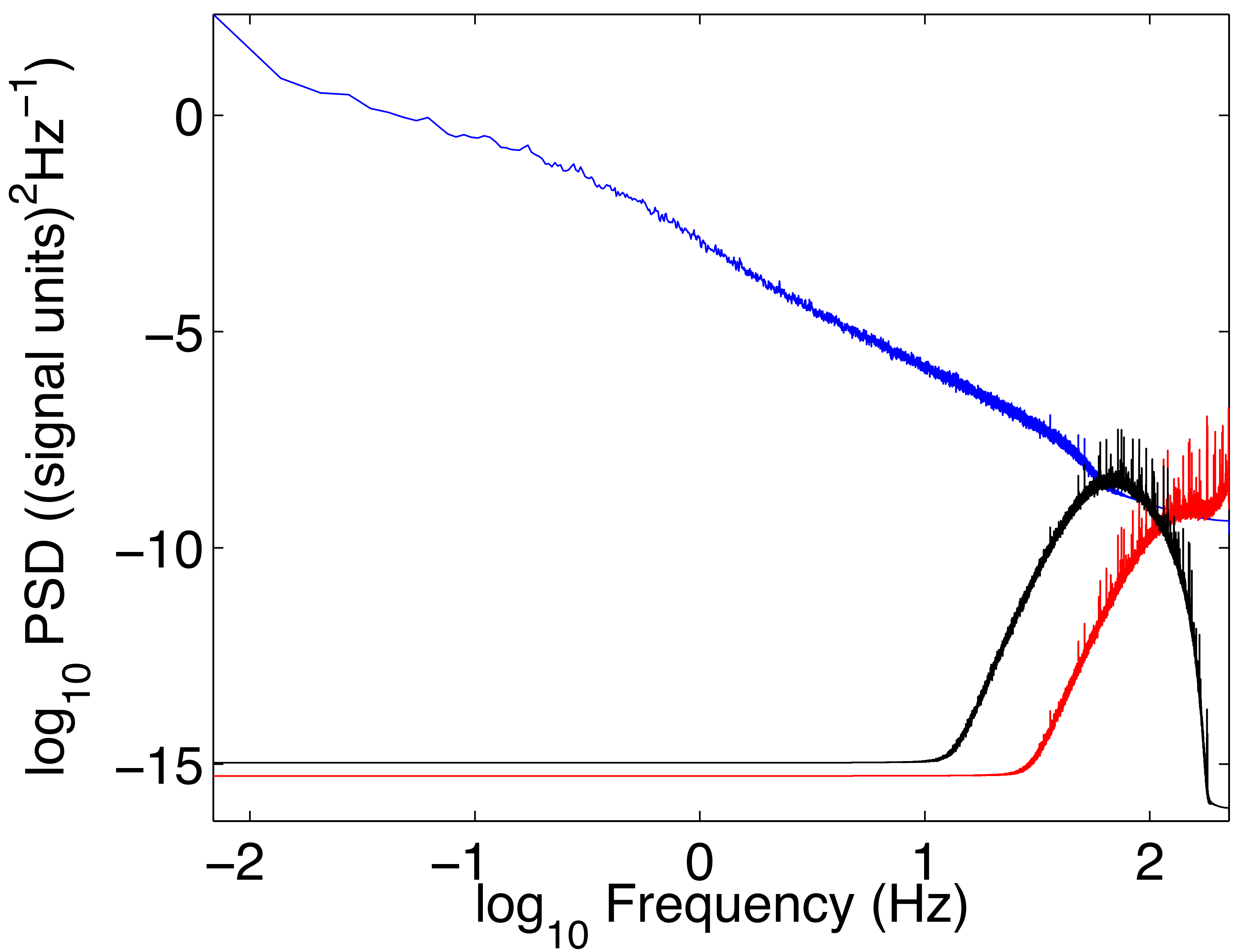

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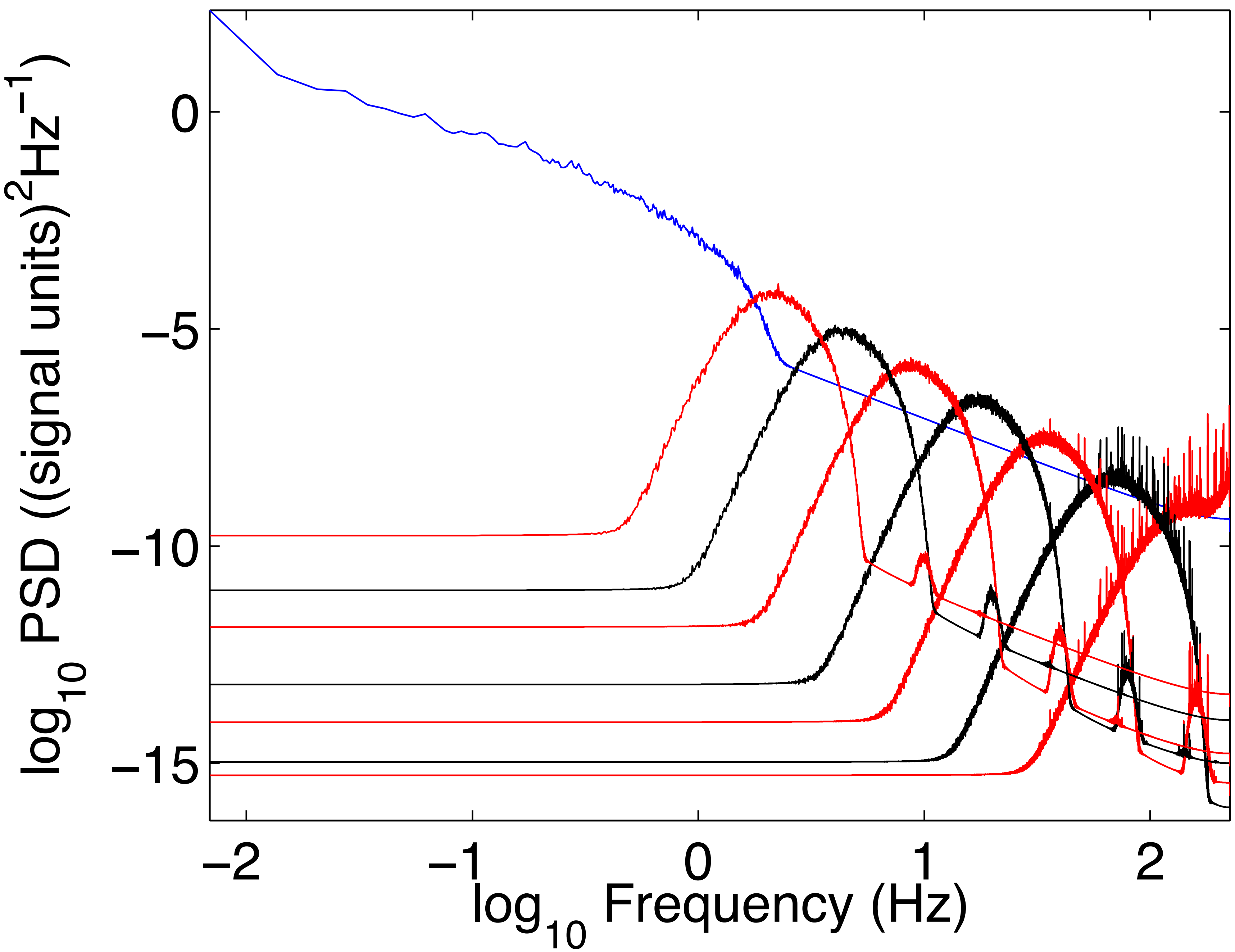
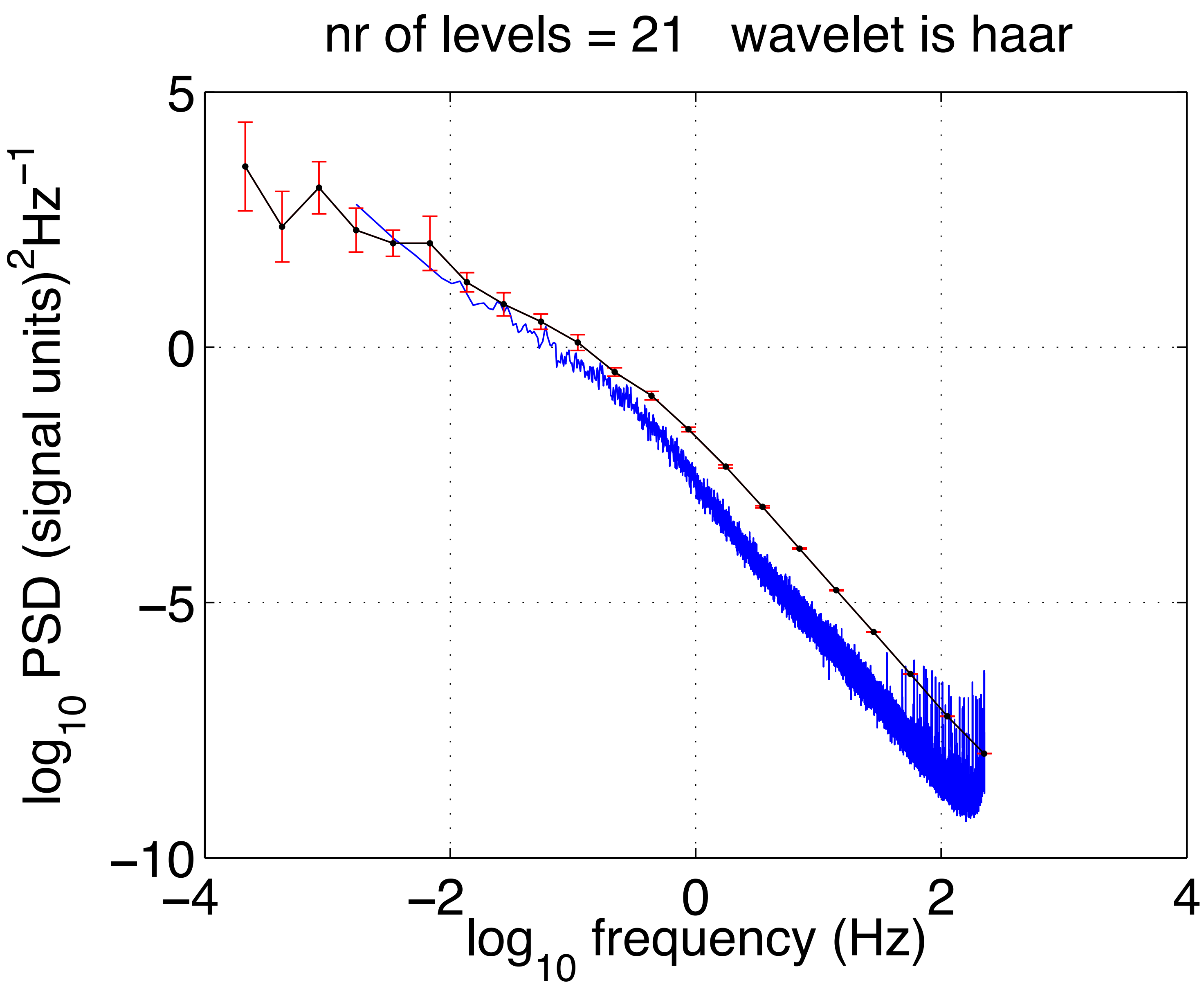


importance of zero moments



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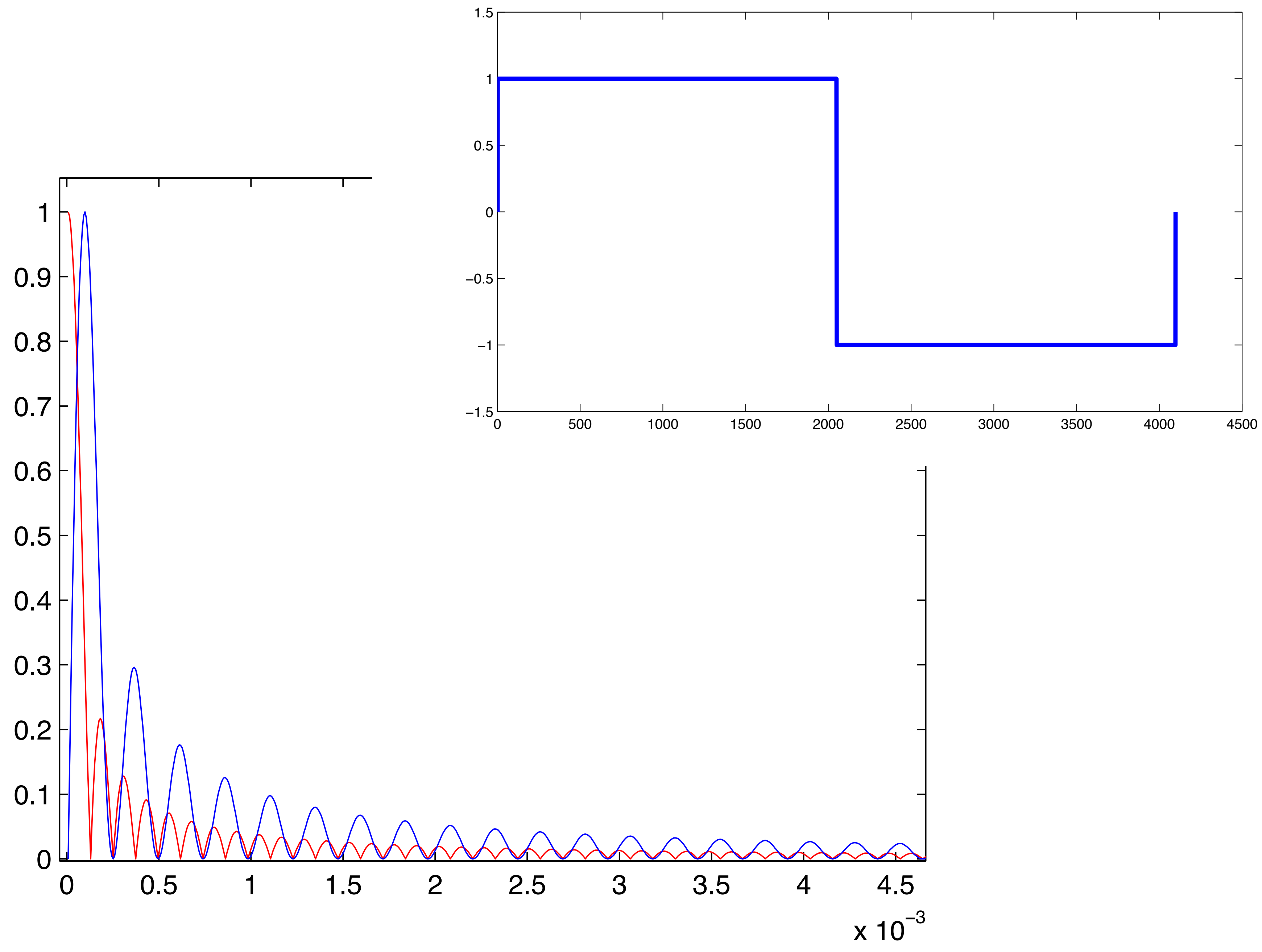
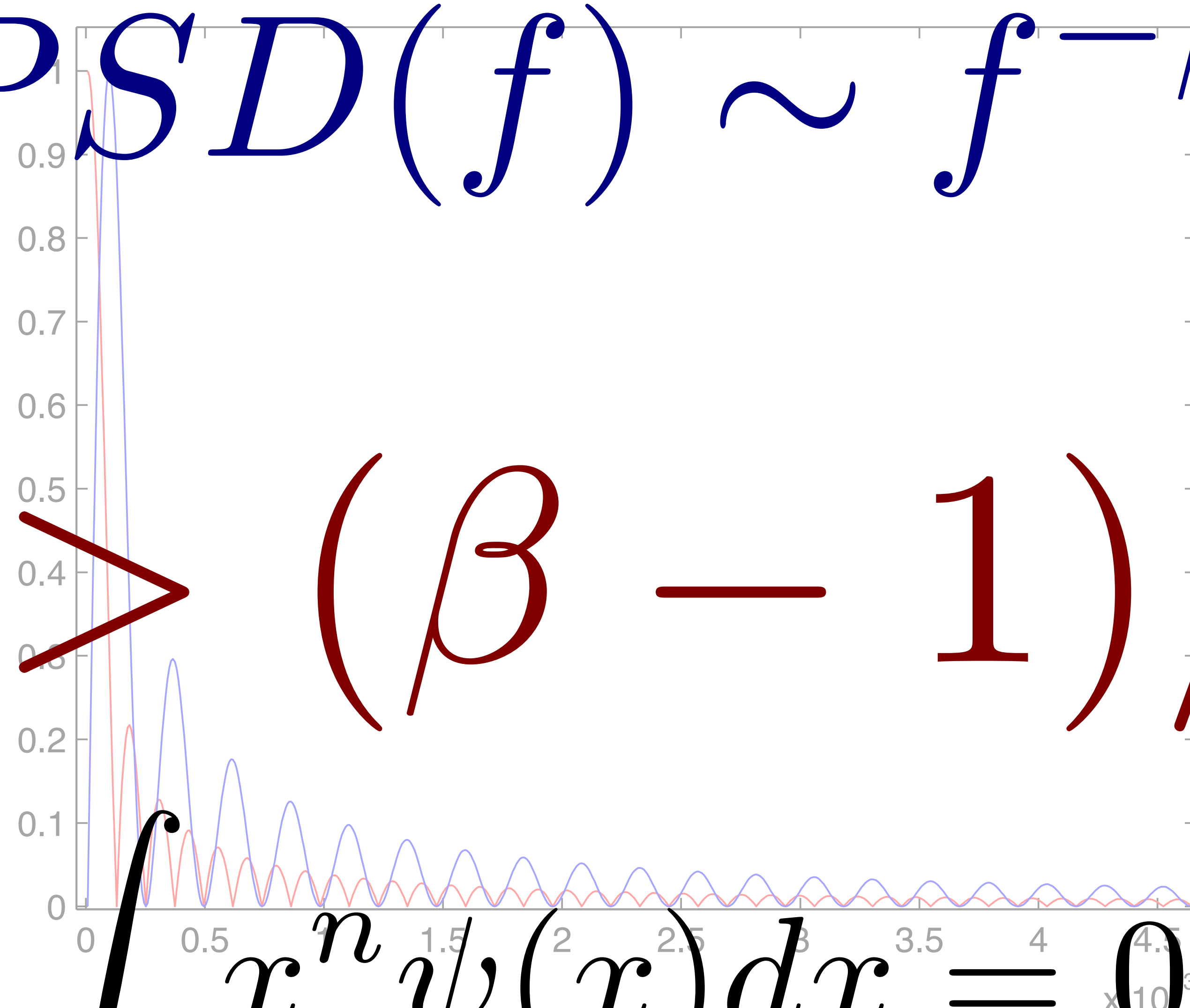


importance of zero moments

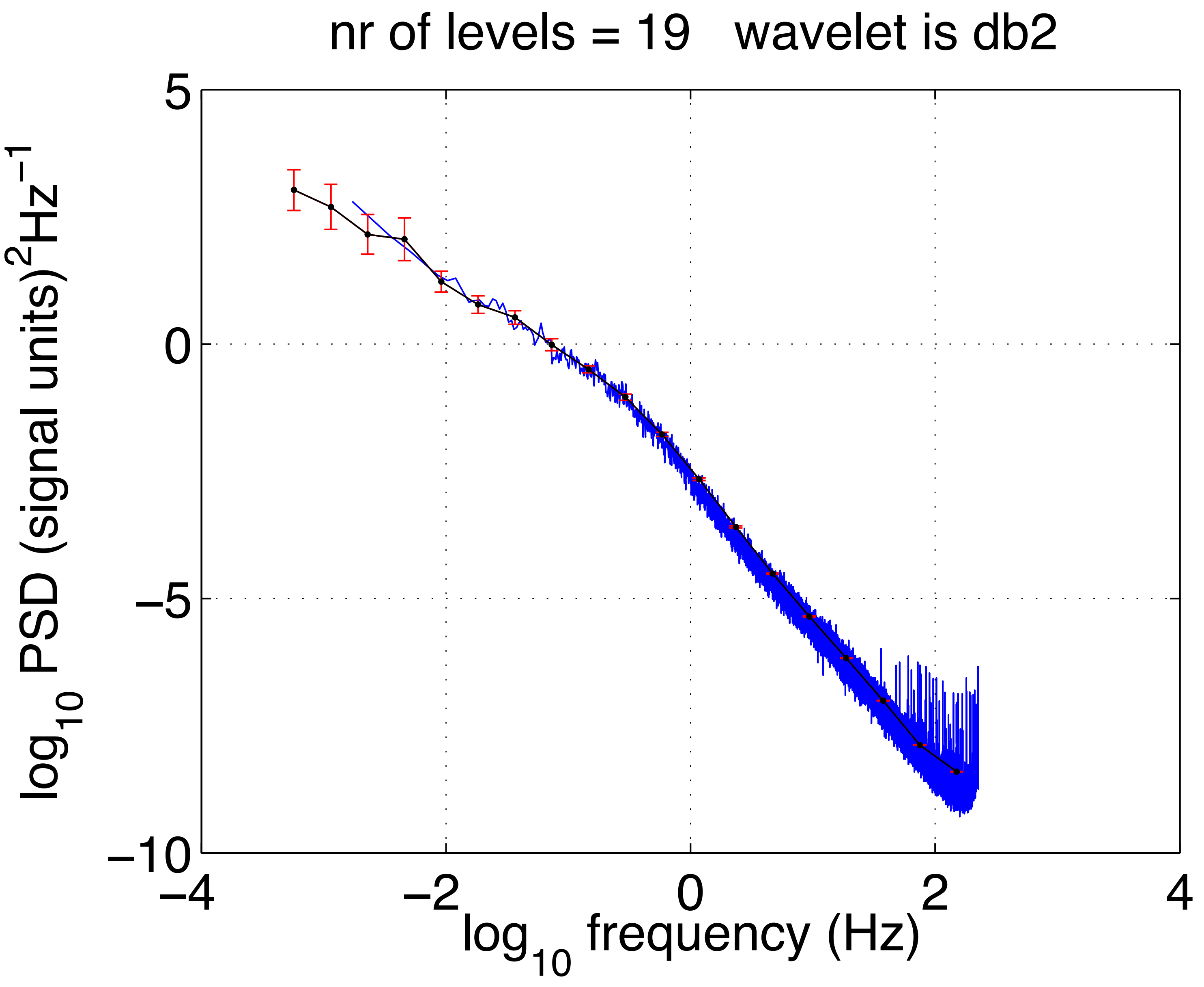
$$PSD(f) \sim f^{-\beta}$$



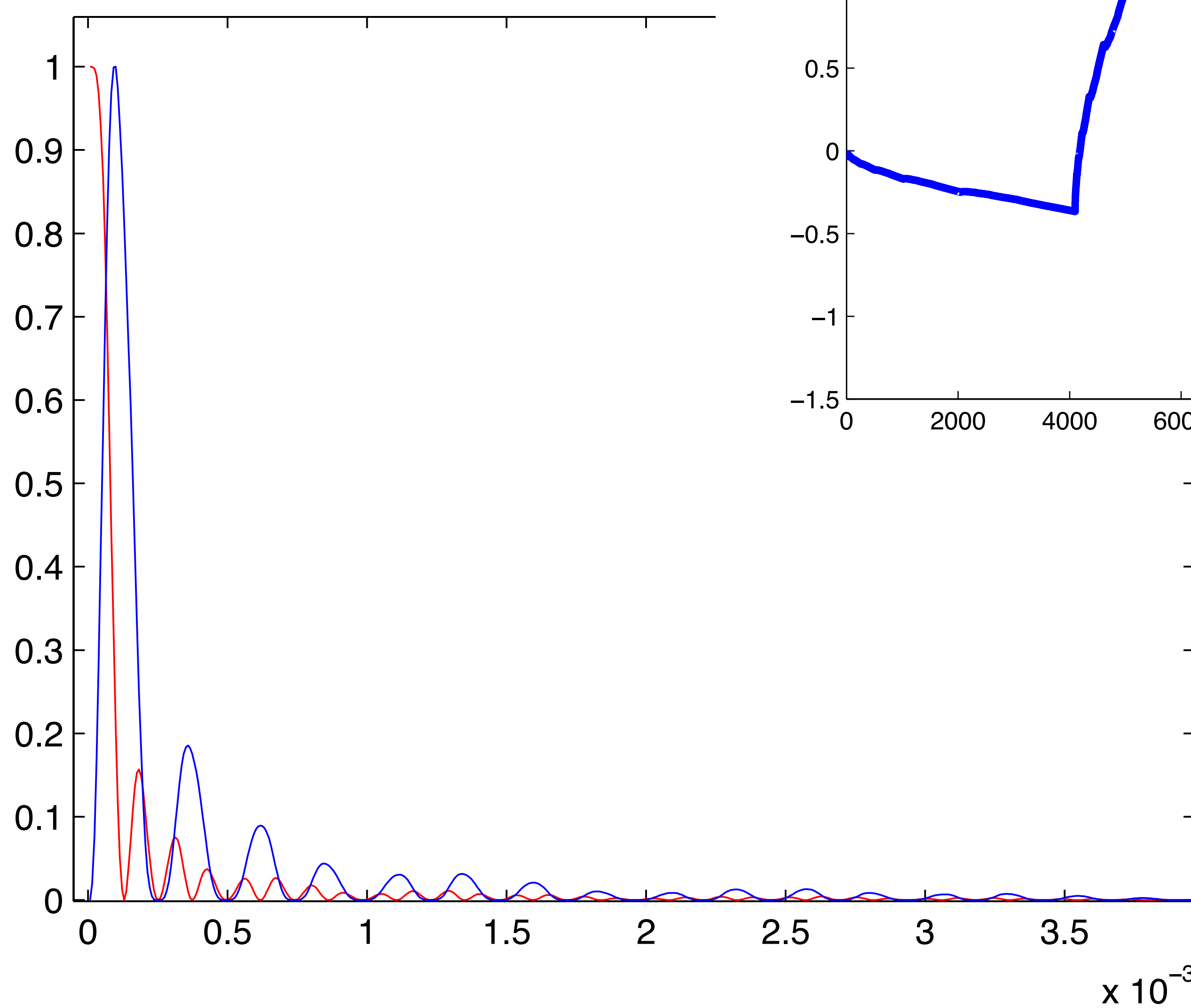
$$n > (\beta - 1)/2$$

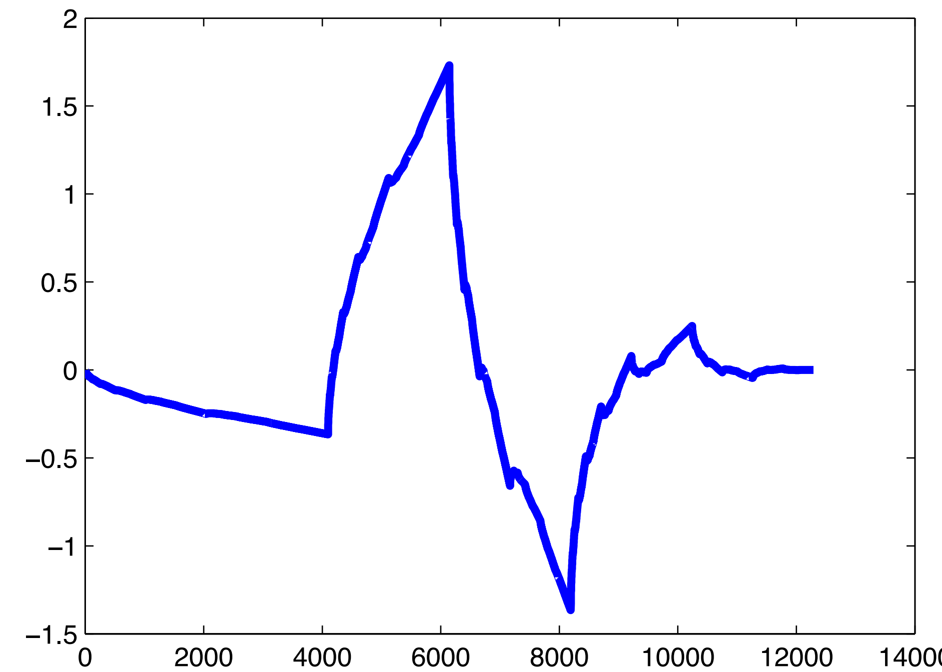
$$\int x^n \psi(x) dx = 0$$

importance of zero moments



importance of zero moments





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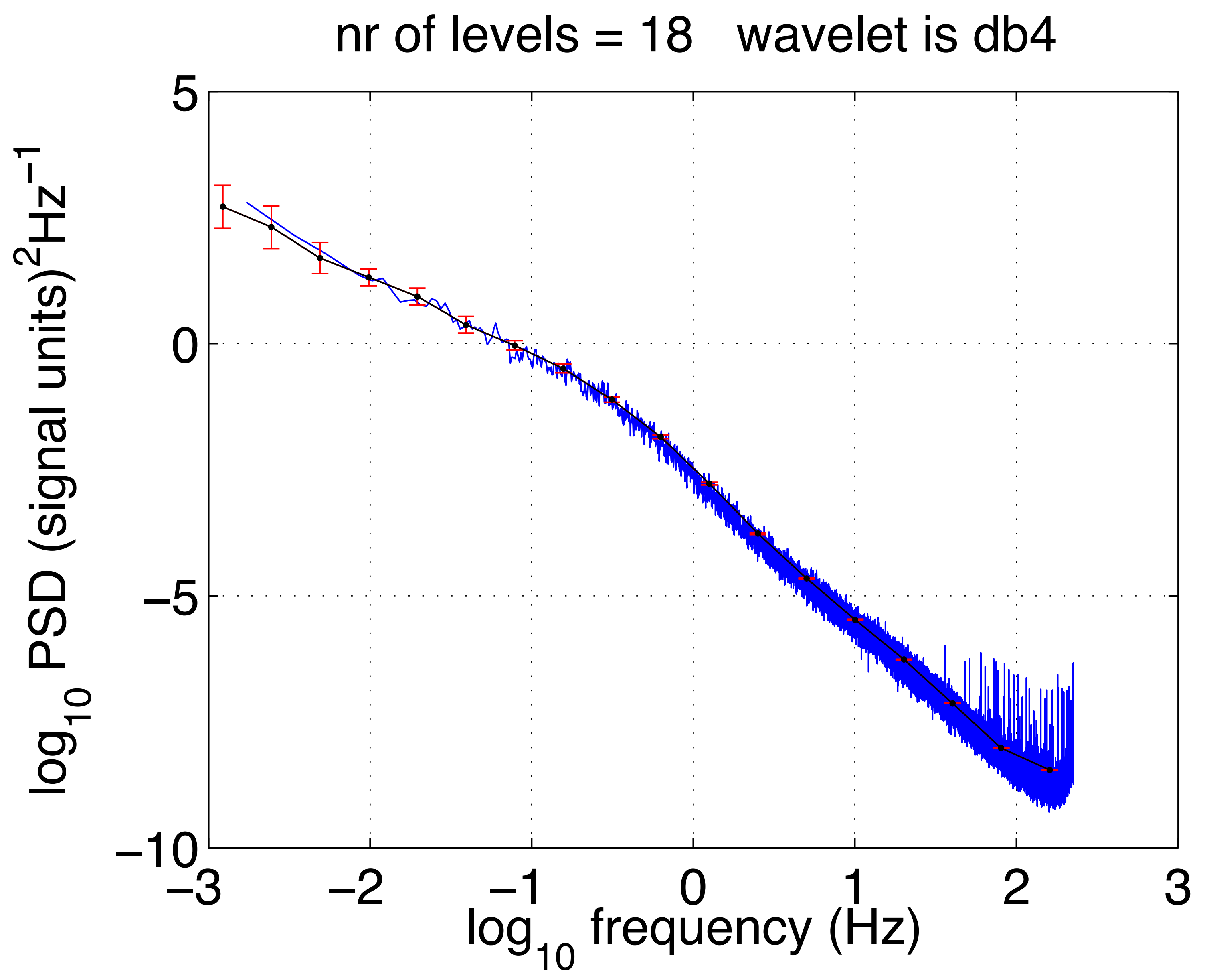

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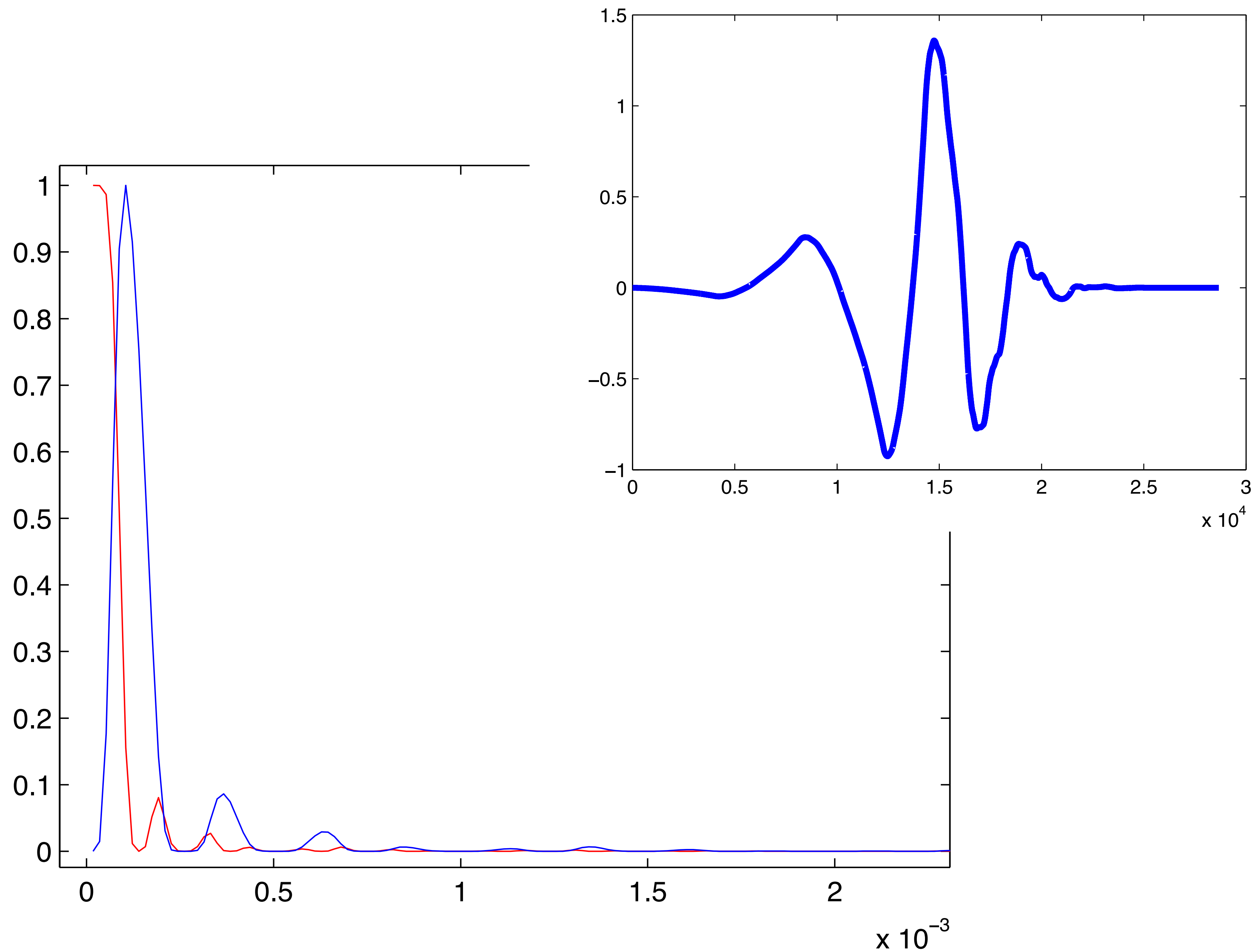

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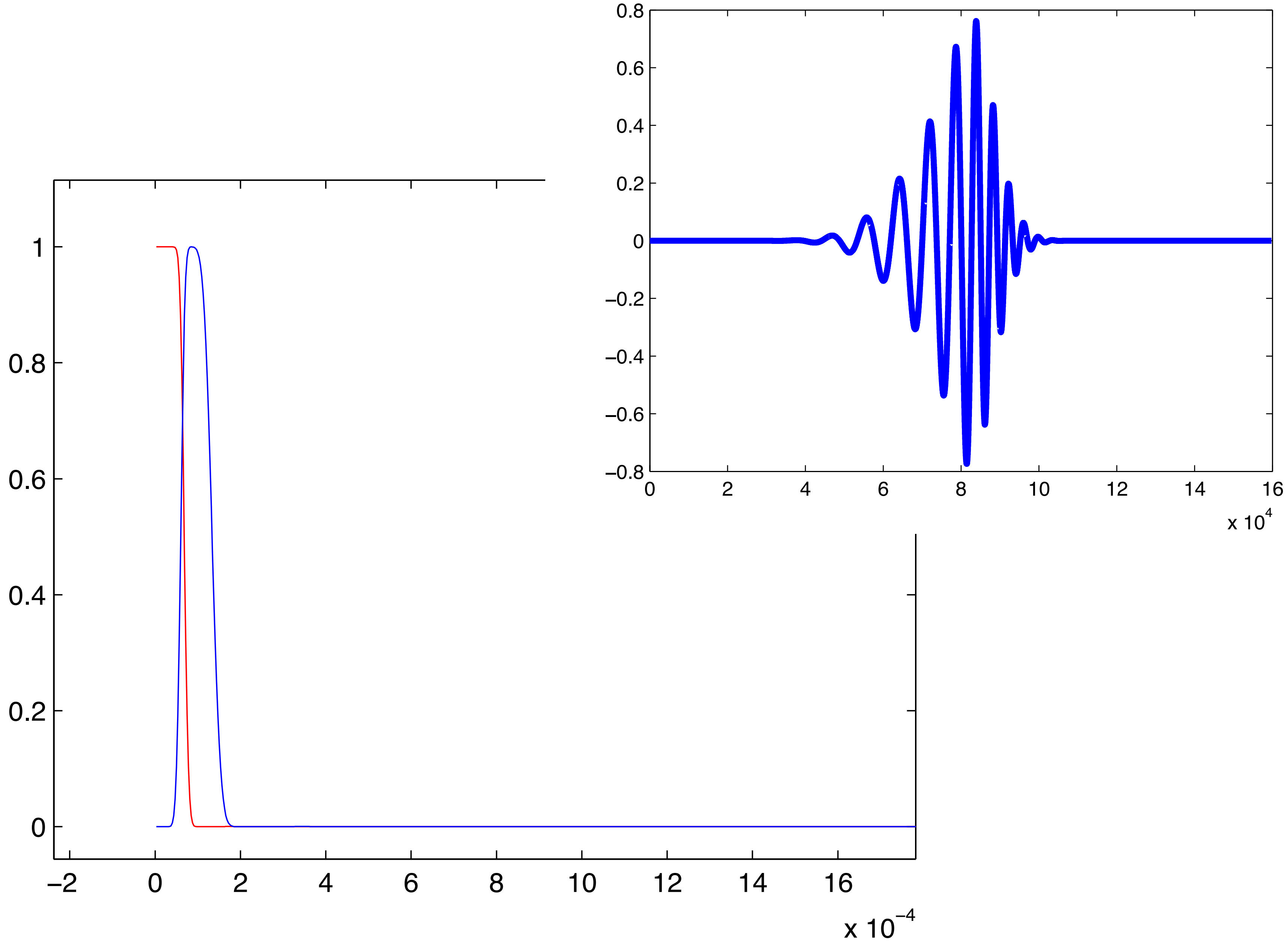


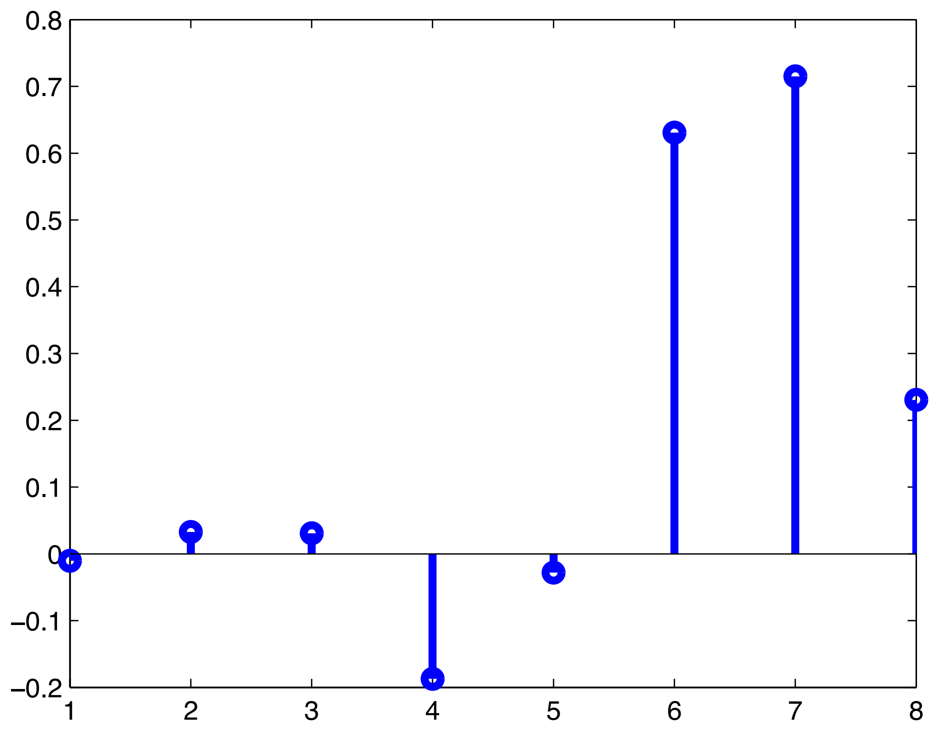
choice of wavelet transform & filters

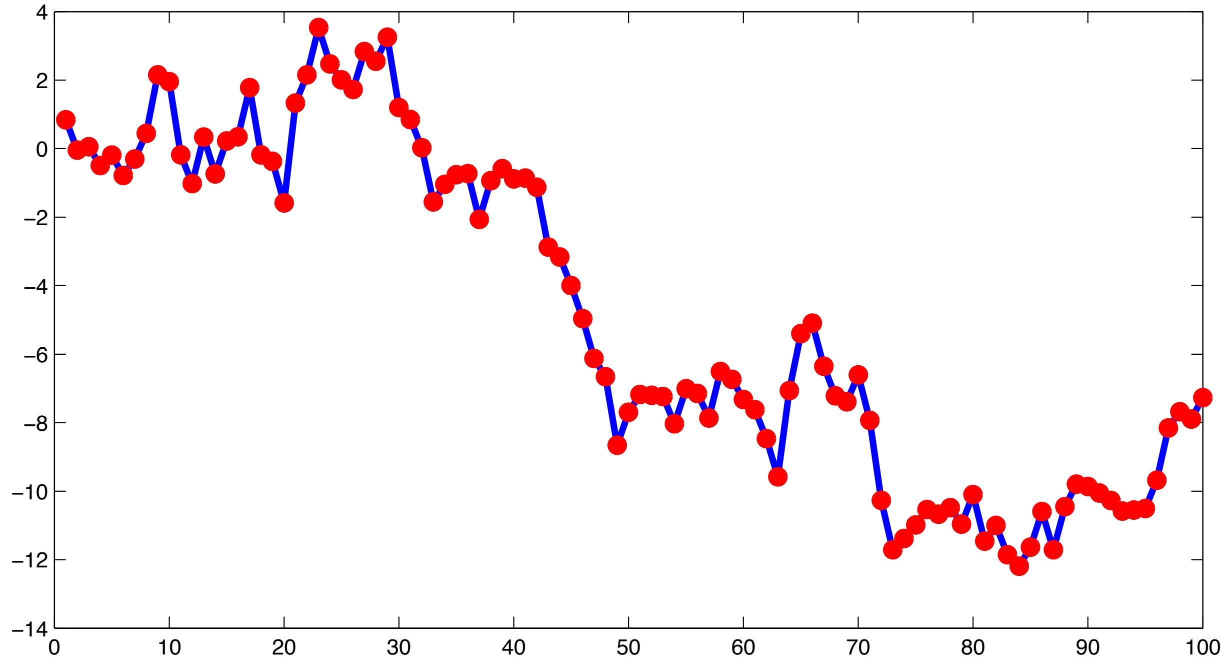
1. Self-consistent high/low pass filters [Quadrature Mirror Filters], full reconstruction -- can’t use continuous wavelets.
2. Time event preservation -- can’t use normal discrete wavelets.
3. Zero or linear phase (symmetric filters) -- for exact time localisation of events.
4. Sharp Fourier frequency resolution for studying spectra -- need higher order wavelets.
5. Time domain implementation with compact digital filters to minimise edge effects, increase time locality and reduce computational expense -- wavelet order can’t be too high; again can’t use continuous wavelets.

choice of wavelet transform & filters

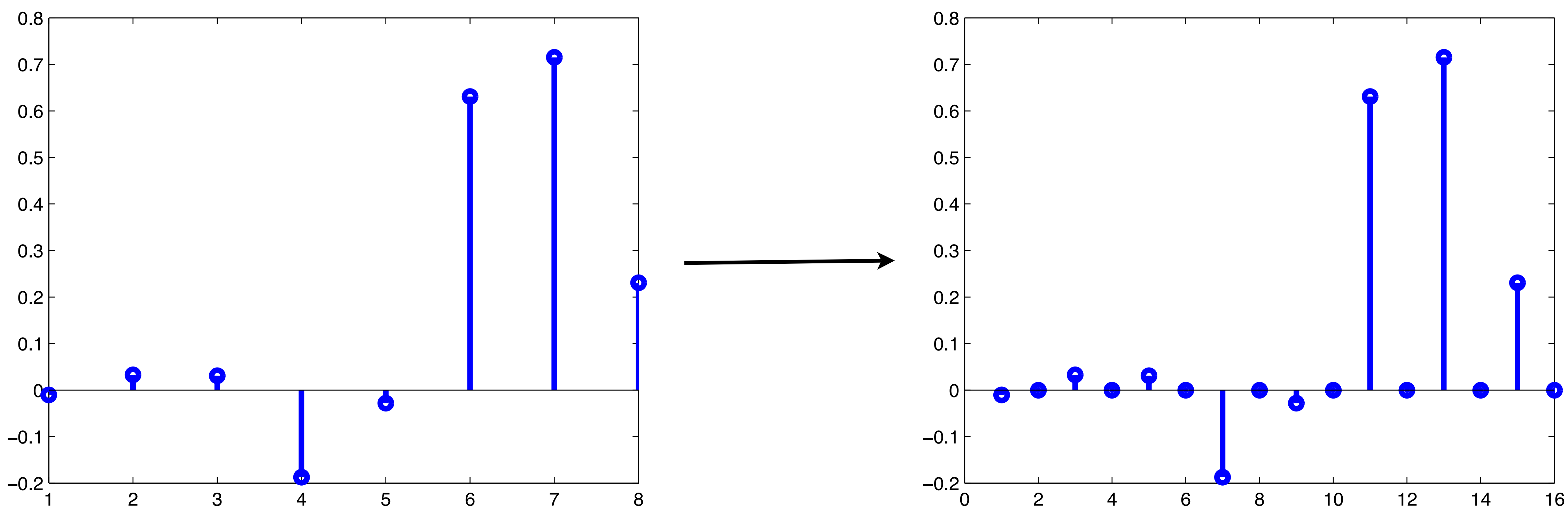
Solution: Undecimated discrete wavelet transform

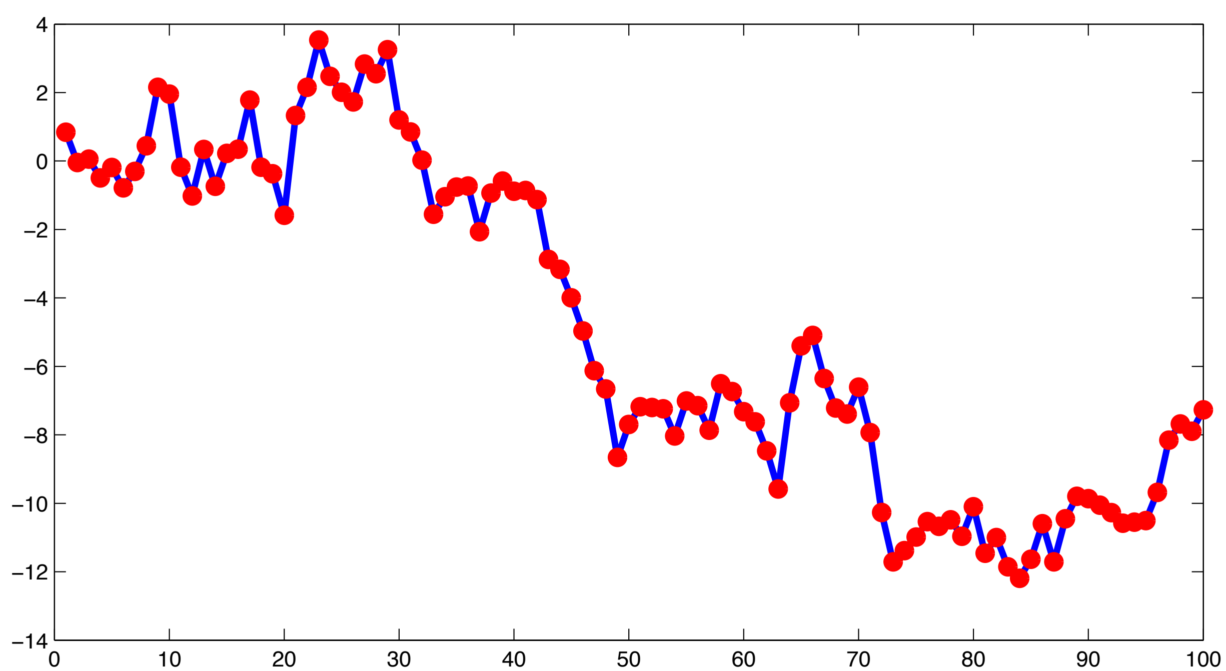
undecimated discrete wavelet transform (udwt)





undecimated discrete wavelet transform (udwt)





<u>UDWT steps:</u>

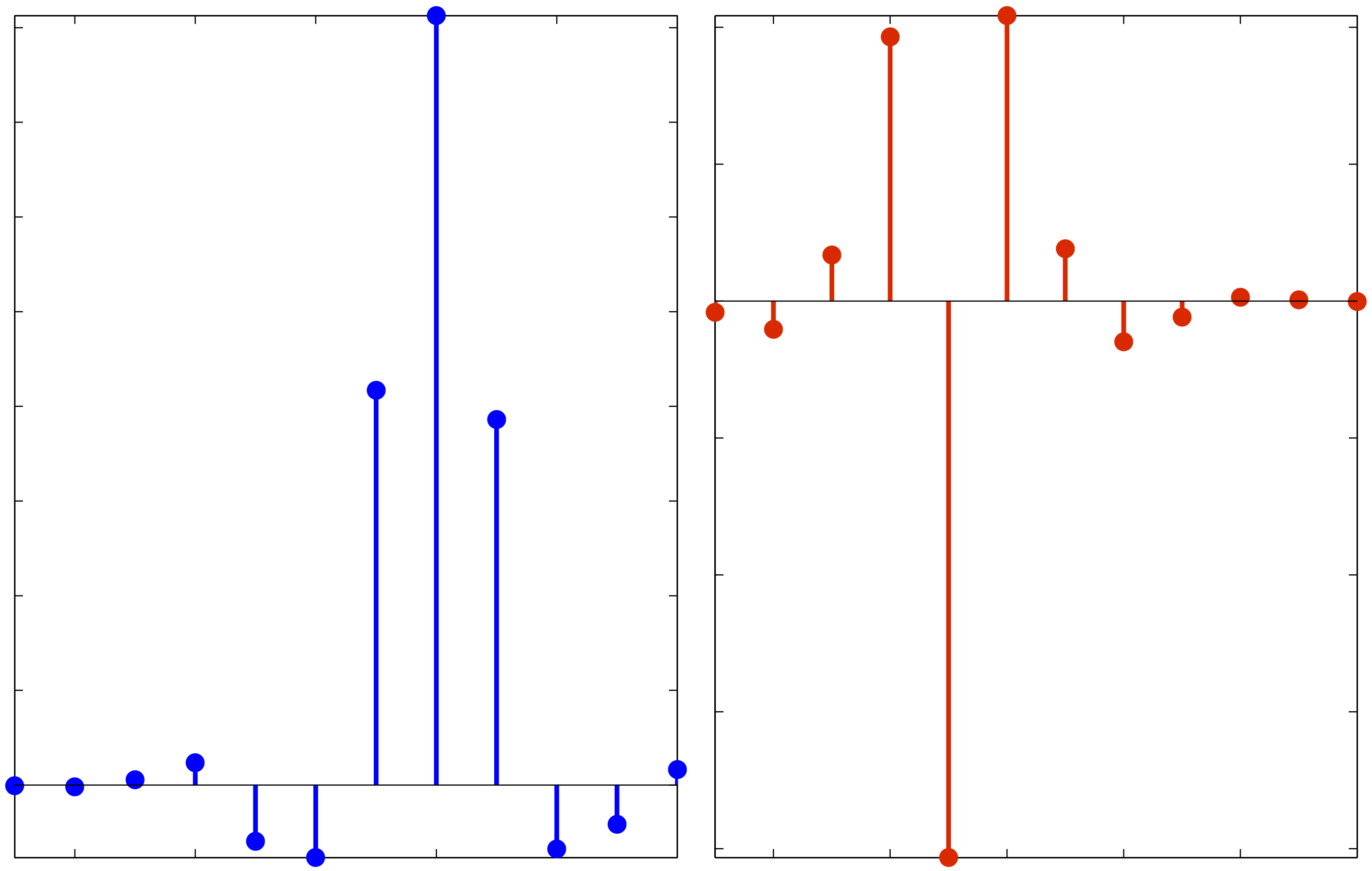
Leave signal as it is

Upsample filter by 2

choice of wavelet transform & filters

Solution: <u>Undecimated discrete wavelet transform</u>

scaling (low-pass) filter

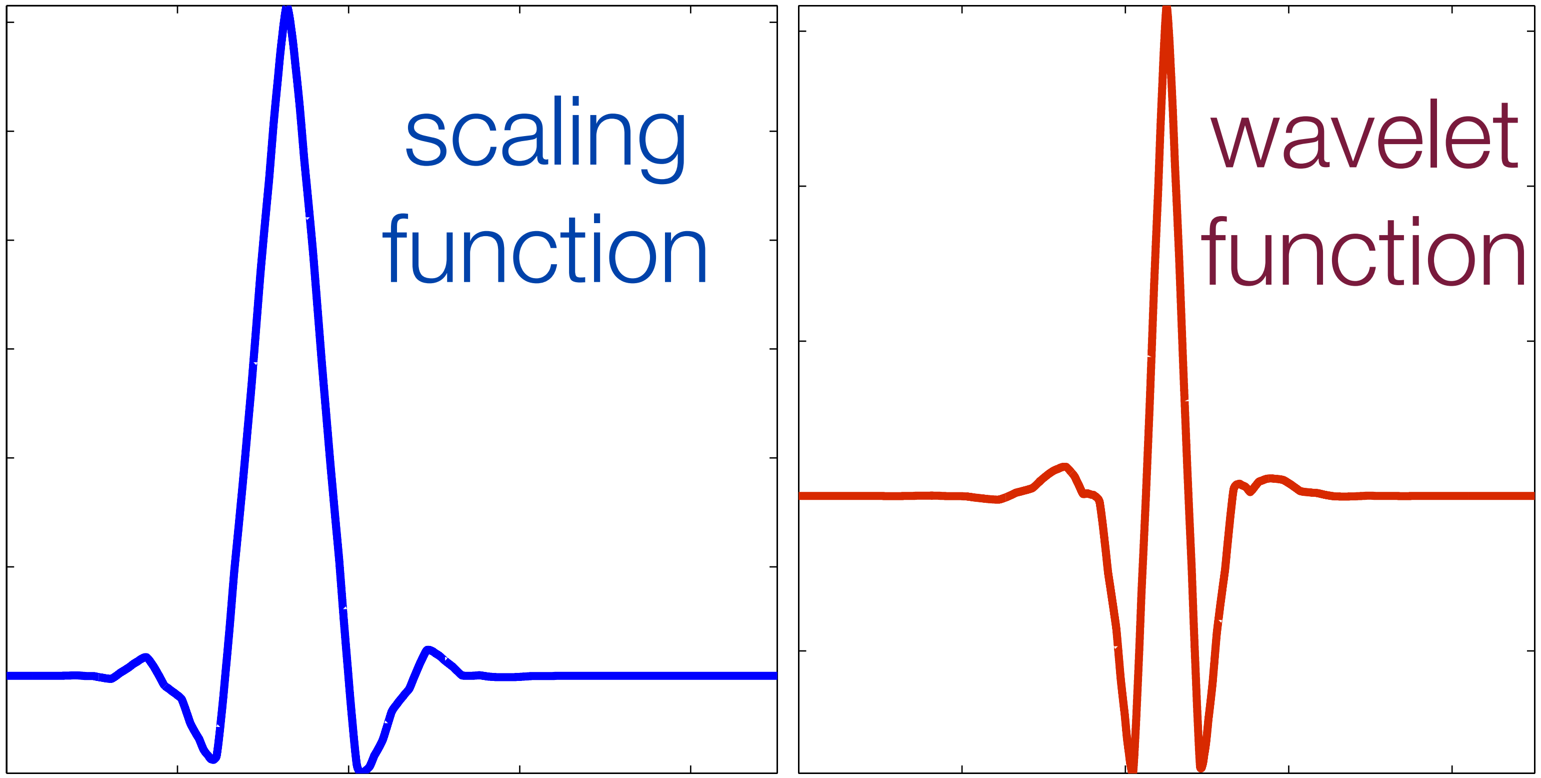


wavelet (high-pass) filter

[*Coiflet-2* wavelet and scaling (digital) time-domain filters]

choice of wavelet transform & filters

Solution: <u>Undecimated discrete wavelet transform</u>



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choice of wavelet transform & filters

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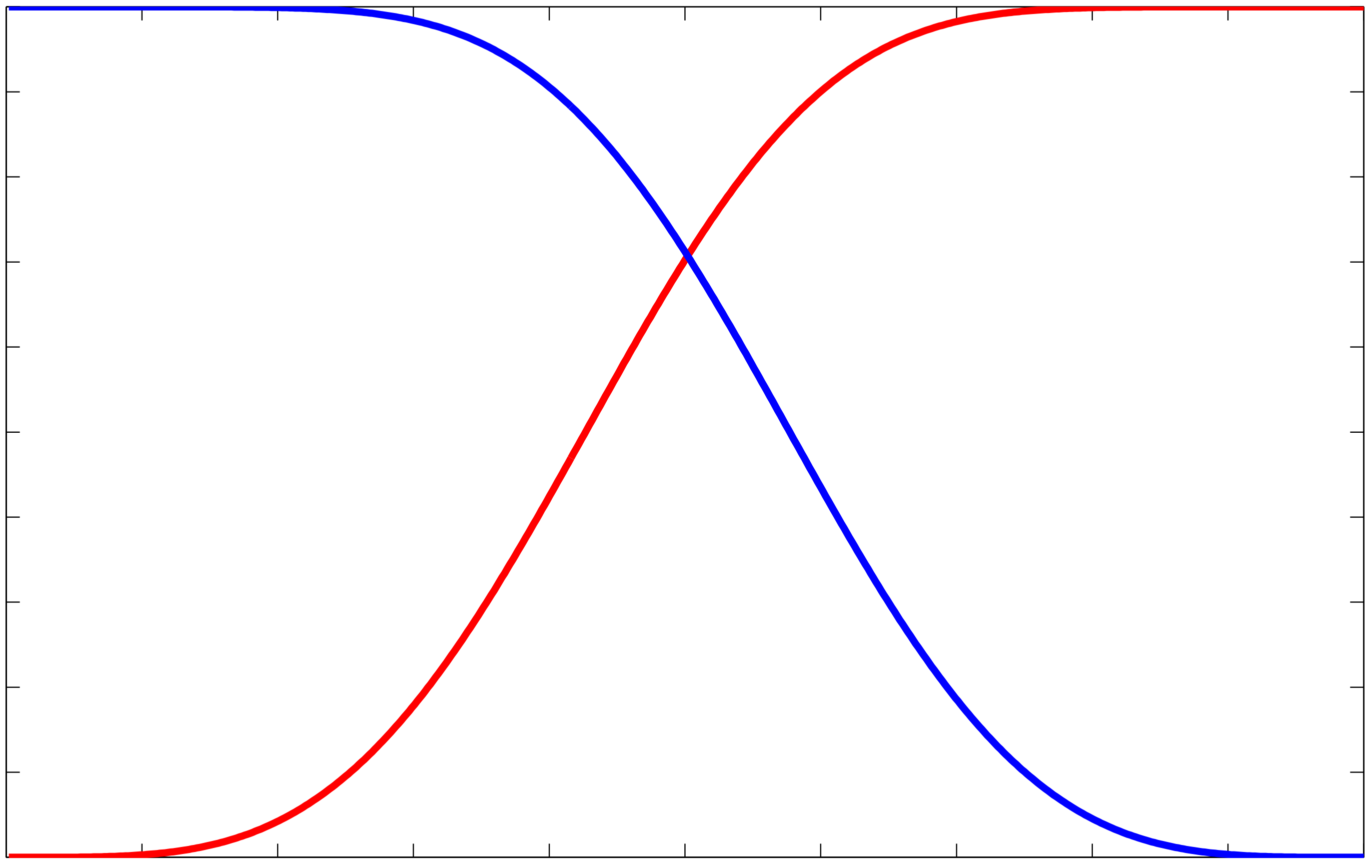

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choice of wavelet transform & filters

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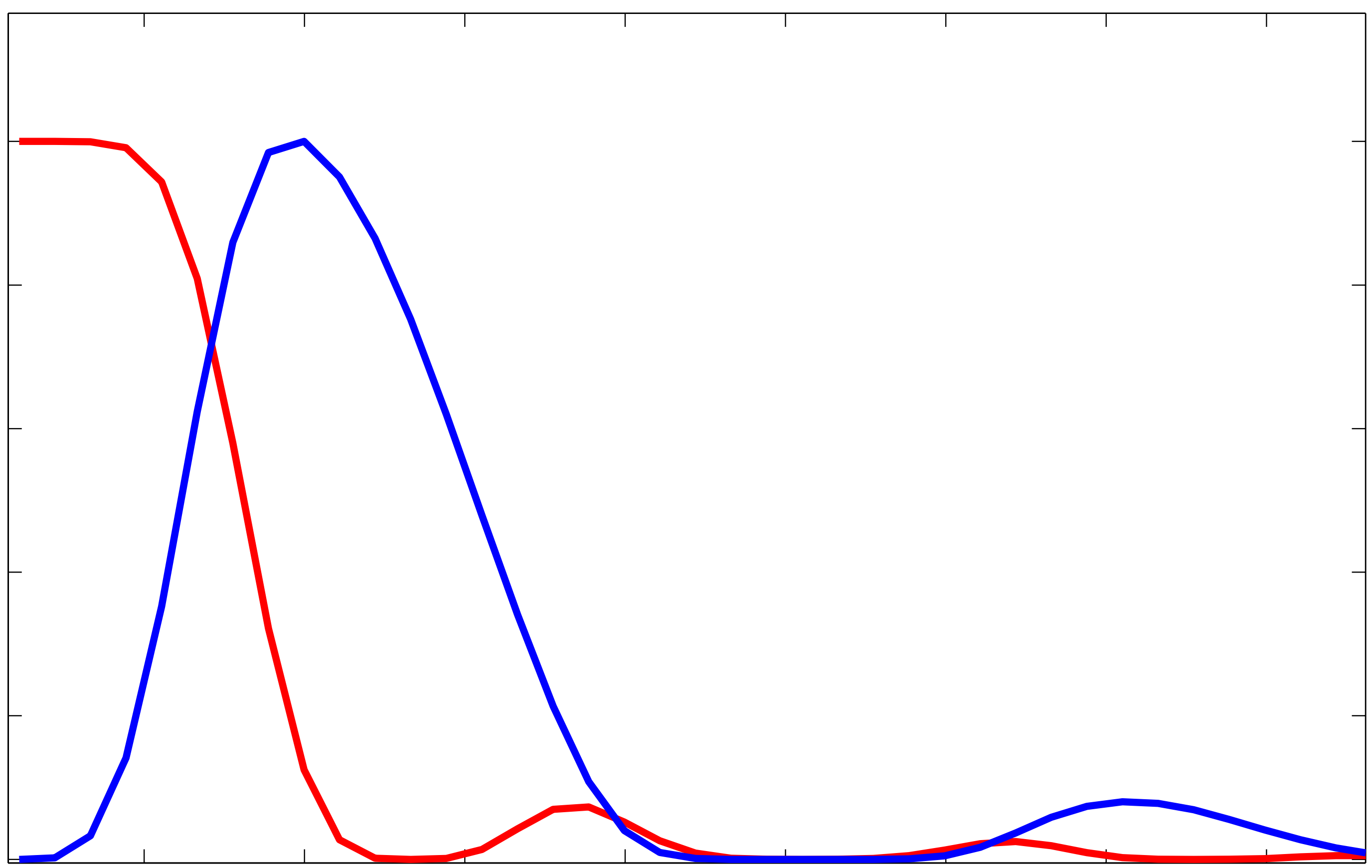

[*Coiflet-2* wavelet and scaling (digital) time-domain filters]

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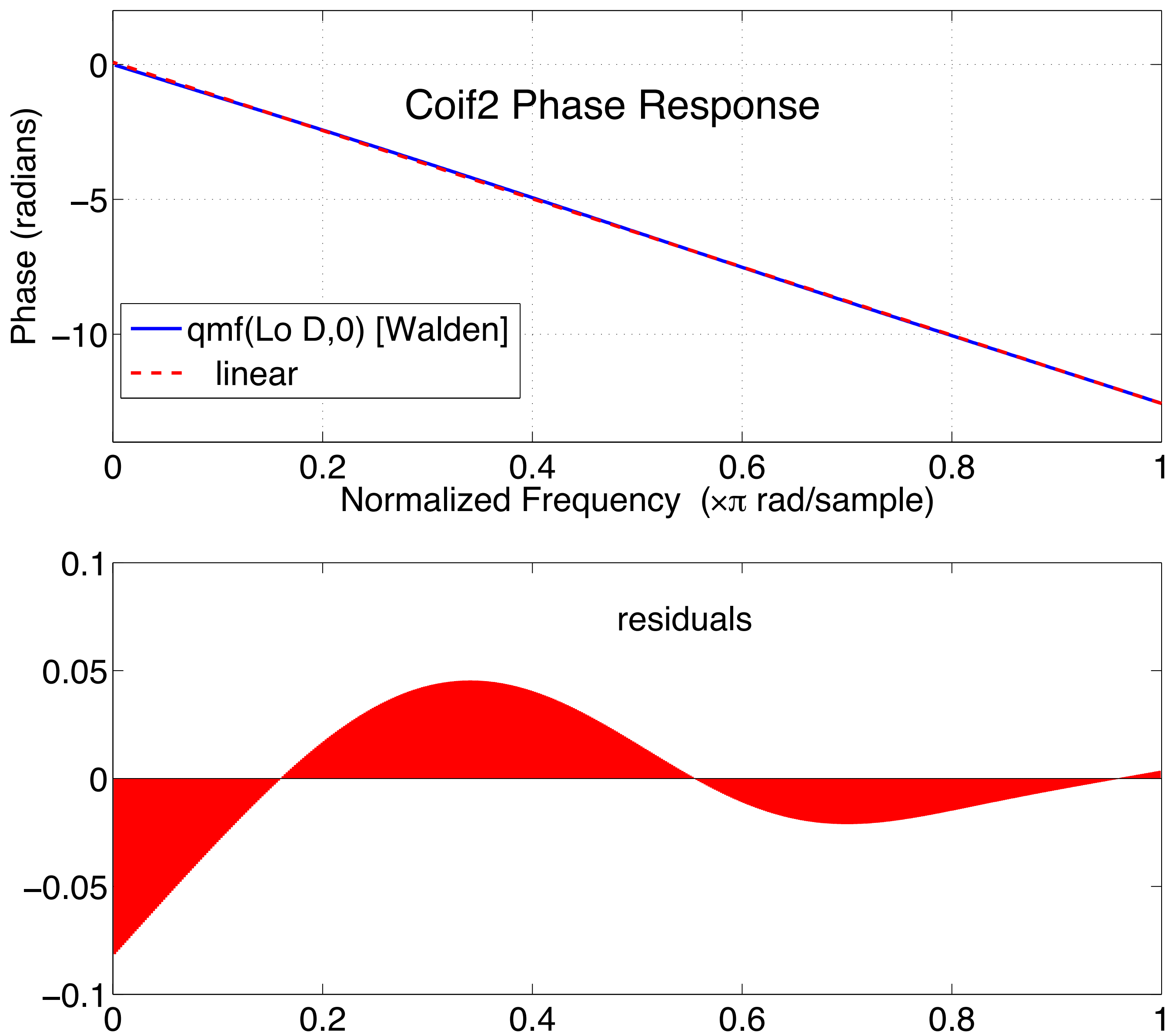
Solution: Undecimated discrete wavelet transform



choice of wavelet transform & filters

Solution: Undecimated discrete wavelet transform

Rice wavelet toolbox for Matlab (C mex files)

https://github.com/ricedsp/rwt

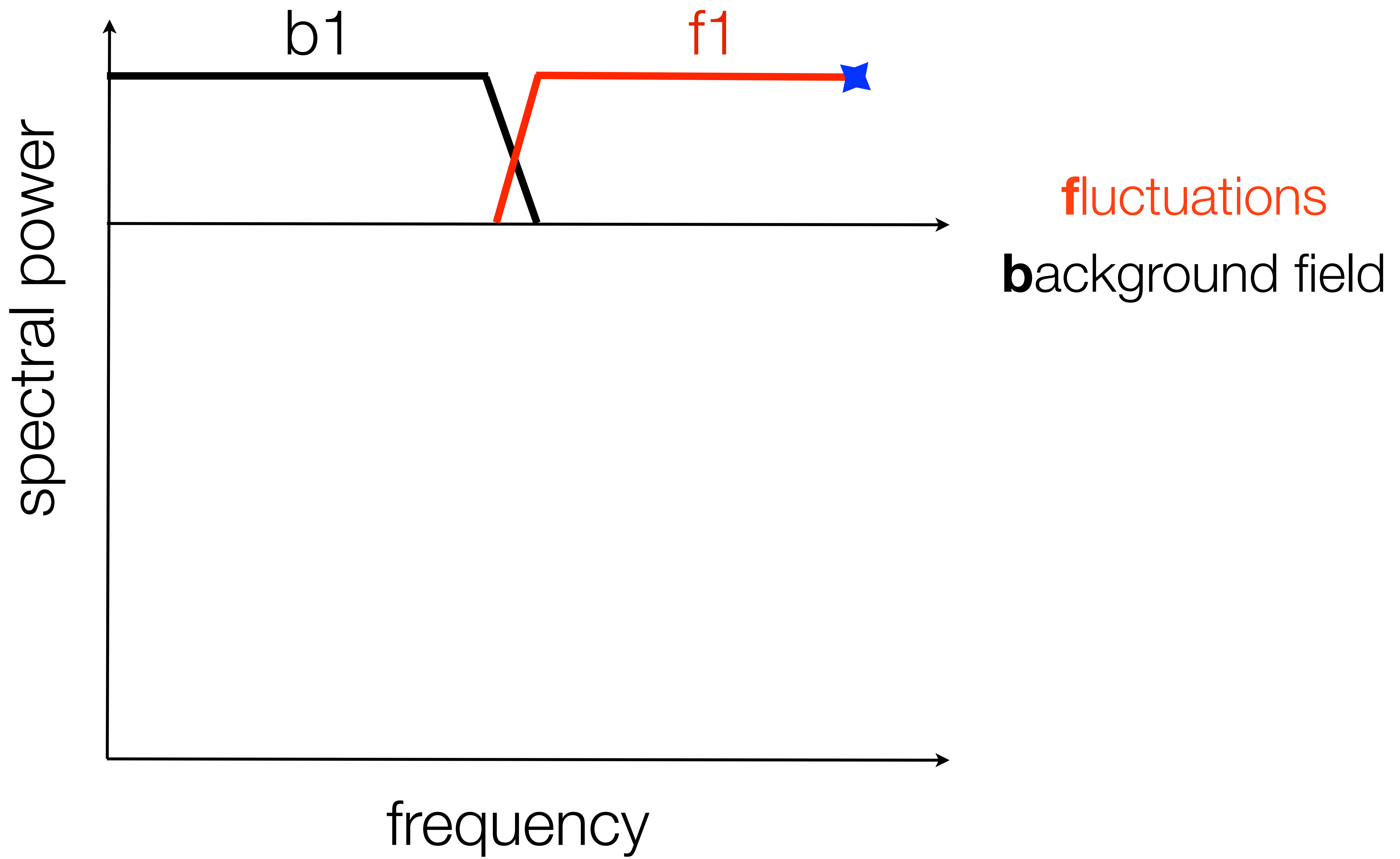
http://dsp.rice.edu/software/rice-wavelet-toolbox

PyWavelets (Cython)

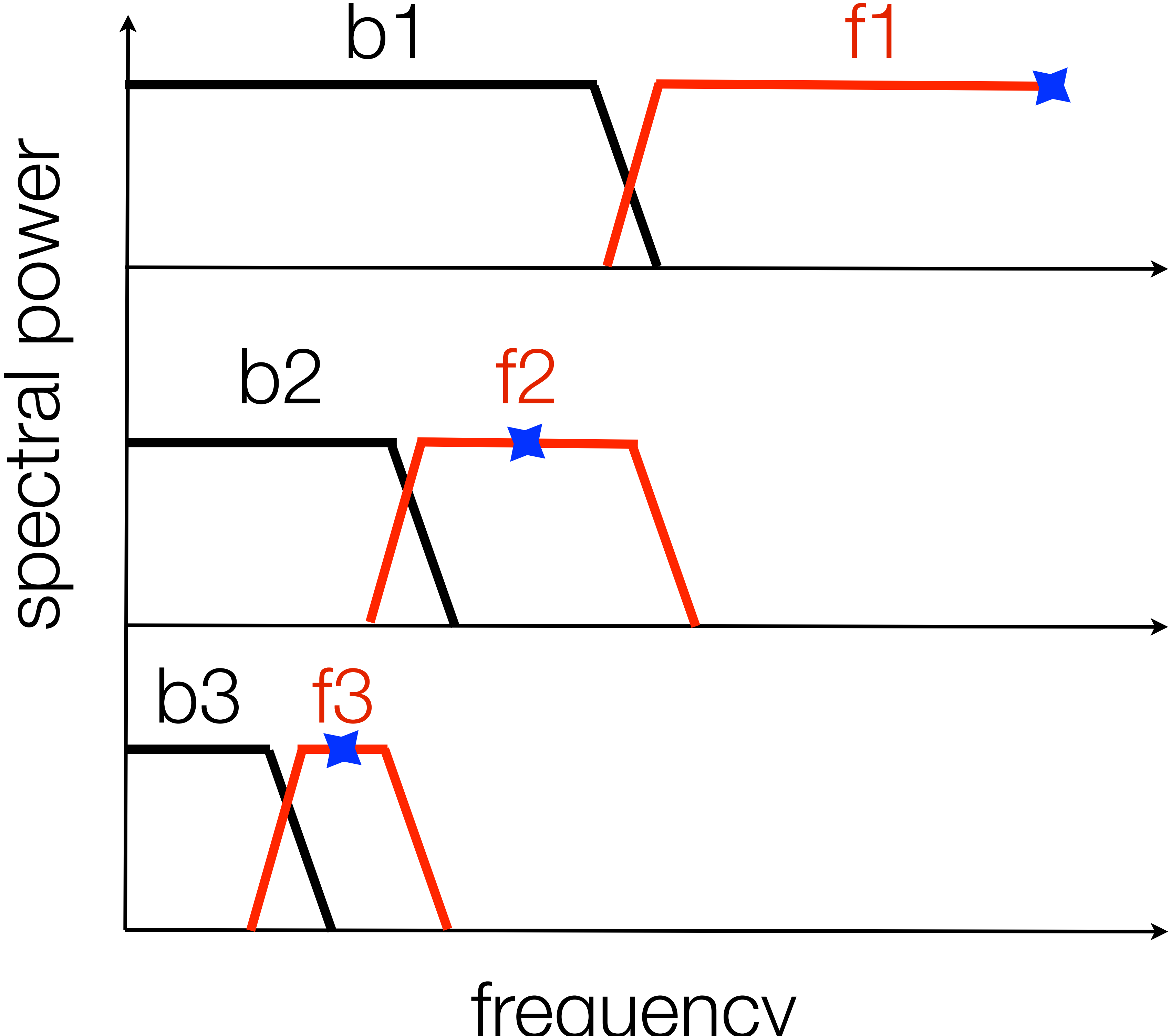
http://www.pybytes.com/pywavelets/

Anisotropic plasma turbulence applications

local scale-dependent fluctuations *and* background field using low/high pass filters



local scale-dependent fluctuations *and* background field using low/high pass filters

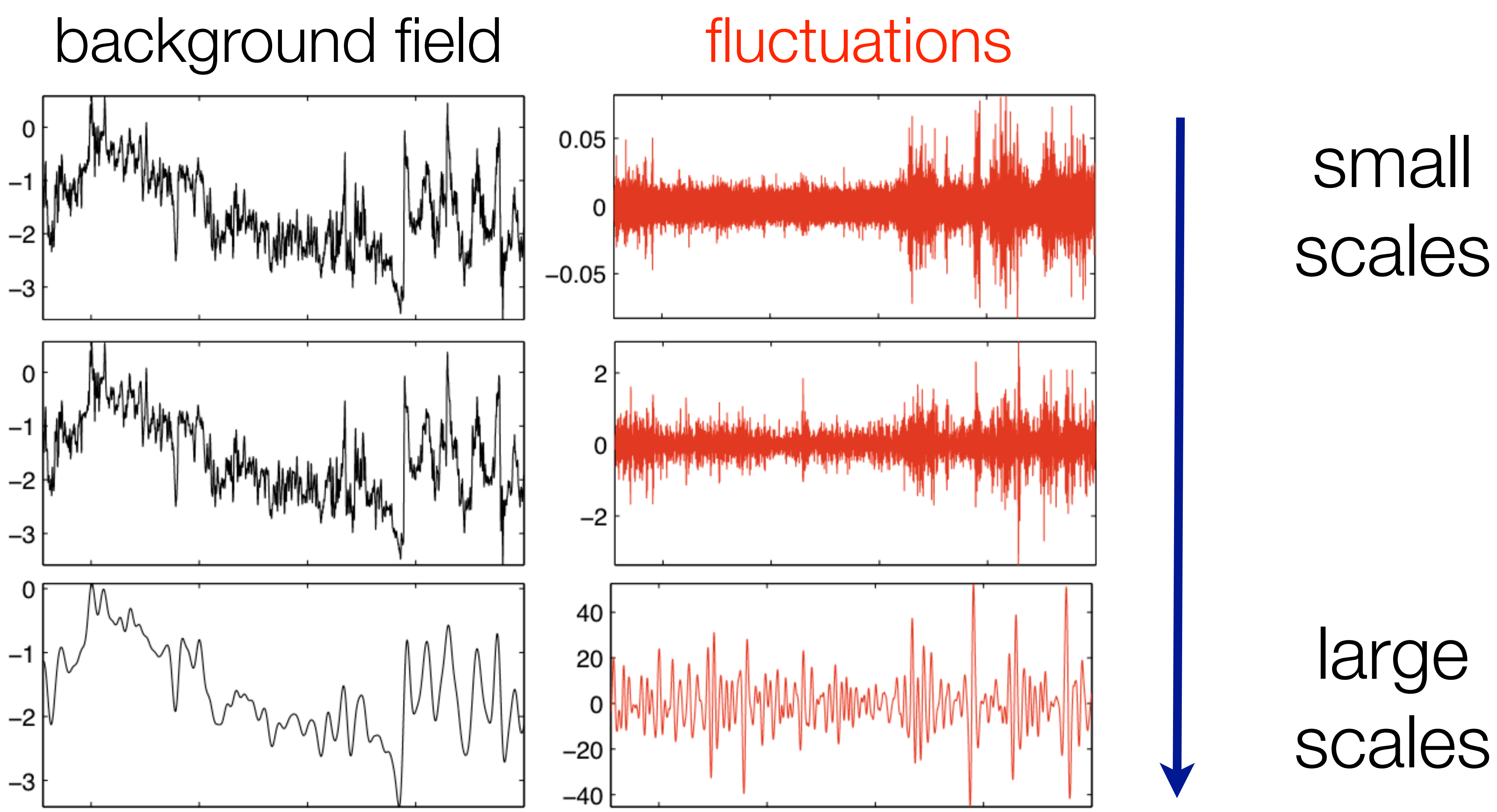


fluctuations

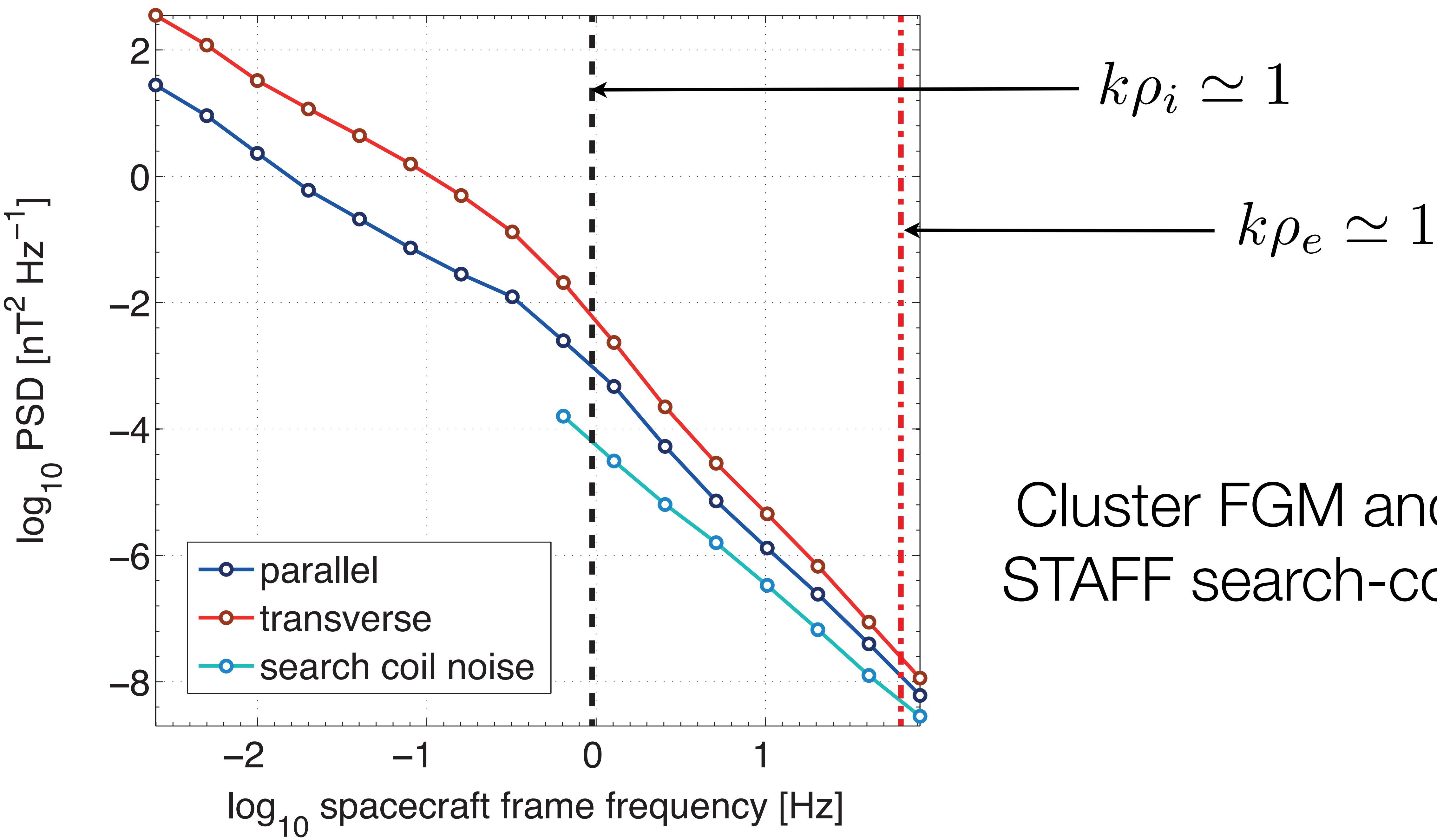
background field

filter bank implementation using quadrature mirror filters

adaptive local scale-dependent fluctuations **and** background guide field



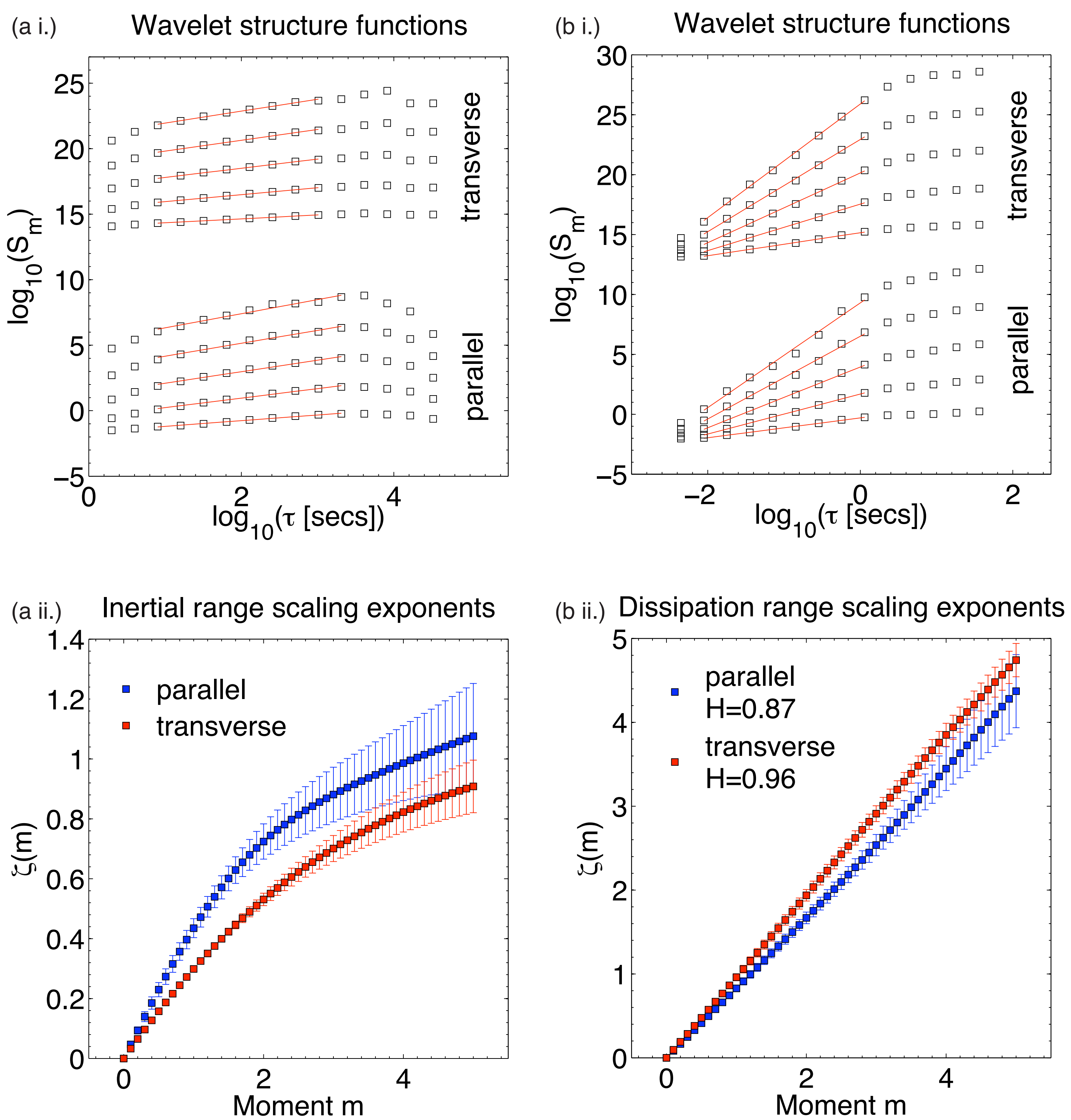
power spectral density



Cluster FGM and STAFF search-coil

Kiyani et al. ApJ (2013)

higher order scaling



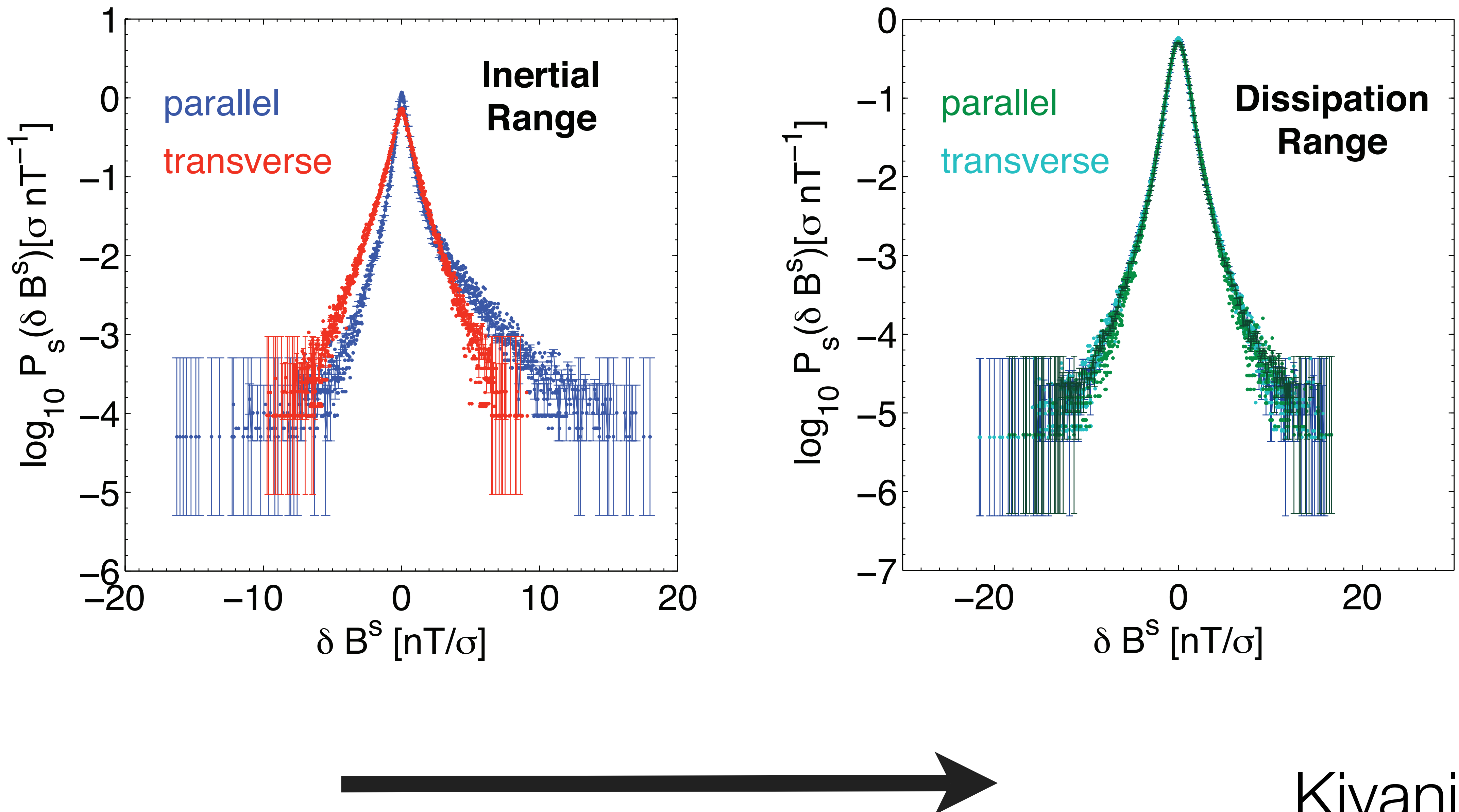
Structure Functions

$$S^{m}_{\parallel(\perp)}(\tau) = \frac{1}{N}\sum_{j=1}^{N}\left|\frac{\delta B_{\parallel(\perp)}(t_j,\tau)}{\sqrt{\tau}}\right|^{m}$$

note: Inertial range data is from ACE

Also see:
K. H. Kiyani et al.
Phys. Rev. Lett.
103, 075006 (2009)

standardised probability densities



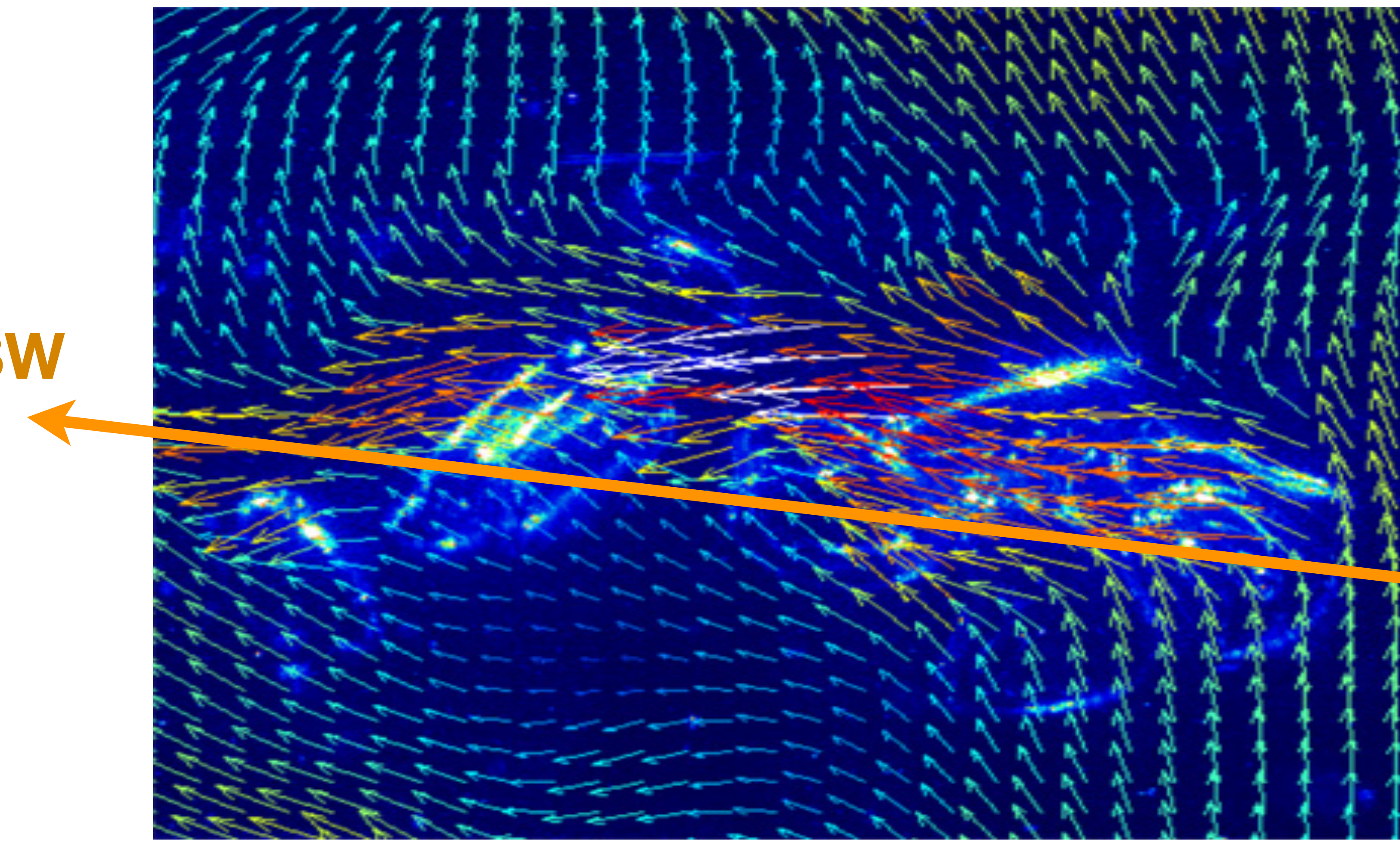
isotropy

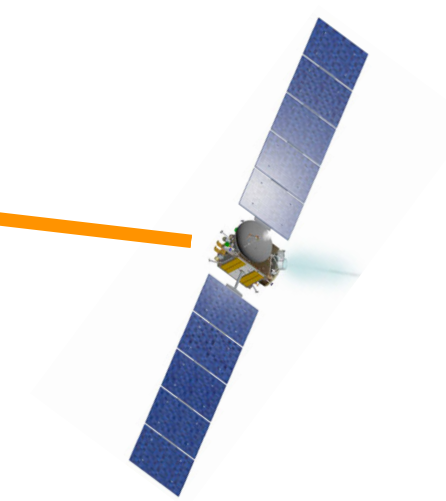
Kiyani et al.
ApJ (2013)

angles of measurement w.r.t. B

$$k = \frac{2\pi f_{sc}}{V_{SW}}$$

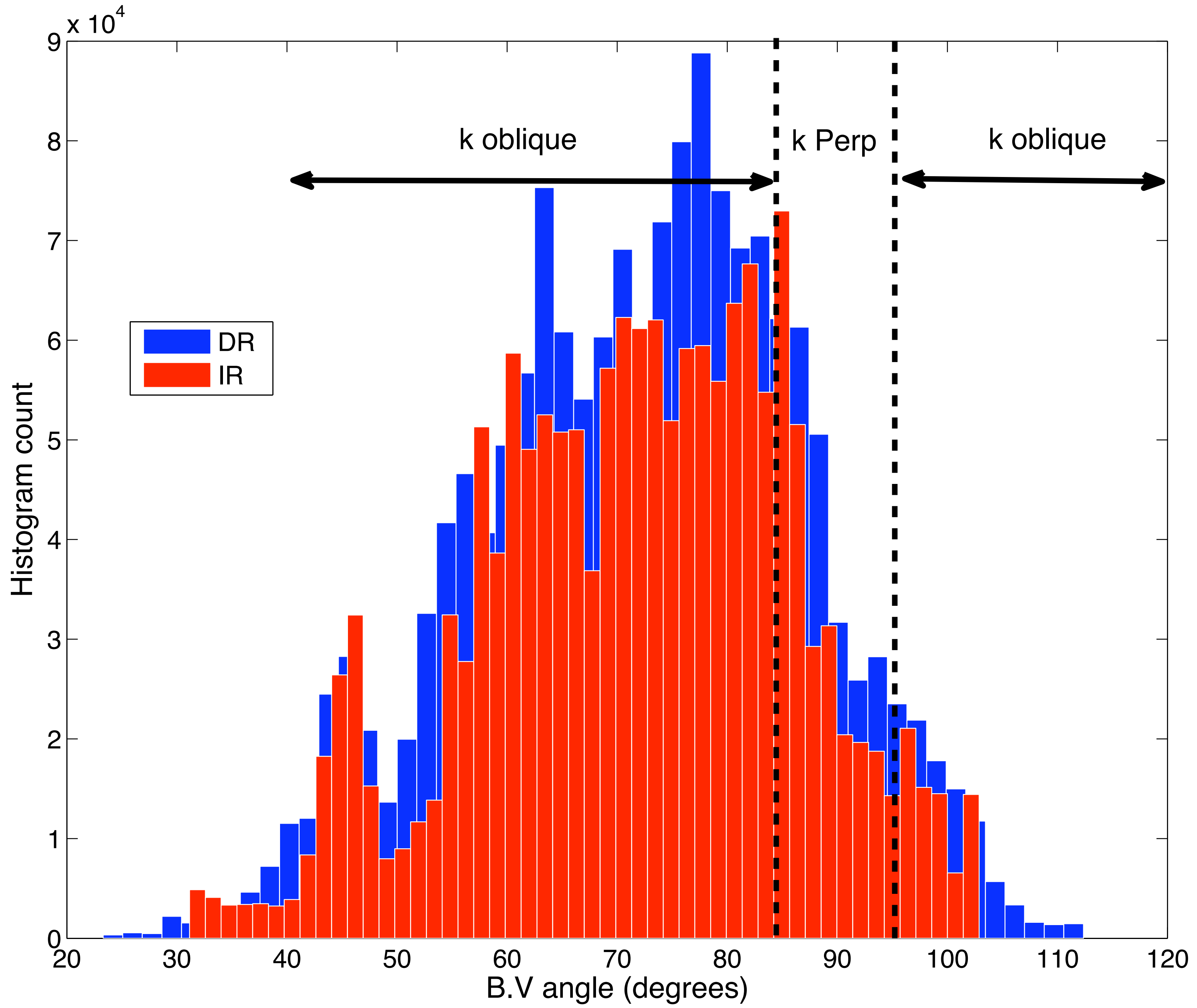
Taylor frozen-in approx,
for low-frequency
phenomena





MHD $\frac{V_A}{V_{SW}} < 1$ else $\frac{V_\phi}{V_{SW}} < 1$

Anisotropy (angle between B and V)



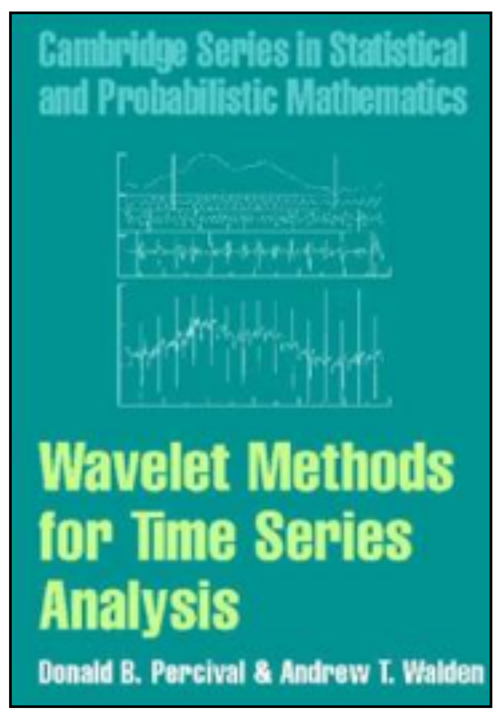
take home message

- Wavelets are excellent tools for studying turbulence and anisotropy
- They are especially useful for both simultaneous event localisation as well as frequency localisation
- Tried extending this to m-band wavelet packets to get better frequency resolution; but generally this is not possible. Recent work on over/undersampling is more promising.

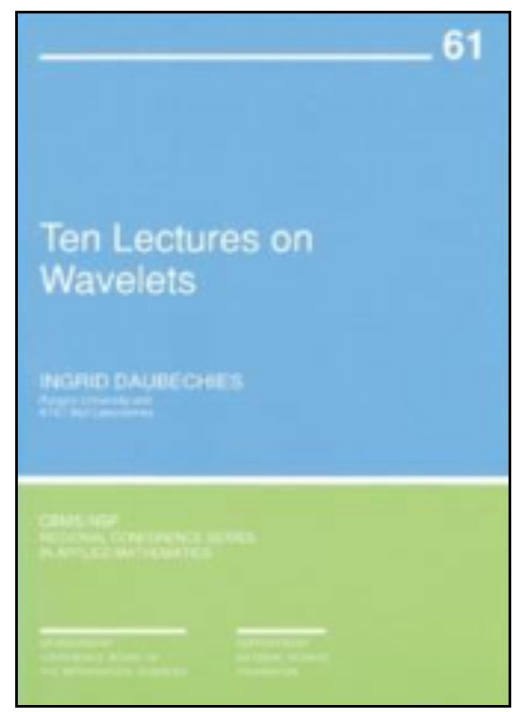
guides and literature

- M. Farge et al., *Wavelets and Turbulence*, Proc. IEEE, **84**(4), 639, (1996)
- A. Walden & A. Cristan, Proc. R. Soc. Lond. A, **454**, 2243 (1998)
- K. Kiyani et al, ApJ, (2013)

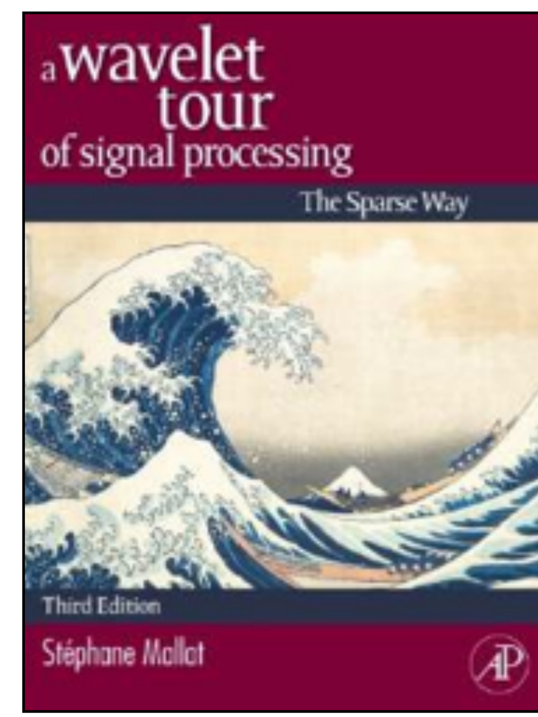
Essential Wavelets for Statistical Applications and Data Analysis
T. Ogden



Wavelet Methods for Time Series Analaysis
D. Percival & A. Walden



Ten Lectures on Wavelets
I. Daubechies



A Wavelet Tour of Signal Processing
S. Mallat