Part two: Kai Schneider

A review on wavelet transforms and their applications to MHD and plasma turbulence II

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In collaboration with
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Methods for Analyzing Turbulence Data

Meudon, 29 May 2015

Fast visible light camera

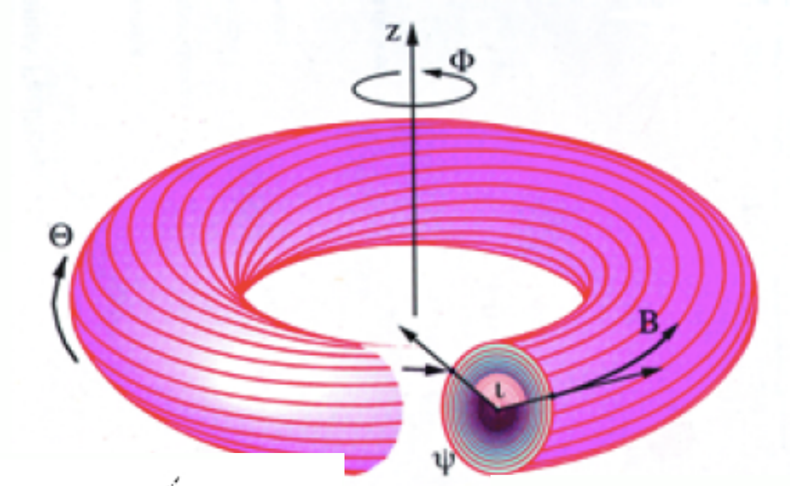
A fast camera from the Nancy team *(G. Bonhomme and F. Brochard)*
was installed on Tore-Supra *(N. Fedorczak and P. Monier-Garbet)*.

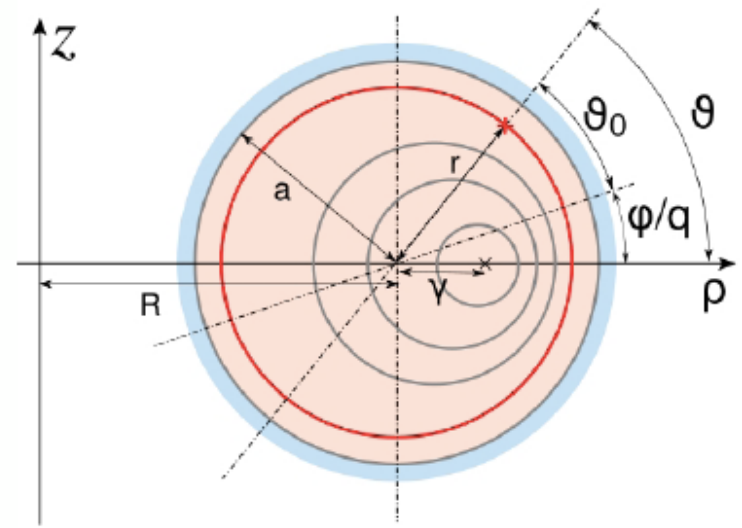
An helical Abel transform relates the plasma light emissivity S
to the integral of the volume emissivity received by the camera I=KS,
where K is a compact continuous operator.
Reconstruction of S from I is an inverse problem
which becomes very difficult
when S is corrupted by noise,
then solving K^{-1} is an ill-posed problem.

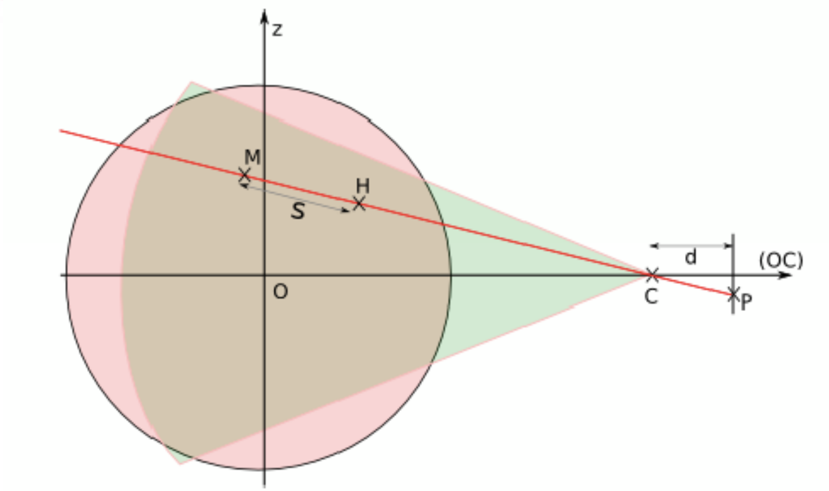
$\Longrightarrow$

Tomographic inversion
using wavelet-vaguelette decomposition
as an alternative to SVD
(Singular Value Decomposition).

Image tomography





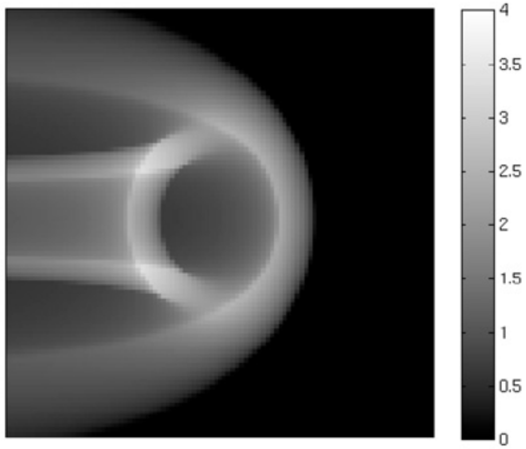


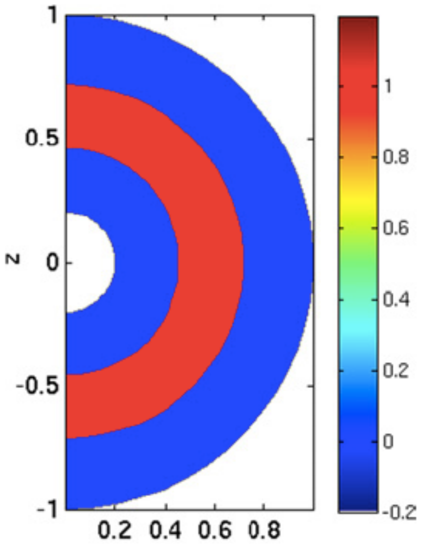
$$I_0(x, y) = \int_{s_C}^{\infty} S_0(\Psi(s), \theta(s), \varphi(s))\, ds$$

Tomography inversion in presence of noise

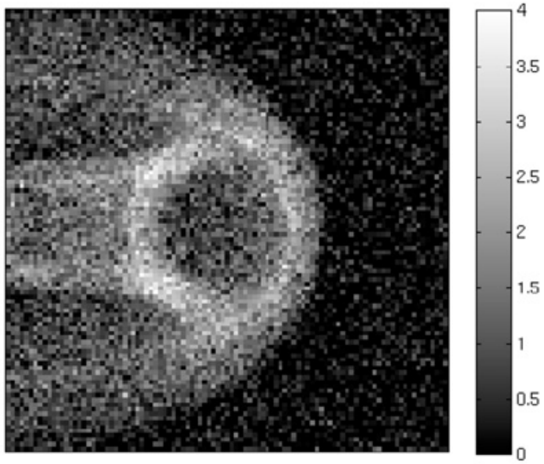
Image received by the camera: integral of the volume emissivity I=KS

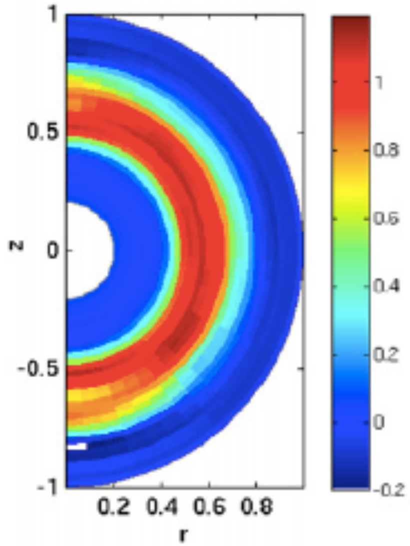
Plasma light emissivity S



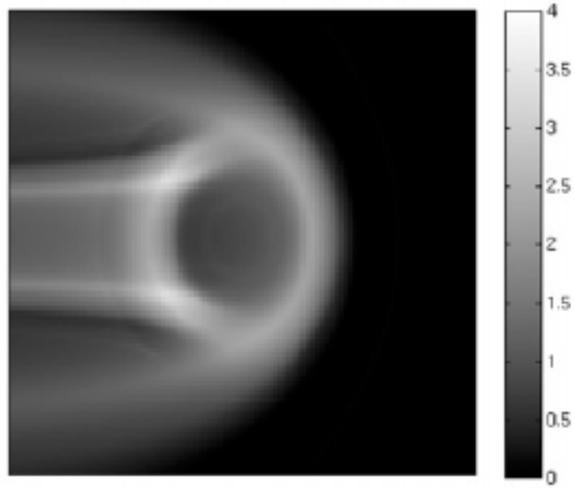


*Nguyen, Fedorczak, Brochard, Bonhomne, Schneider, Farge, Monier-Garbet, Nuclear Fusion, **52**, 2012*

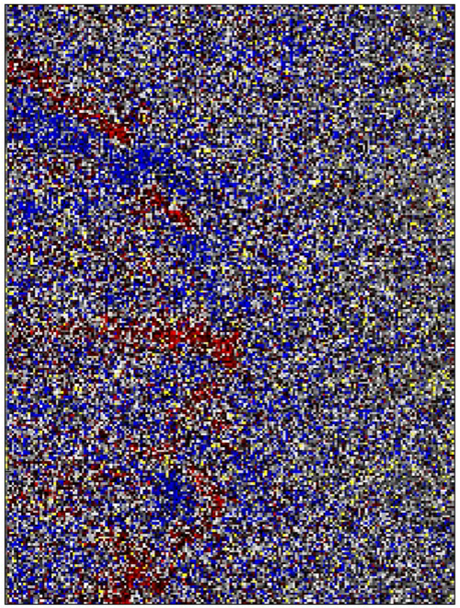


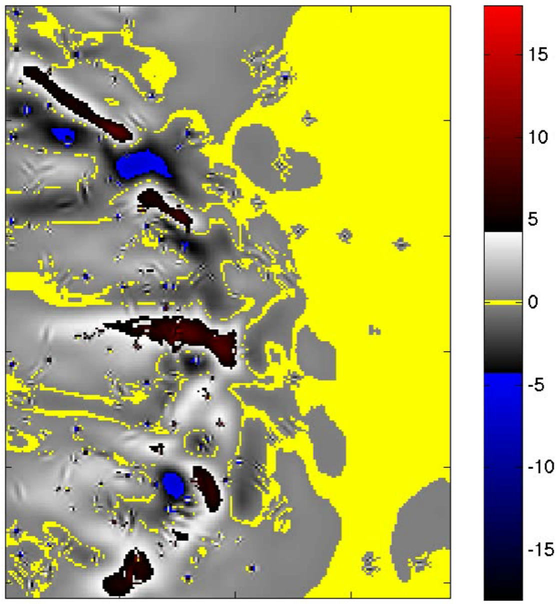


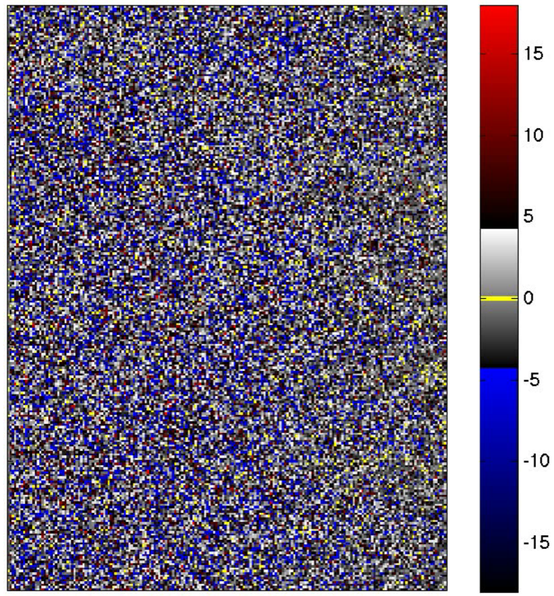
Denoised plasma emissivity



Movie from a fast camera in Tore-Supra tokamak







Noisy images

Coherent structures

Incoherent background

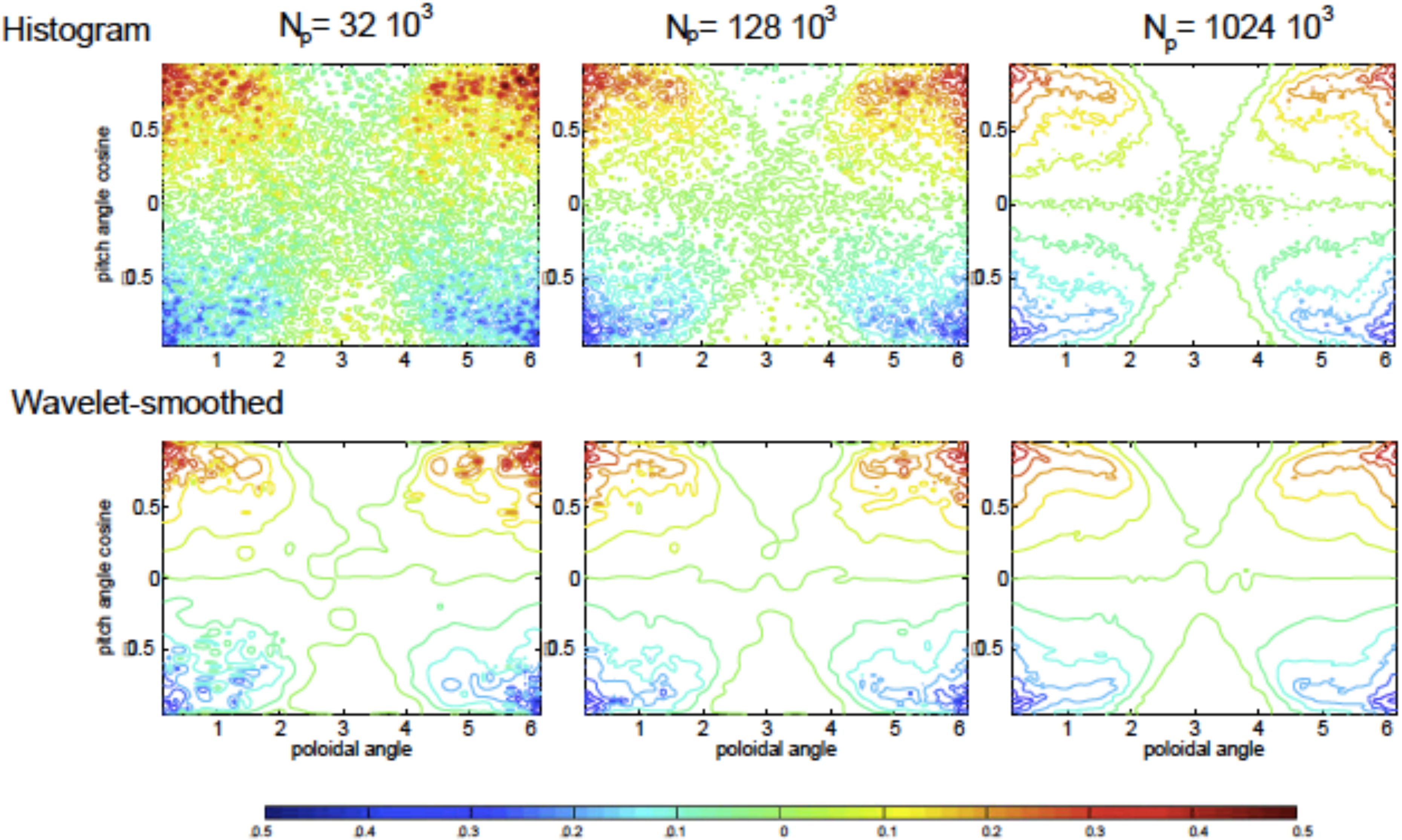
*Nguyen, Fedorczak, Brochard, Bonhomne, Schneider, Farge, Monier-Garbet, Nuclear Fusion, **52**, 2012*

Noise reduction in plasma simulations using particles

- Accuracy of particle simulations is limited by noise (statistical sampling, not enough particles and grid effects)
- Wavelet based density estimation, accurate estimation of distribution functions with localized sharp features
- Preservation of moments in the distribution functions
- No a priori selection of a global smoothing scale
- No constraints on the dimensionality
- Computationally efficient: same order as for finite size particle approach

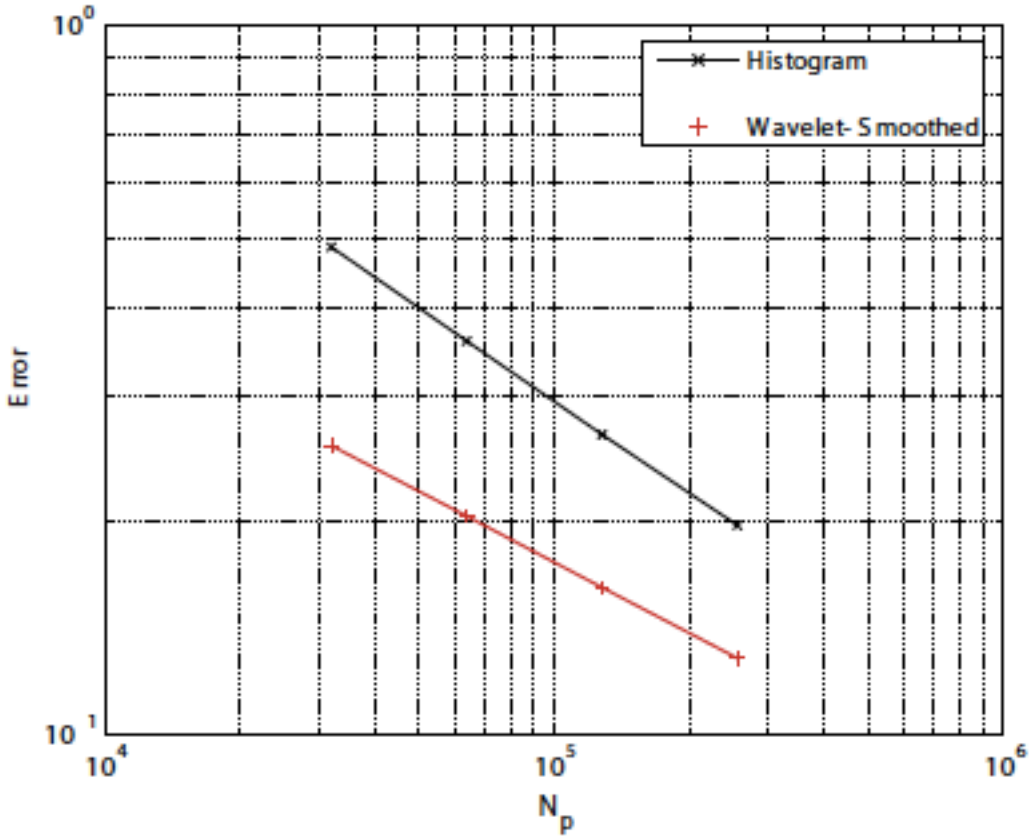
Nguyen van yen, del-Castillo-Negrete, Schneider, Farge and Chen, J. Comput. Phys., ***229****, 2010*

Collisional guiding center transport data (Delta5d)



Noise reduction in plasma simulations using particles

Collisional guiding center transport data (Delta5d)



RMS error estimate with respect to the reference density computed with $N_p = 1024 \times 10^3$.

Error reduction by about a factor 2.

Nguyen van yen,
del-Castillo-Negrete,
Schneider, Farge
and Chen,
J. Comput. Phys.,
***229**, 2010*

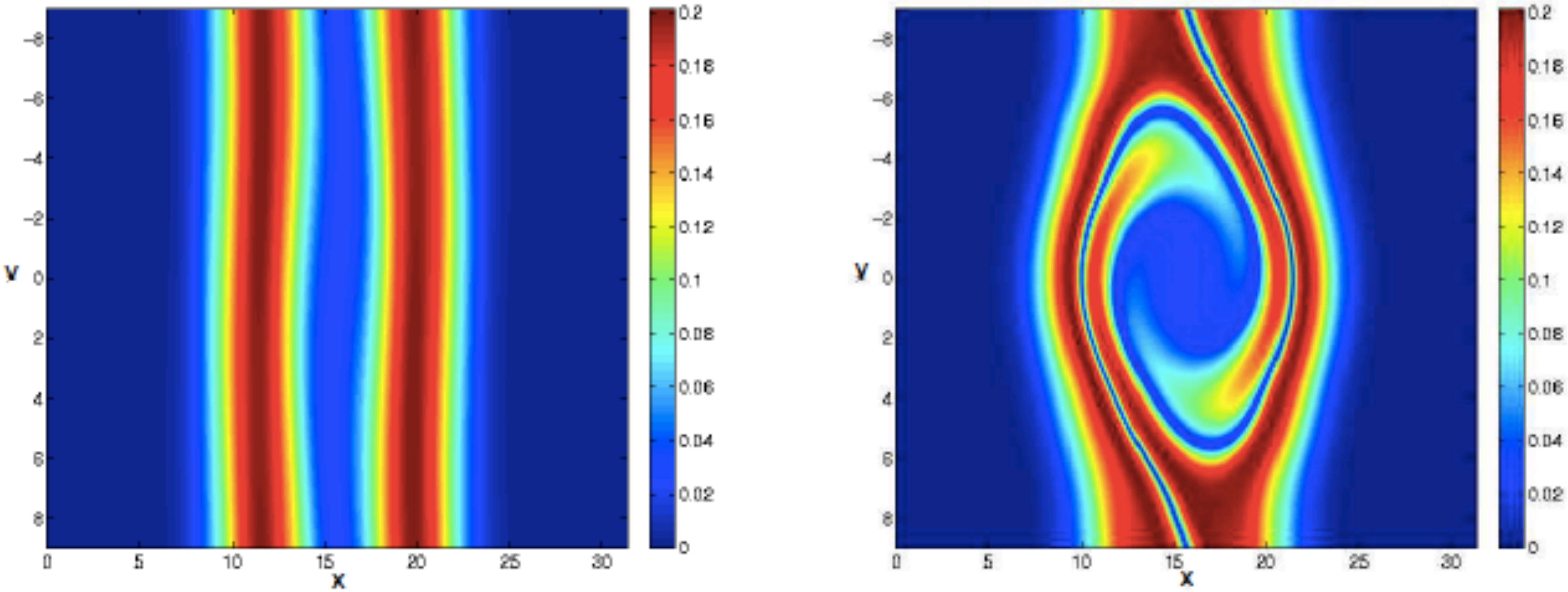
Particle in wavelets scheme for Vlasov-Poisson equation

- Plasma distribution function is discretized using tracer particles
- The charge distribution is reconstructed using wavelet based density estimation
- Wavelet expansion of the Dirac delta functions corresponding to each particle
- Wavelet Galerkin Poisson solver to compute the electric potential from the electron charge density (diagonal preconditioning)
- Improvement of precision compared to a classical PIC scheme for a given number of particles

Nguyen van yen, Sonnendrücker,
Schneider and Farge,
ESAIM Proc., ***32***, *2011*

Particle in wavelets scheme for Vlasov-Poisson equation

Two-stream instability test case

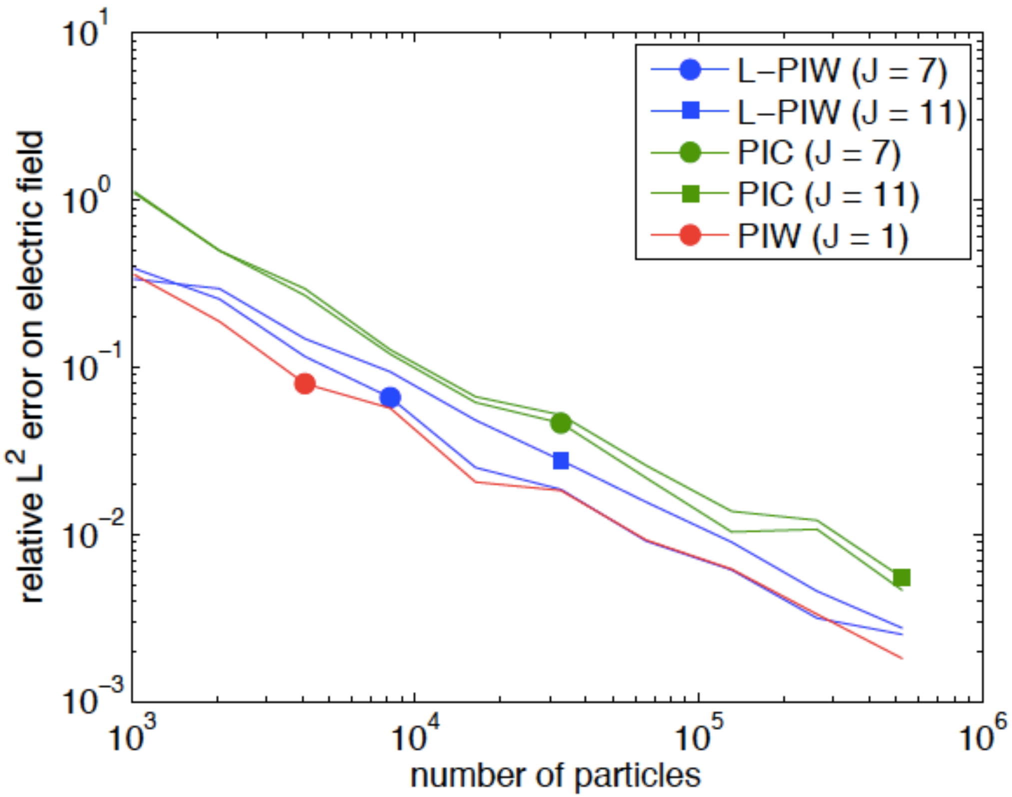


Particle distribution function at t=10 (left) and t=30 (right).

*Nguyen van yen, Sonnendrücker, Schneider and Farge, ESAIM Proc., **32**, 2011*

Particle in wavelets scheme for Vlasov-Poisson equation

L^2 error on the electric field at $t = 30$,
as a function of number of particles



Nguyen van yen, Sonnendrücker,
Schneider and Farge,
ESAIM Proc., ***32****, 2011*

Papers on applications to tokamaks
http://wavelets.ens.fr

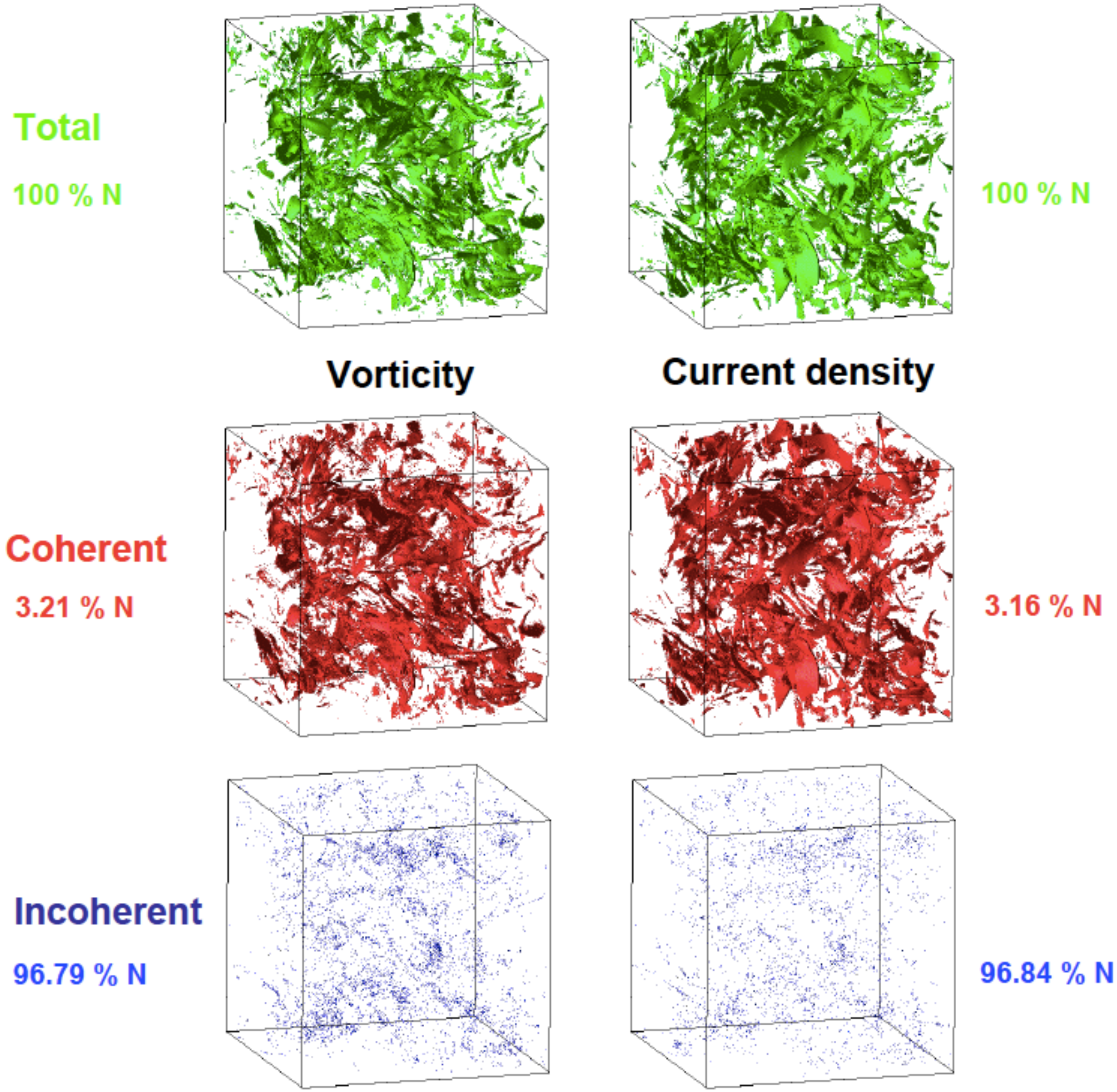
Marie Farge, Kai Schneider and Pascal Devynck, 2006
Extraction of coherent bursts in turbulent edge plasma using orthogonal wavelets
*Physics of Plasmas, **13**(2), 042304, 1-11*

Romain Nguyen van yen, Diego del Castilo–Negrete, Kai Schneider, Marie Farge and Guangye Chen, 2010
Wavelet–based density estimation for noise reduction in plasma simulation using particles
*J. Comput. Phys., **229**(8), 2821-2839*

Romain Nguyen van yen, Eric Sonnendrücker, Kai Schneider and Marie Farge, 2011
Particle-in-wavelet scheme for the 1D Vlasov-Poisson equations
*ESAIM Proc., **32**, 134-148*

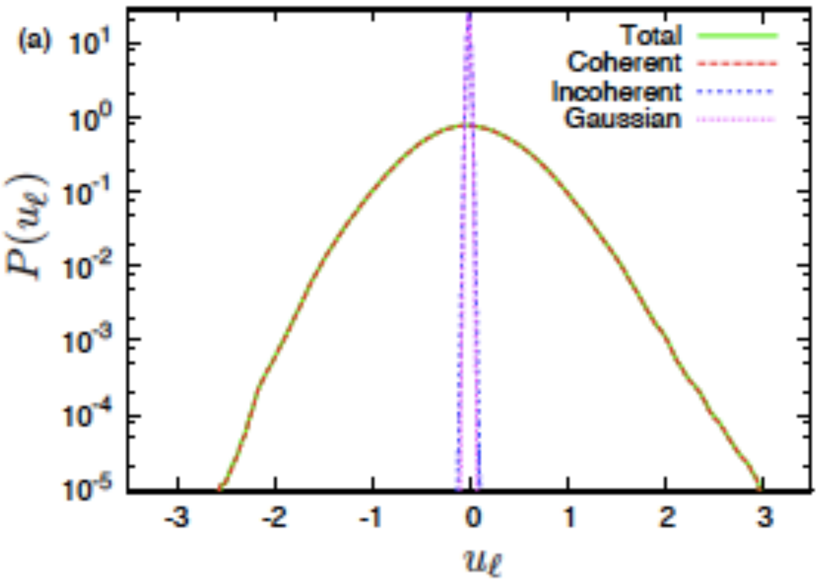
Romain Nguyen van yen, Nicolas Fedorczak, Frédéric Brochard, Kai Schneider, Marie Farge and Pascale Monier-Garbet, 2012
Tomographic reconstruction of tokamak edge turbulence light emission from single image using wavelet-vaguelette decomposition
*Nuclear Fusion, IAEA (International Atomic Energy Agency), **52**, 013005, 1-11*

Coherent structures extraction in 3D MHD flow

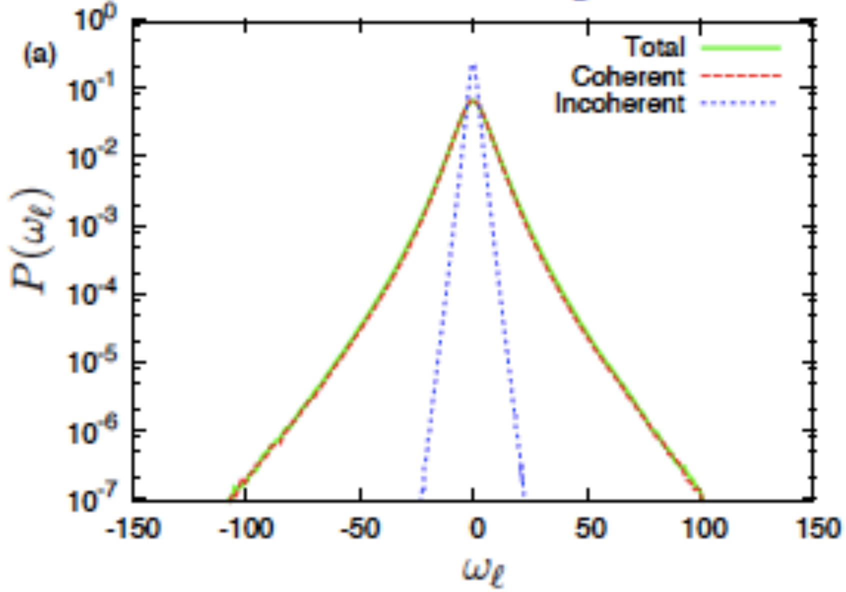


Yoshimatsu, Kondo, Schneider, Okamoto, Hagiwara & Farge Phys. Plasmas, ***16****, 2009*

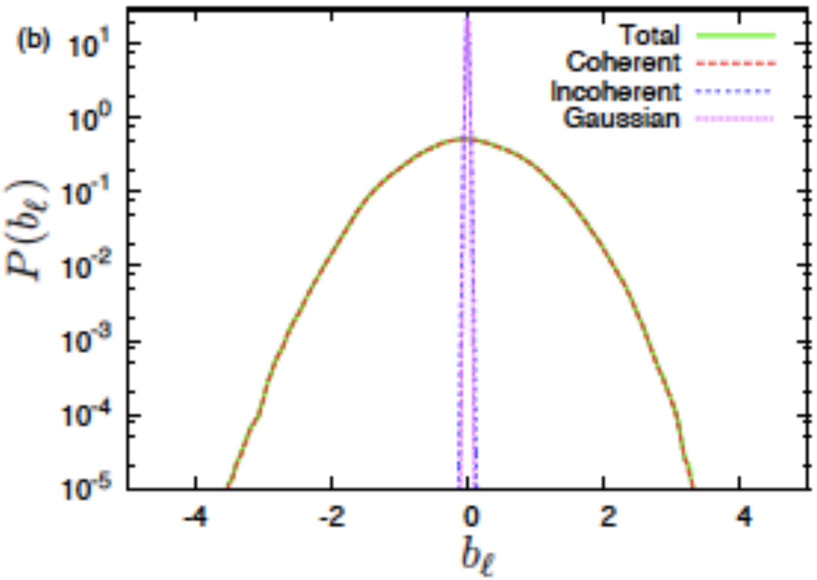
Velocity



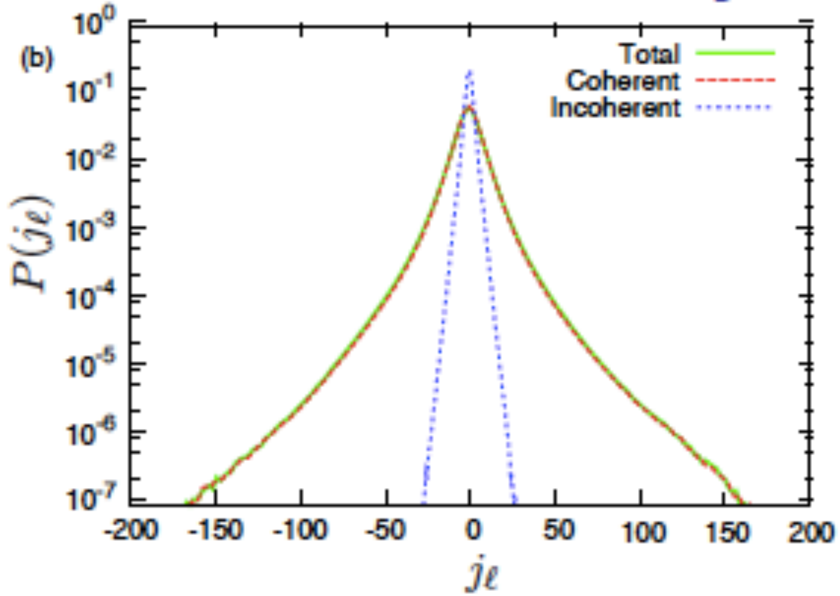
Vorticity



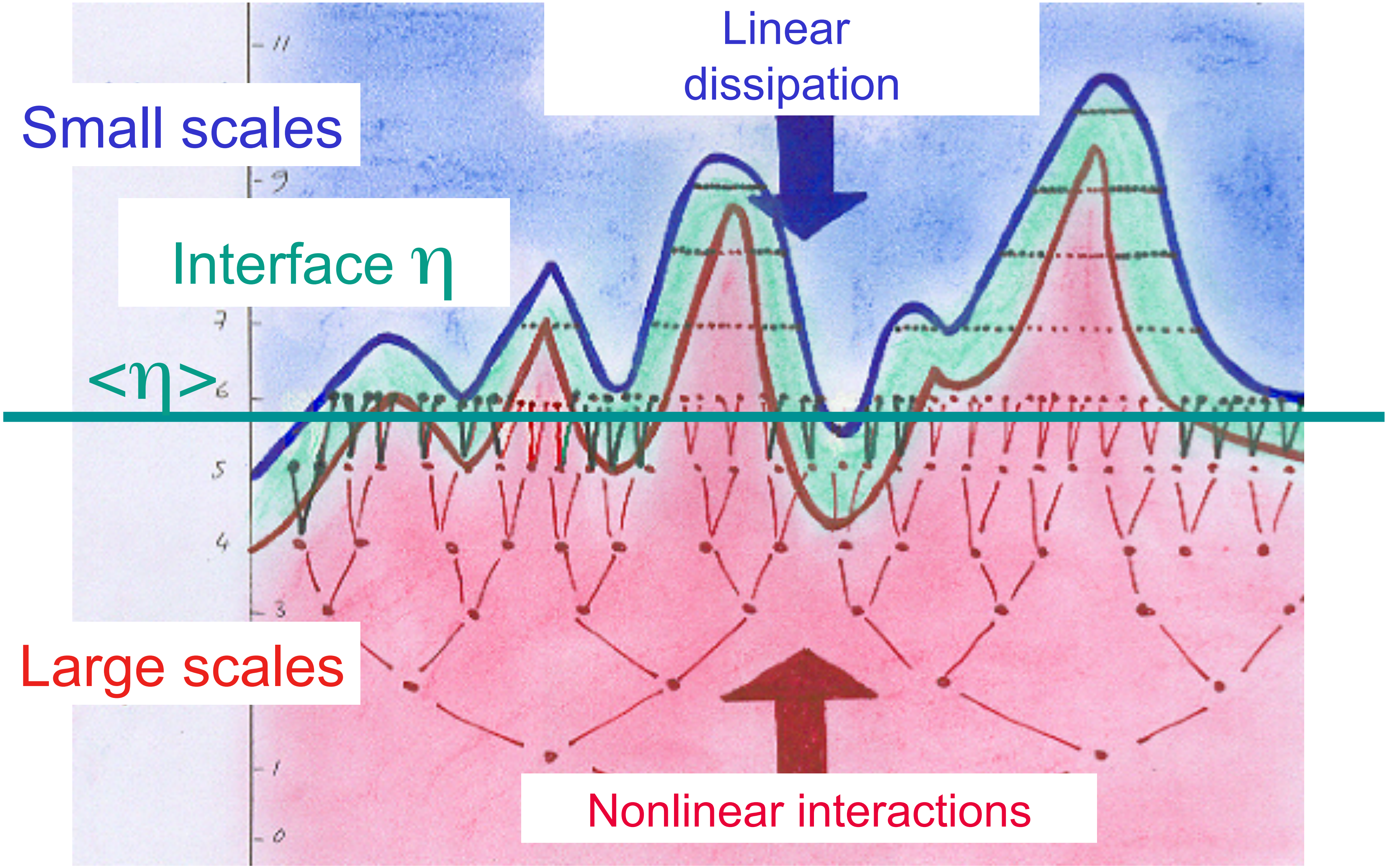
Magnetic field



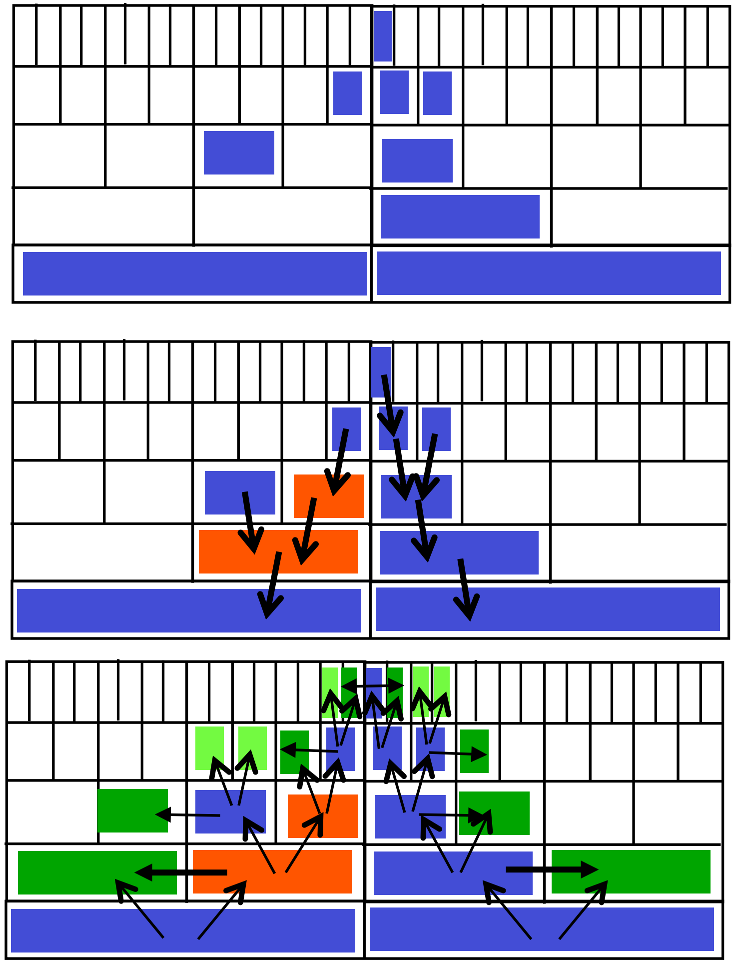
Current density



Coherent Vorticity Simulation (CVS)



Coherent Vortex Simulation (CVS)



Schneider & Farge, 2000,
Comp. Rend. Acad. Sci. Paris, 328

1. Selection of the wavelet coefficients whose modulus is larger than the threshold.

2. Construction of a 'graded-tree' which defines the 'interface' between the coherent and incoherent wavelet coefficients.

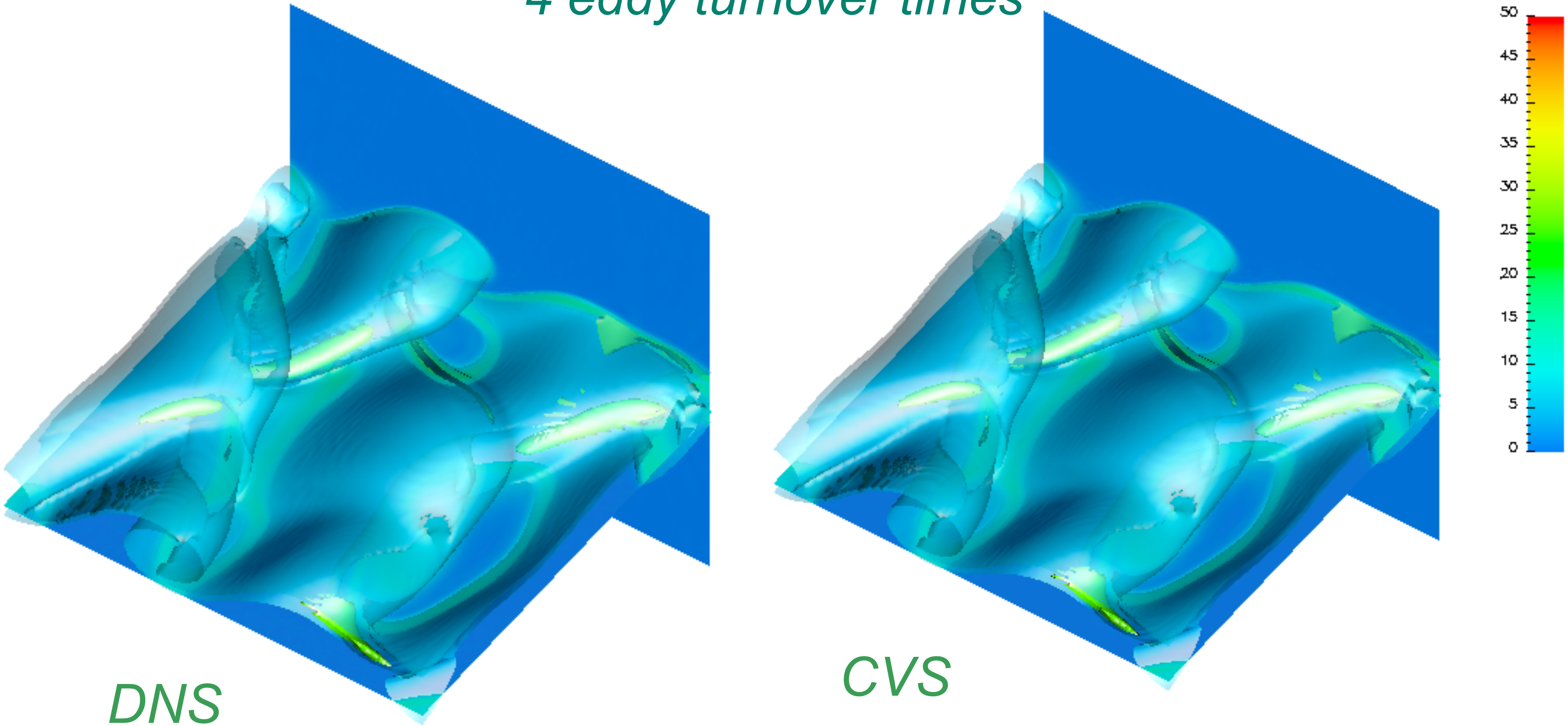
3. Addition of a 'security zone' which corresponds to dealiasing.

Schneider & Farge, 2002,
Appl. Comput. Harmonic Anal., 12

Schneider, Farge et al., 2005,
J. Fluid Mech., 534(5)

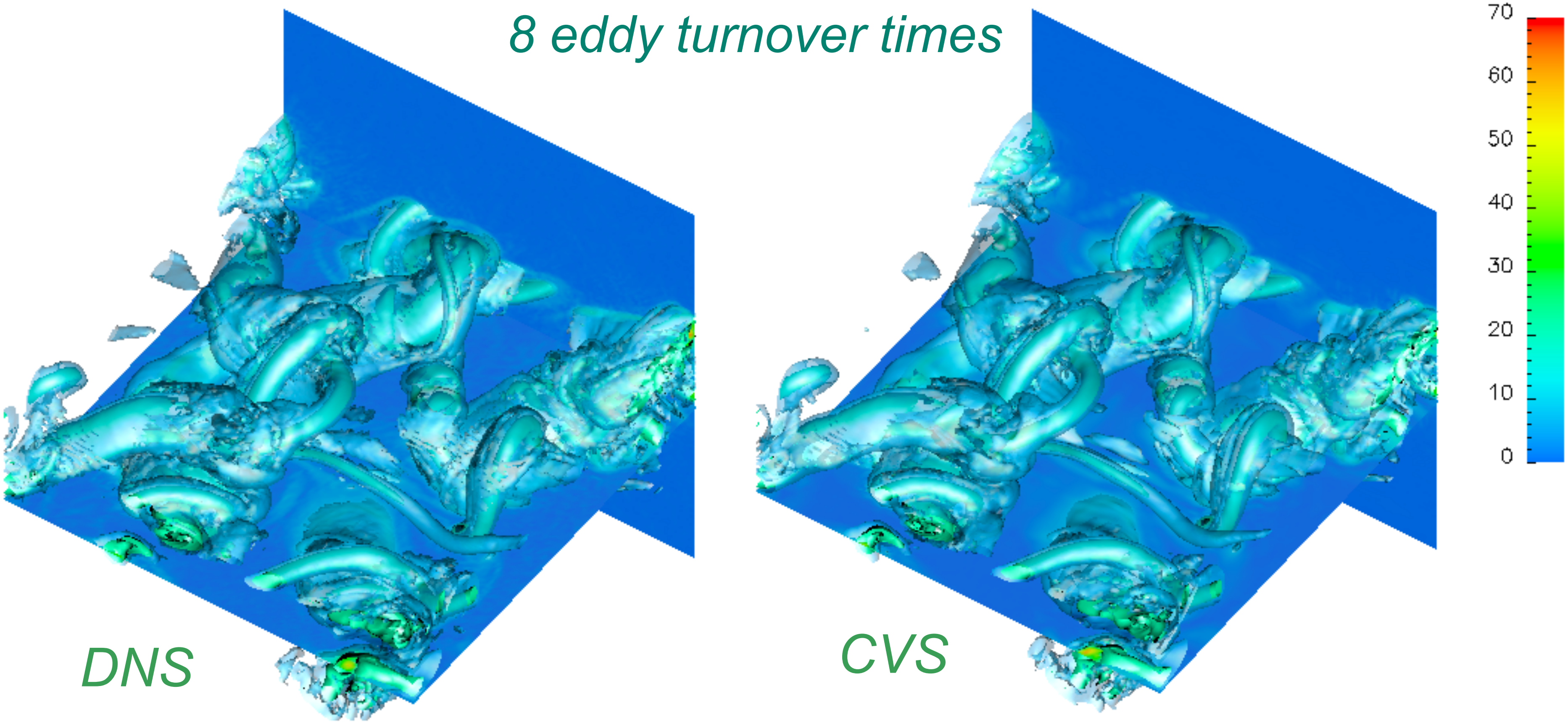
3D turbulent mixing layer

4 eddy turnover times



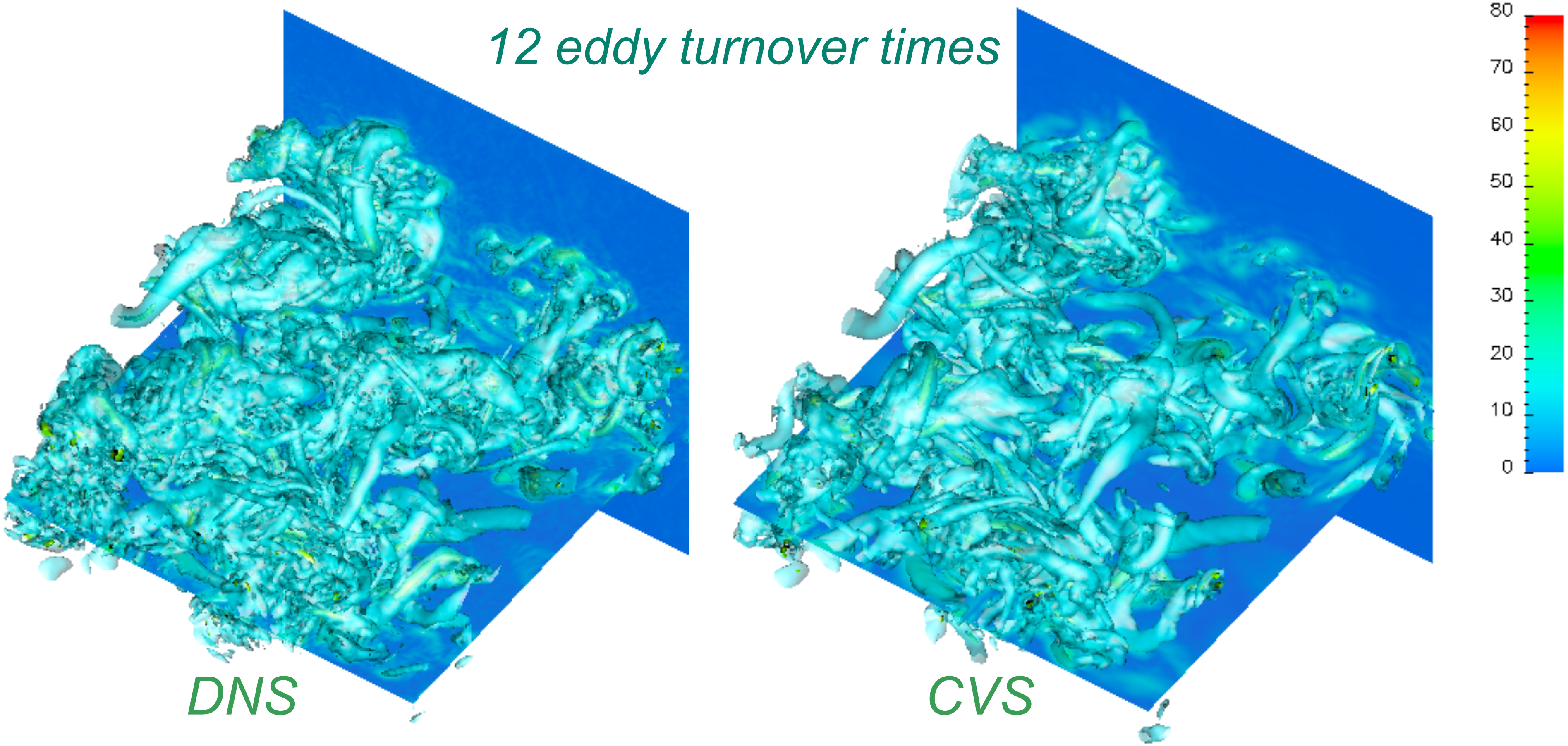
Schneider, Farge, Pellegrino, Rogers 2005, J. Fluid Mech., 534(5)

3D turbulent mixing layer



Schneider, Farge, Pellegrino, Rogers 2005, J. Fluid Mech., 534(5)

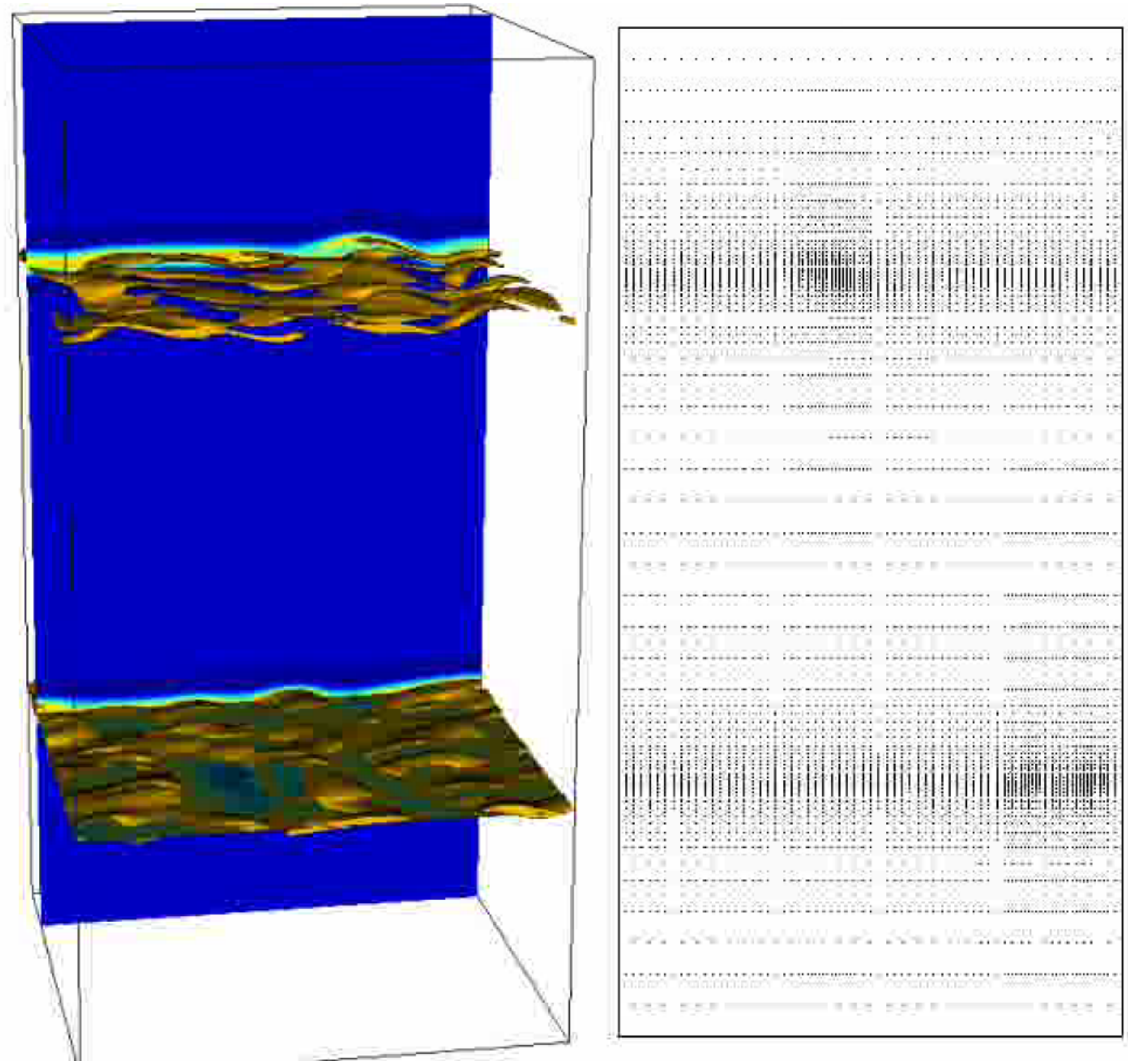
3D turbulent mixing layer



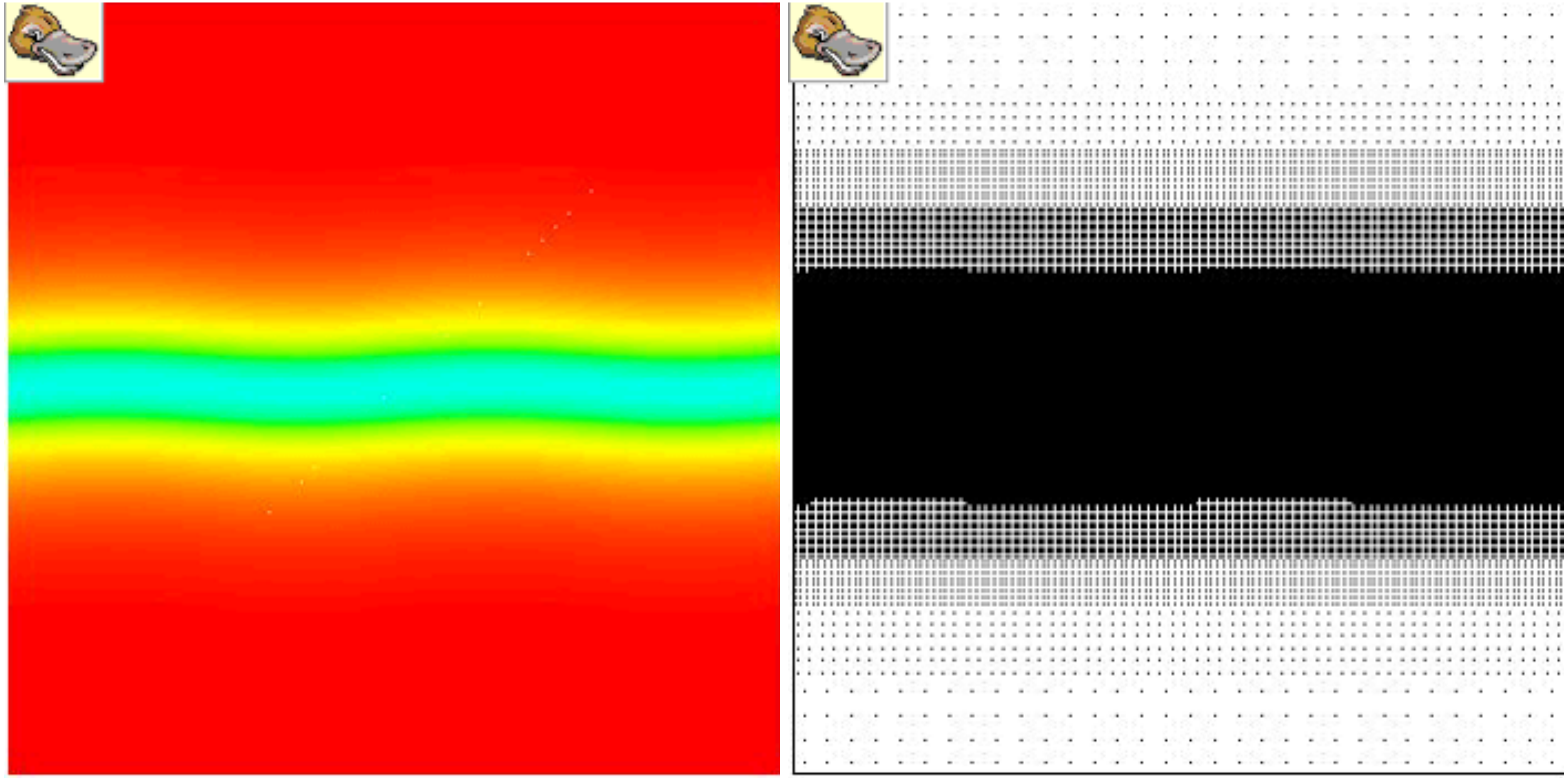
Schneider, Farge, Pellegrino, Rogers 2005, J. Fluid Mech., 534(5)

Adaptive computation using wavelets

Koster, Schneider, Griebel, Farge
Numerical Flow Simulation II,
***75**, Springer, 2001*



Adaptive computation using wavelets



Roussel and Schneider,
***75**, 2000*

Intermittency and geometrical statistics of 3d homogeneous magnetohydrodynamic turbulence using wavelets

Kai Schneider

Aix-Marseille University, France

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[1]*Nagoya University, Japan* [2]*ENS Paris, France*

Turbulence workshop 2015: Analyzing turbulence data
May 29, 2015, Observatoire de Paris, Meudon, France

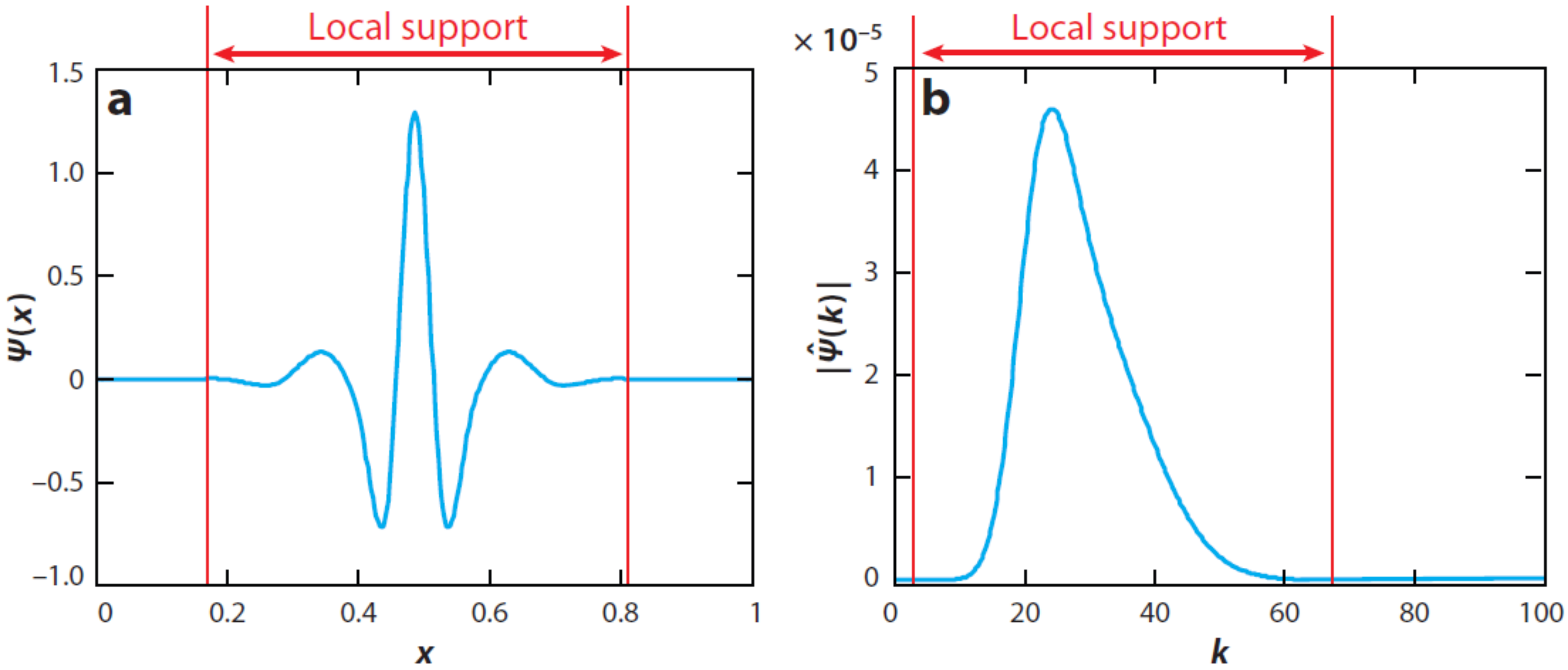
Motivation

Different wavelet based tools to examine scale-dependent statistics.

What are similarities and differences of small-scale intermittency
in HD and MHD turbulence?

Here we use and generalize the diagnostics introduced in
Yoshimatsu et al., Phys. Rev. E, 79, 2009, which are based
on the orthonormal wavelet decomposition.

Orthogonal wavelets



Ref.: M. Farge. *Annu. Rev. Fluid Mech.*, 24, 1992
K. Schneider, O. Vasilyev. *Annu. Rev. Fluid Mech.*, 42, 2010

Wavelet decomposition

The vector field $\boldsymbol{v}$, having a mean value $\langle \boldsymbol{v} \rangle$ (which vanishes in the present applications for all components), can be decomposed into an orthogonal wavelet series

$$\boldsymbol{v}(\boldsymbol{x}) = \langle \boldsymbol{v} \rangle + \sum_{j=0}^{J-1} \boldsymbol{v}_j(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{v}_j$ is the contribution of $\boldsymbol{v}$ at scale 2^{-j} defined by

$$\boldsymbol{v}_j(\boldsymbol{x}) = \sum_{\mu=1}^{7} \sum_{i_1,i_2,i_3=0}^{2^j-1} \tilde{\boldsymbol{v}}_{\mu,\lambda} \psi_{\mu,\lambda}(\boldsymbol{x}). \tag{2}$$

with the wavelet coefficients $\tilde{\boldsymbol{v}}_{\mu,\lambda} = \langle \boldsymbol{v}, \psi_{\mu,\lambda} \rangle$

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Scale dependent statistics (I)

Scale dependent energy

$$e_j^v = \langle \boldsymbol{v}_j, \boldsymbol{v}_j \rangle / 2 \quad \text{with} \quad \bar{E}^v = \sum_{j=0}^{J-1} e_j^v$$

Wavelet energy spectrum

$$\tilde{E}_j^v = \frac{1}{\Delta k_j} \langle e_j^{v,\ell} \rangle_c \quad \text{with } k_j = k_\psi 2^j \text{ and } k_\psi = 0.77$$

$$\Delta k_j = k_j \ln 2$$

Spatial variability of the energy spectrum

$$\tilde{\sigma}_j^v = \frac{1}{\Delta k_j} \sqrt{\langle (e_j^{v,\ell})^2 \rangle_c - (\langle e_j^{v,\ell} \rangle_c)^2}$$

Scale dependent statistics (II)

Scale dependent flatness

$$F[\boldsymbol{v}_j] = \frac{\langle (v_j^\ell)^4 \rangle_c}{\{\langle (v_j^\ell)^2 \rangle_c\}^2}$$

Link with wavelet energy spectrum and its standard deviation

$$F[\boldsymbol{v}_j] = \left(\frac{\tilde{\sigma}_j^v}{\tilde{E}_j^v} \right)^2 + 1$$

Ref.: W. Bos, L. Liechtenstein and K. Schneider. *Phys. Rev. E*, 76, 2007.

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Scale dependent statistics (III)

Scale dependent helicites (kinetic, cross, magnetic)

$$H_j^{\mathrm{K}}(\boldsymbol{x}) = \boldsymbol{u}_j \cdot \boldsymbol{\omega}_j \quad H_j^{\mathrm{C}}(\boldsymbol{x}) = \boldsymbol{u}_j \cdot \boldsymbol{b}_j \quad H_j^{\mathrm{M}}(\boldsymbol{x}) = \boldsymbol{a}_j \cdot \boldsymbol{b}_j$$

Scale dependent relative helicites

$$h_j^{\mathrm{K}}(\boldsymbol{x}) = \frac{H_j^{\mathrm{K}}}{|\boldsymbol{u}_j||\boldsymbol{\omega}_j|} \qquad h_j^{\mathrm{C}}(\boldsymbol{x}) = \frac{H_j^{\mathrm{C}}}{|\boldsymbol{u}_j||\boldsymbol{b}_j|} \qquad h_j^{\mathrm{M}}(\boldsymbol{x}) = \frac{H_j^{\mathrm{M}}}{|\boldsymbol{a}_j||\boldsymbol{b}_j|}$$

DNS of MHD Turbulence

Forced 3D incompressible MHD turbulence without mean magnetic field in a 2π periodic box Ω.

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\frac{1}{\rho_0}\nabla P + \boldsymbol{j} \times \boldsymbol{b} + \nu \Delta \boldsymbol{u} + \boldsymbol{f},$$

$$\partial_t \boldsymbol{b} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{b} = (\boldsymbol{b} \cdot \nabla)\boldsymbol{u} + \eta \Delta \boldsymbol{b},$$

$$\nabla \cdot \boldsymbol{u} = 0,$$

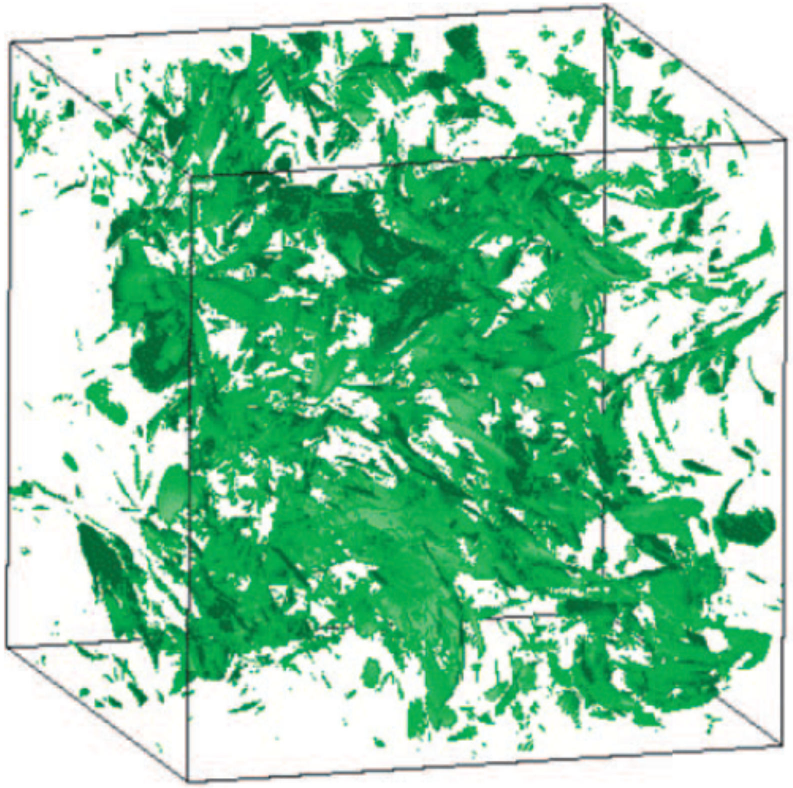
$$\nabla \cdot \boldsymbol{b} = 0,$$

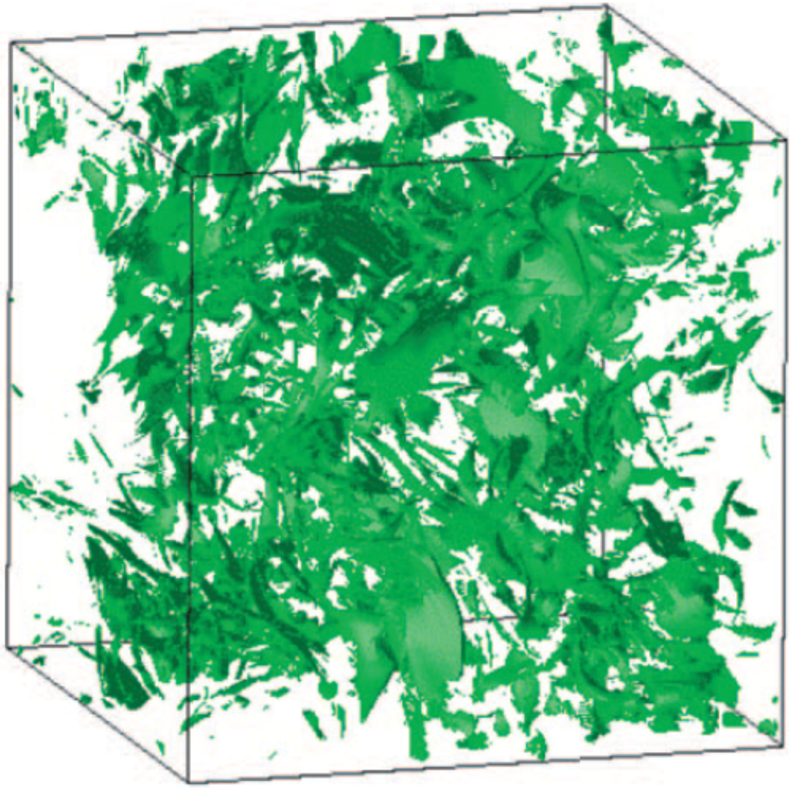
where t is time, $\boldsymbol{f}$ is an external force, P is the pressure, ν is the kinematic viscosity, η is the magnetic diffusivity, and $\partial_t = \partial / \partial t$. The Prandtl number Pr is set to 1, i.e., $\eta = \nu$.

092304-8 Yoshimatsu *et al.* Phys. Plasmas **18**, 092304 (2011)

Vorticity and current density

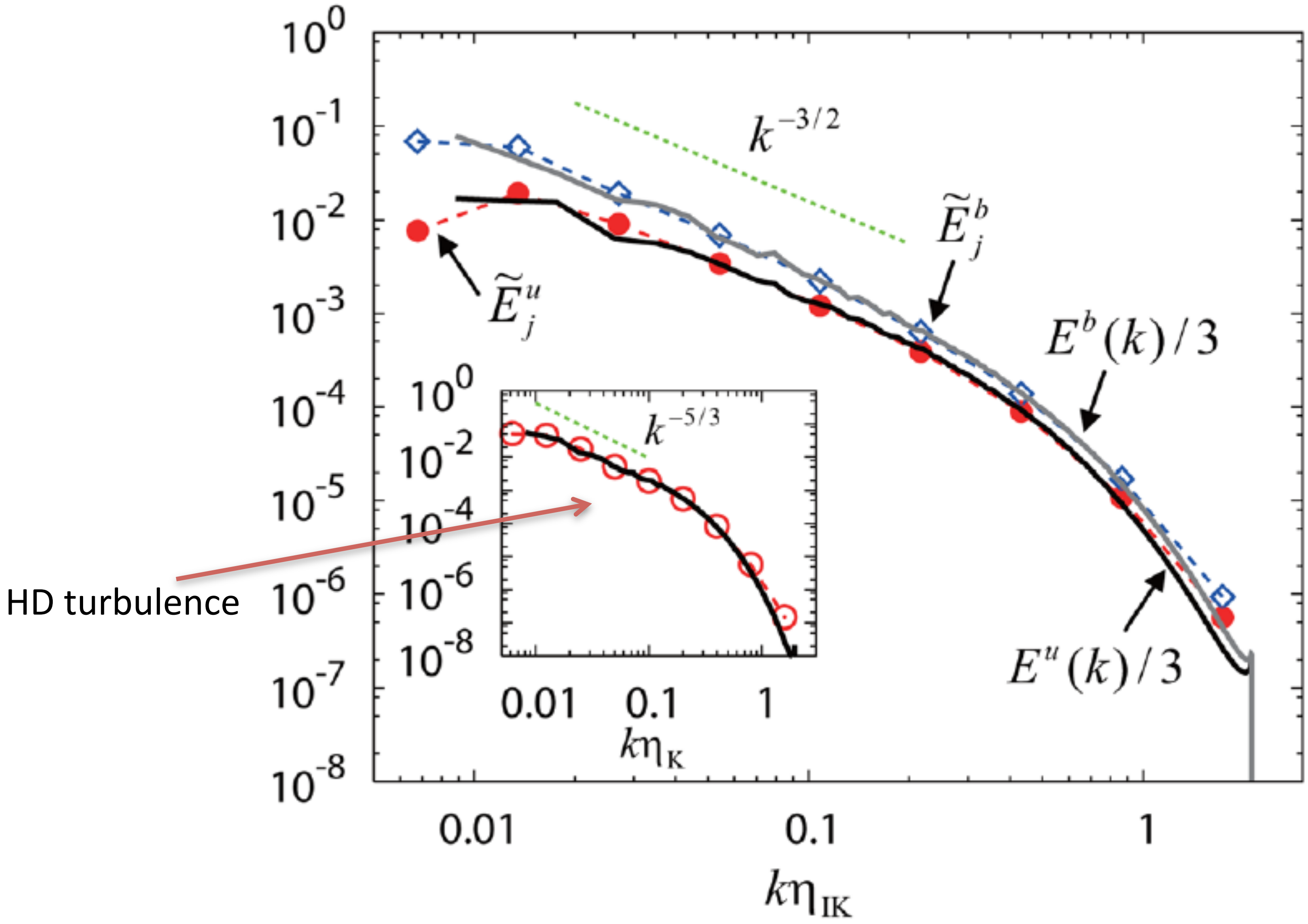
$\bar{E}^u$	$\bar{E}^b$	$\bar{H}^{\mathrm{K}}(\times 10^{-3})$	$\bar{H}^{\mathrm{C}}(\times 10^{-3})$	$\bar{H}^{\mathrm{M}}$	$\eta_{\mathrm{IK}}(\times 10^{-3})$	R^u_λ	R^b_λ
0.238	0.618	7.16	−5.77	0.503	8.79	150	306



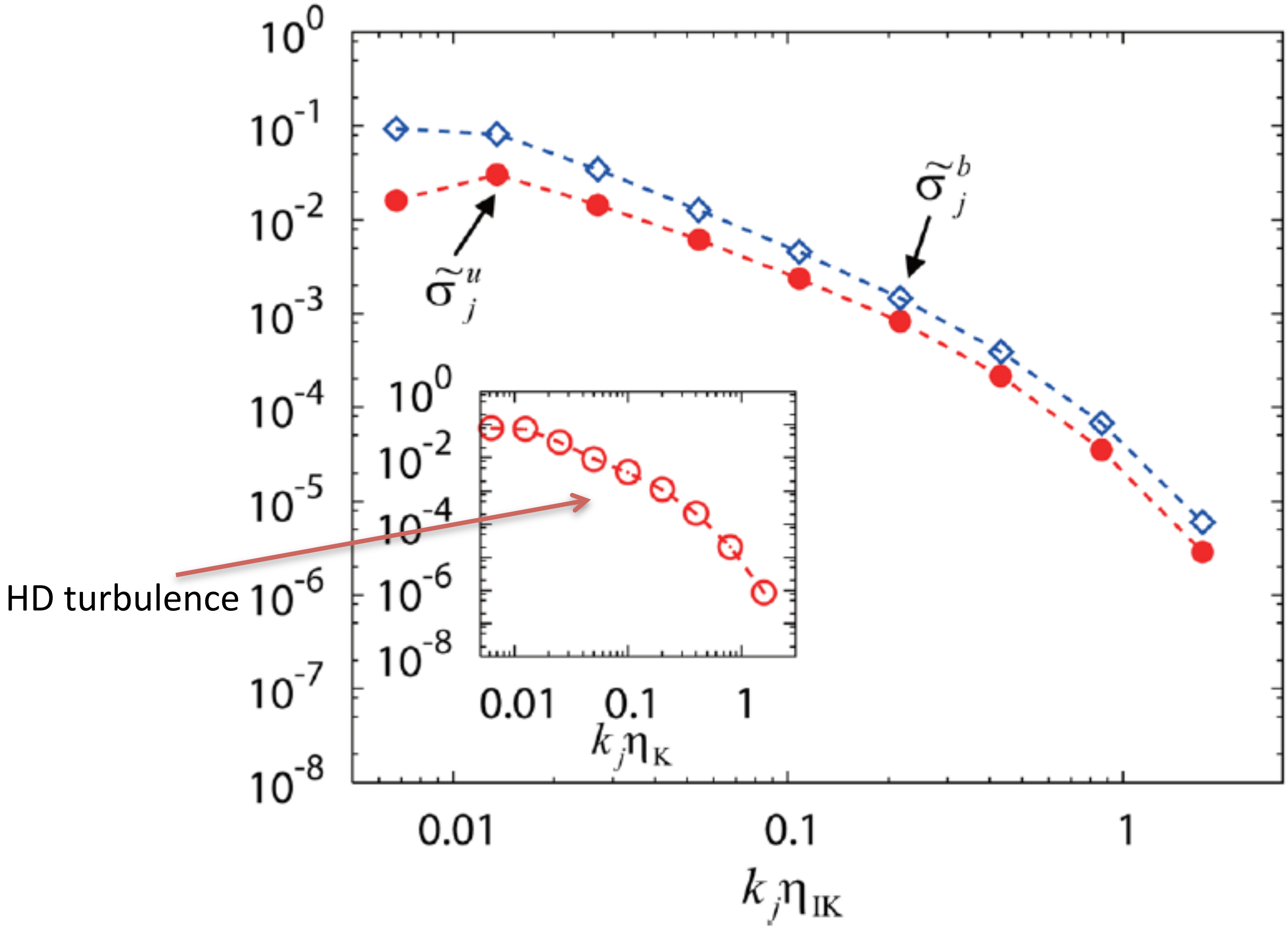


Resolution N= 512^3

Wavelet mean kinetic and magnetic energy spectra

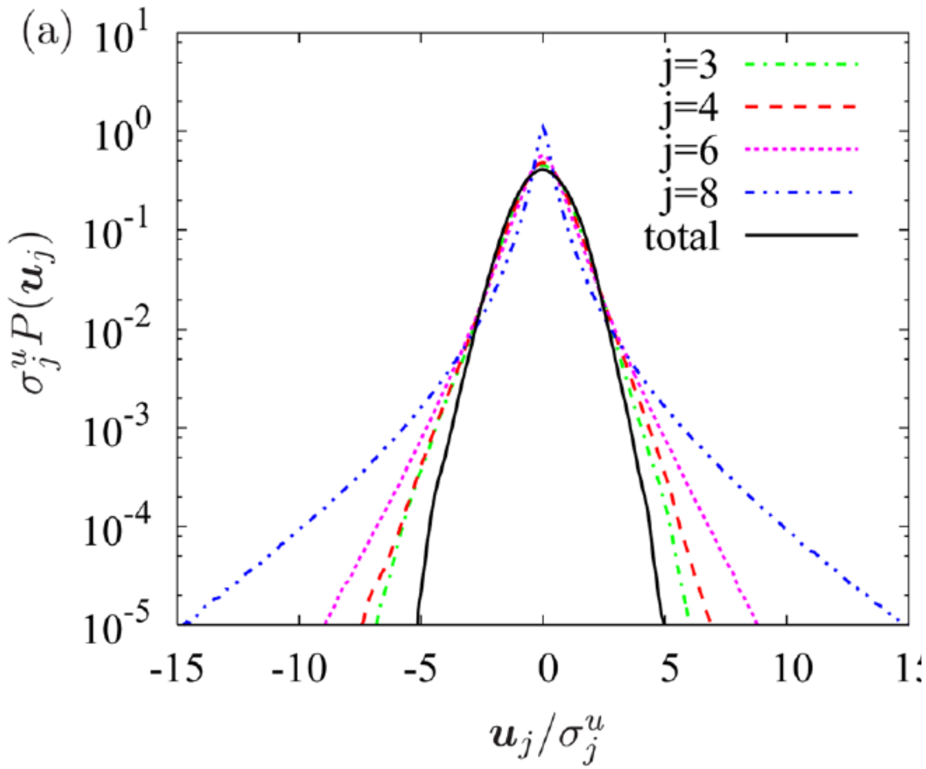


Spatial variability of the kinetic and magnetic energy spectra

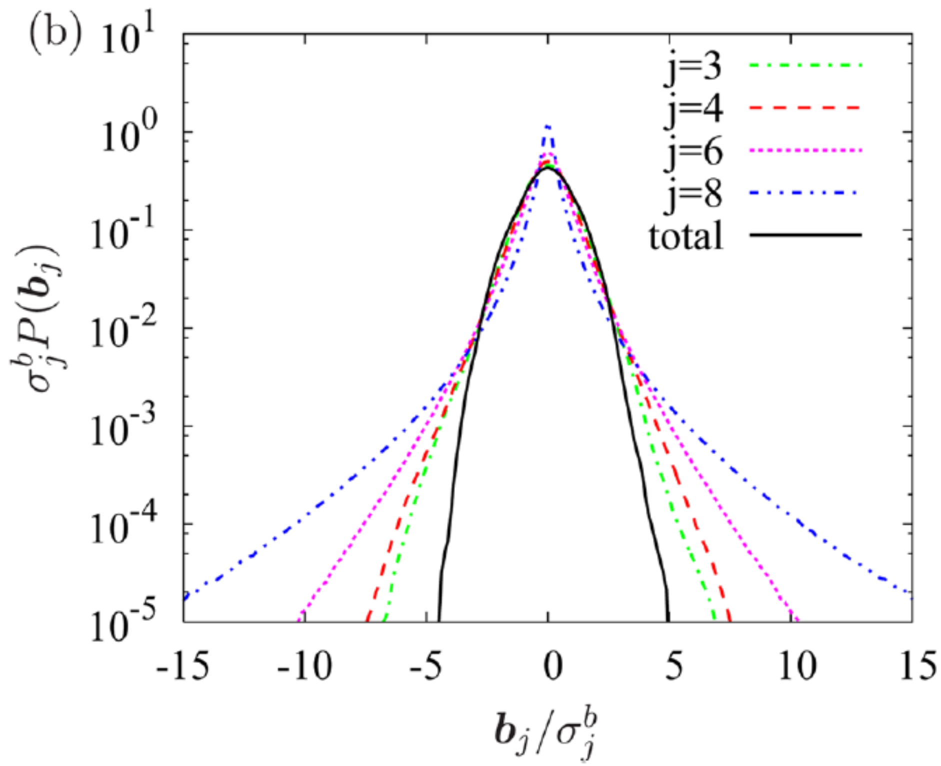


Scale dependent PDFs

Velocity



Magnetic field



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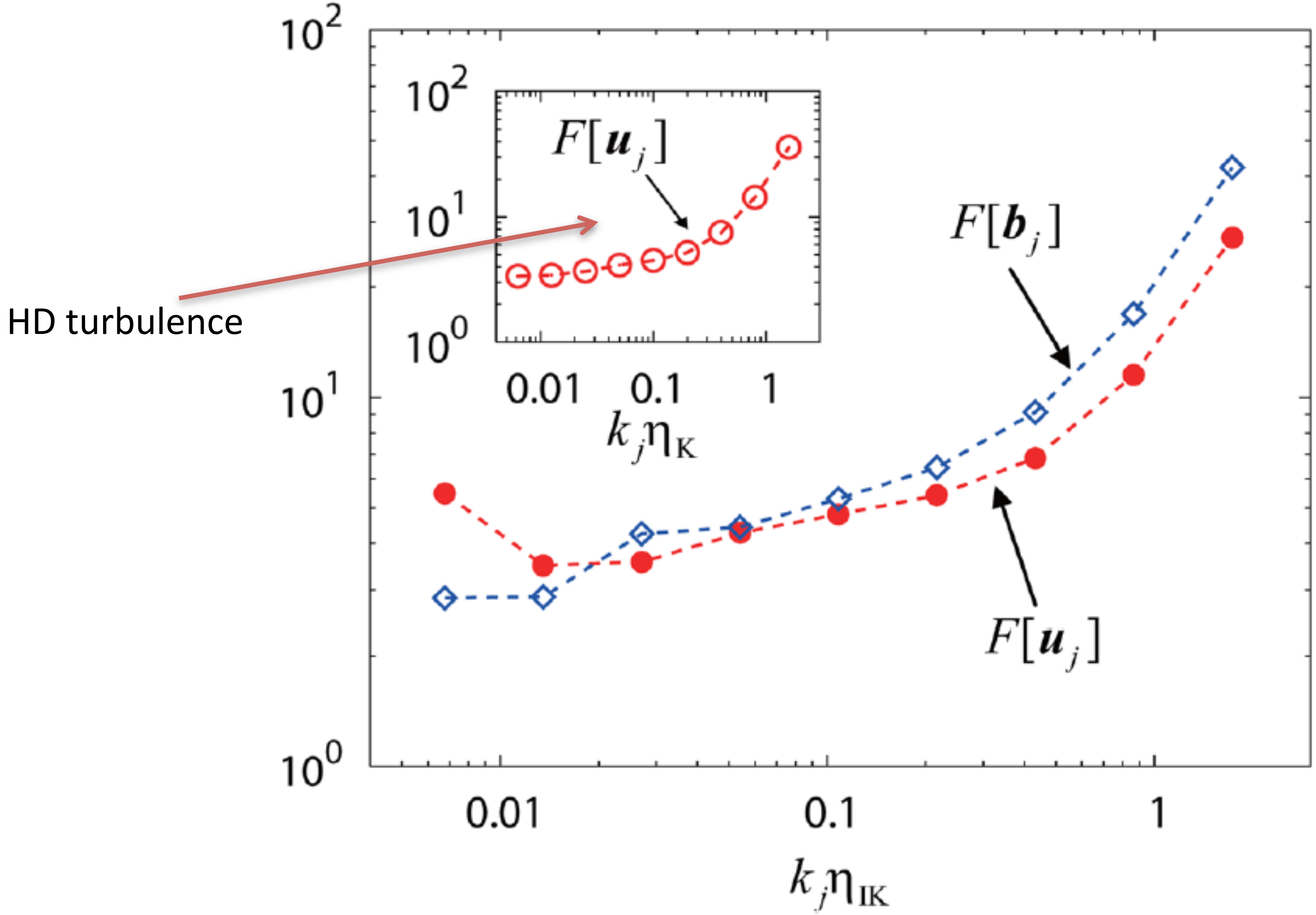
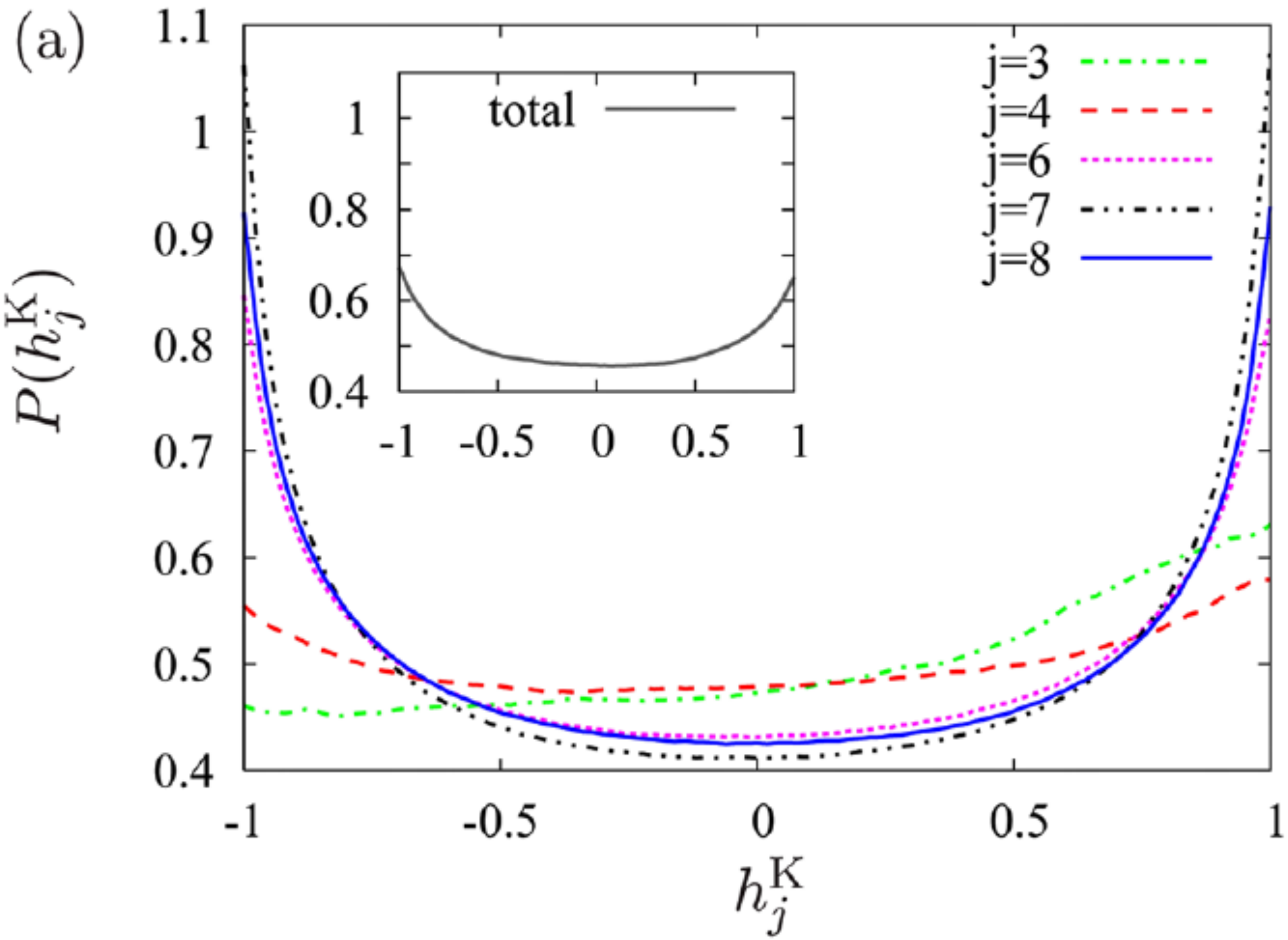
Scale dependent flatness of velocity and magnetic field



Scale dependent PDFs of relative kinetic helicity

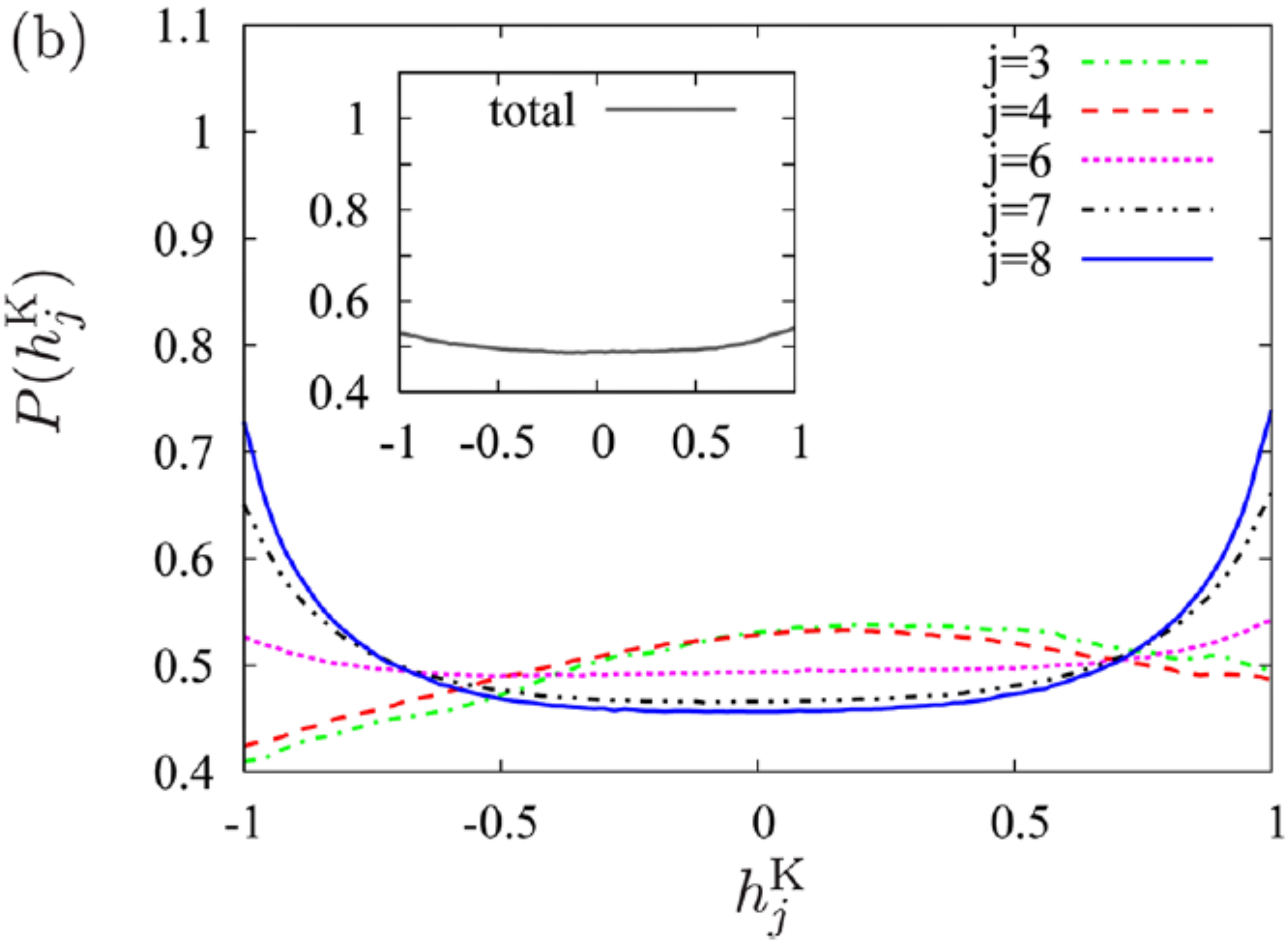
MHD turbulence



092304-8 Yoshimatsu *et al.* Phys. Plasmas **18**, 092304 (2011)

Scale dependent PDFs of relative kinetic helicity

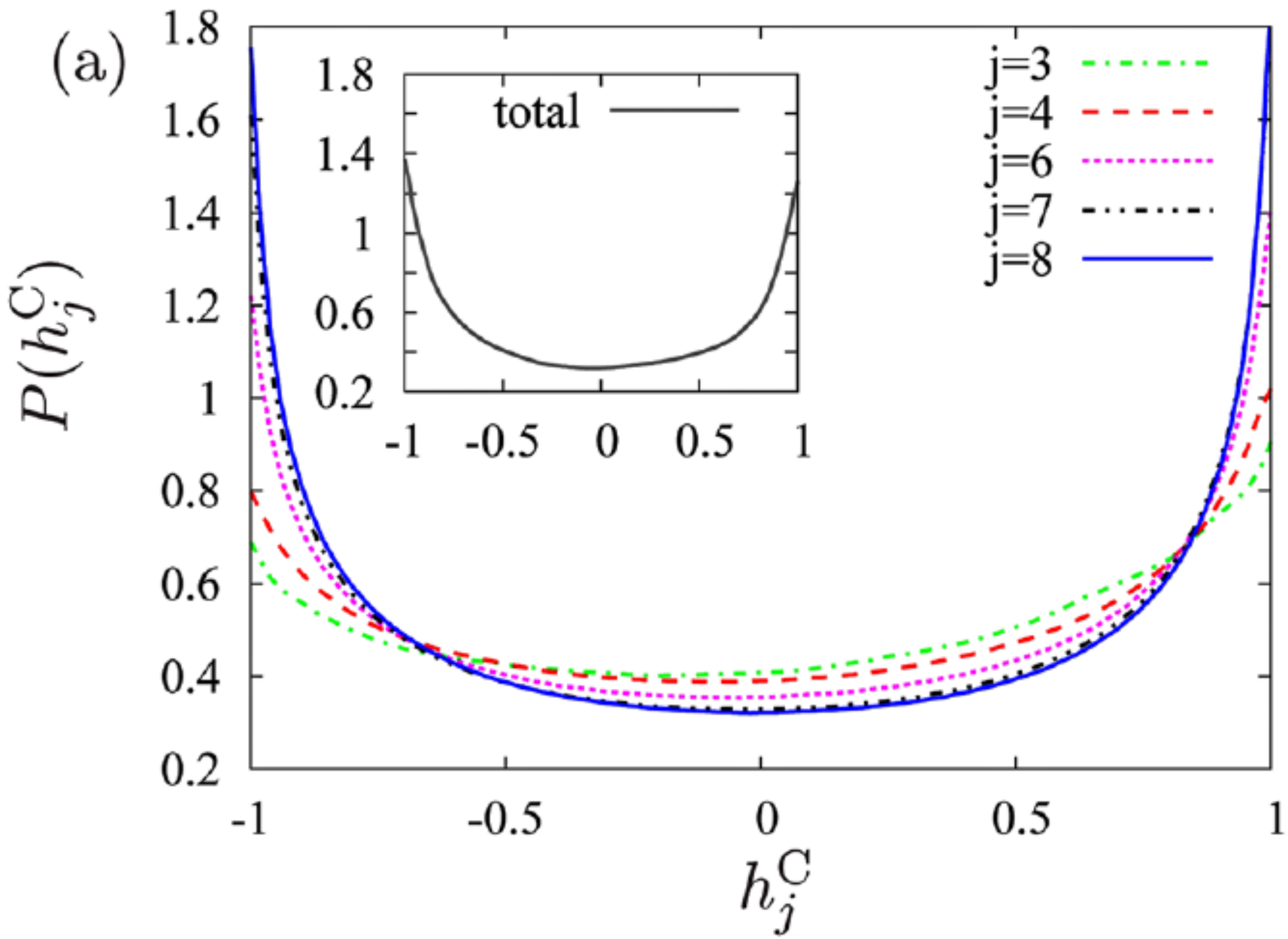
HD turbulence



 Phys. Plasmas **18**, 092304 (2011)

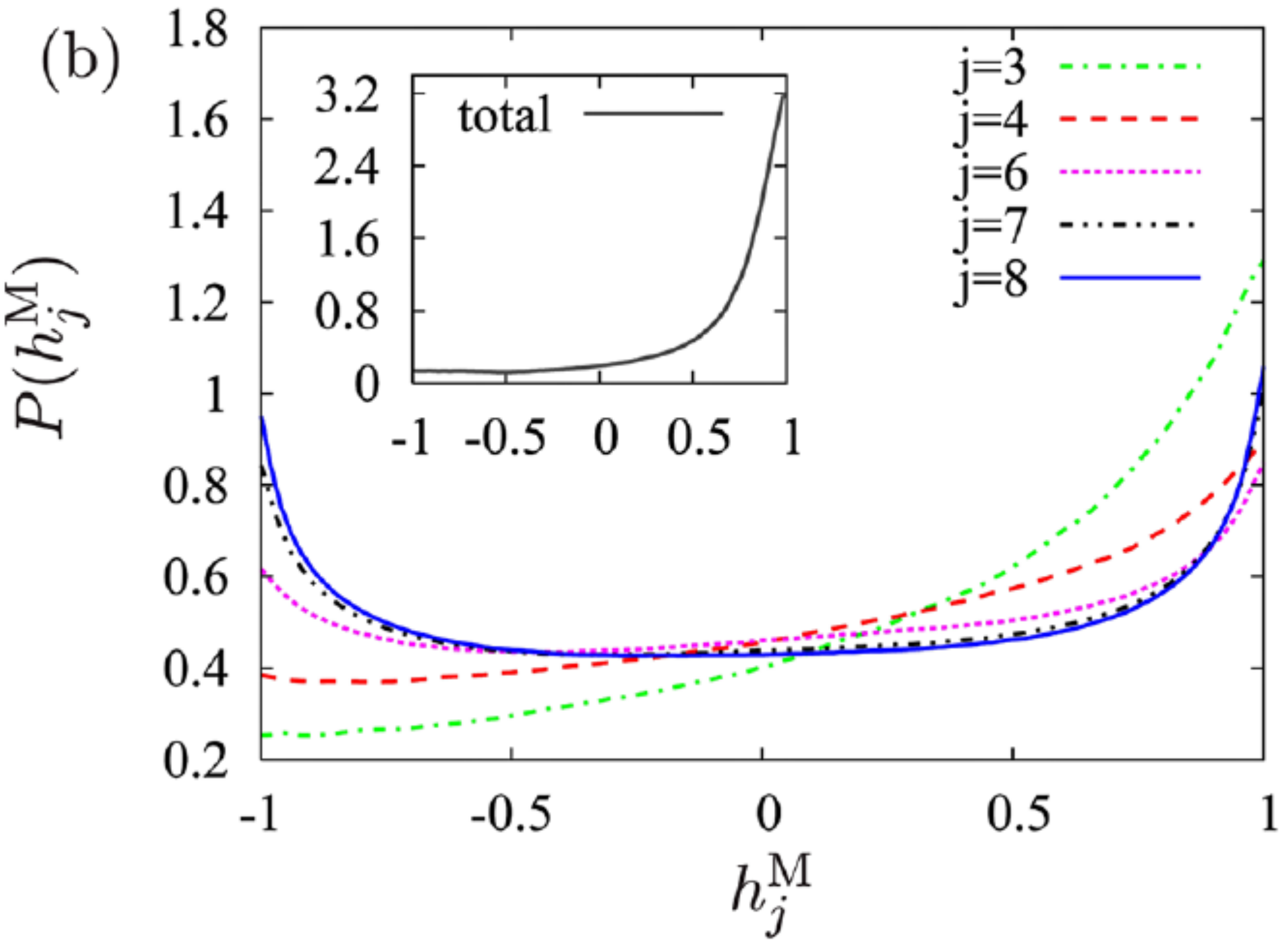
Scale dependent PDFs of relative cross helicity

MHD turbulence



Scale dependent PDFs of relative magnetic helicity

MHD turbulence



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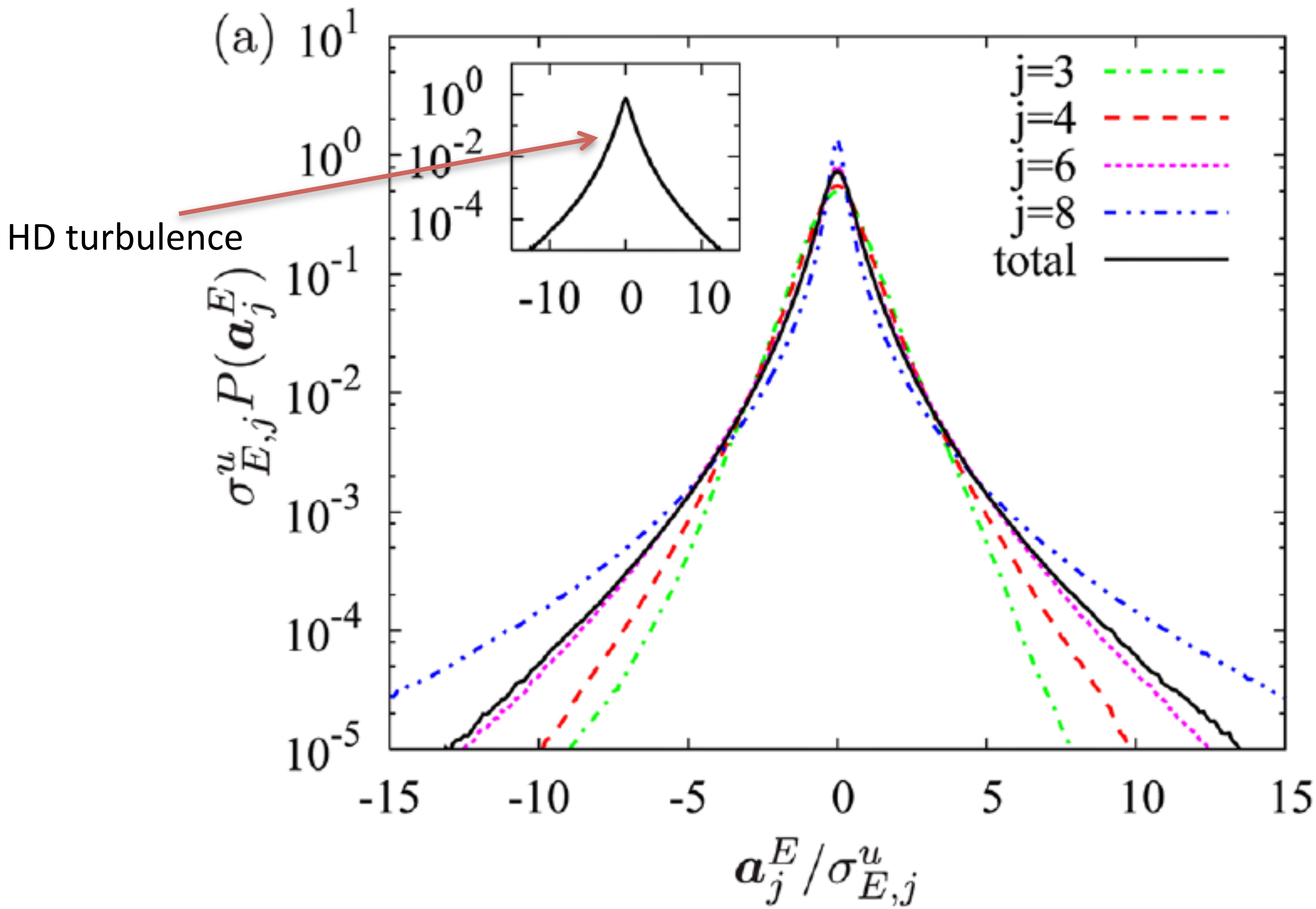
Eulerian and Lagrangian acceleration

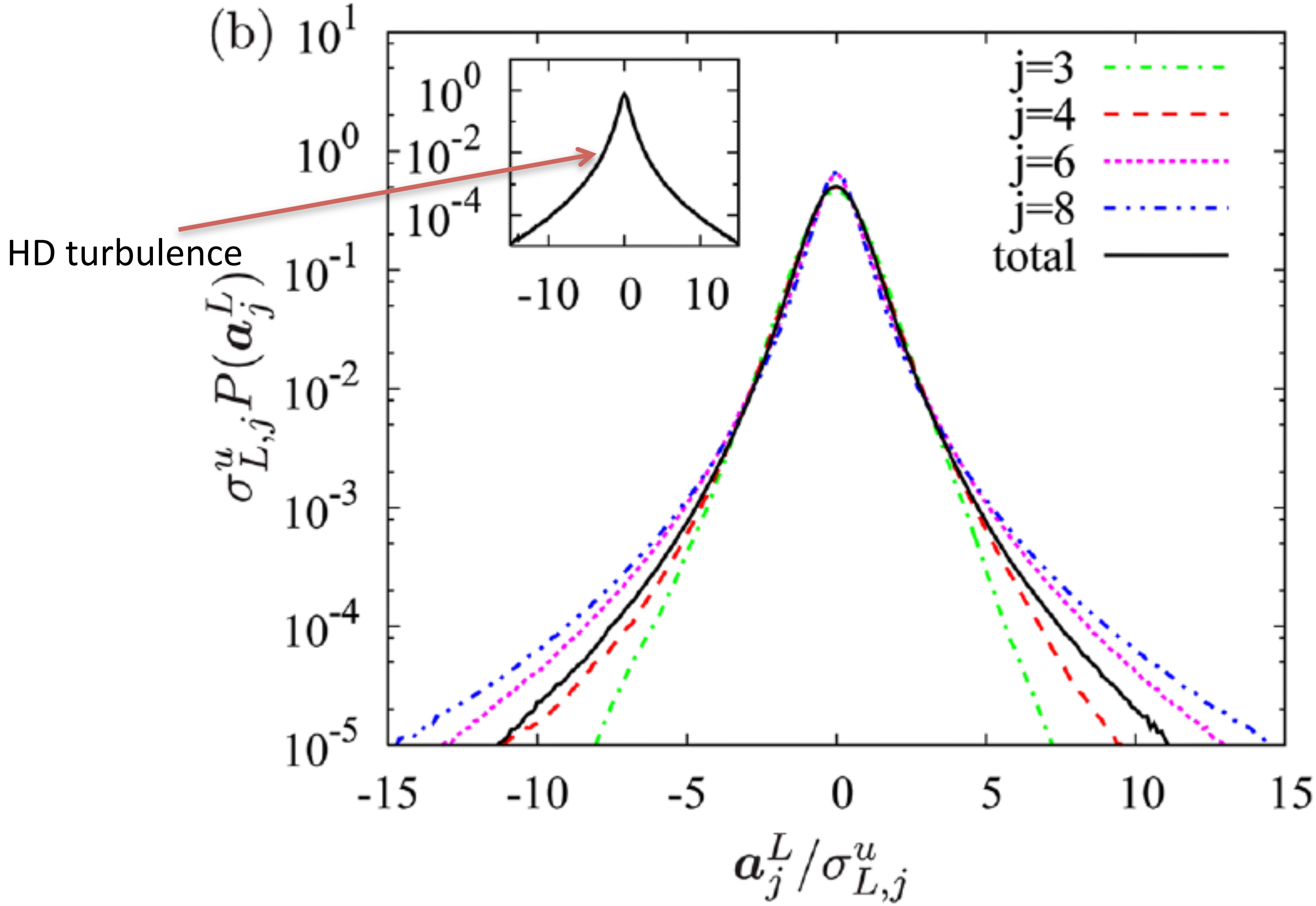
$$\boldsymbol{a}^E = -(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} - \frac{1}{\rho_0}\nabla P + \boldsymbol{j} \times \boldsymbol{b} + \nu\nabla^2\boldsymbol{u},$$

$$\boldsymbol{a}^L = -\frac{1}{\rho_0}\nabla P + \boldsymbol{j} \times \boldsymbol{b} + \nu\nabla^2\boldsymbol{u},$$

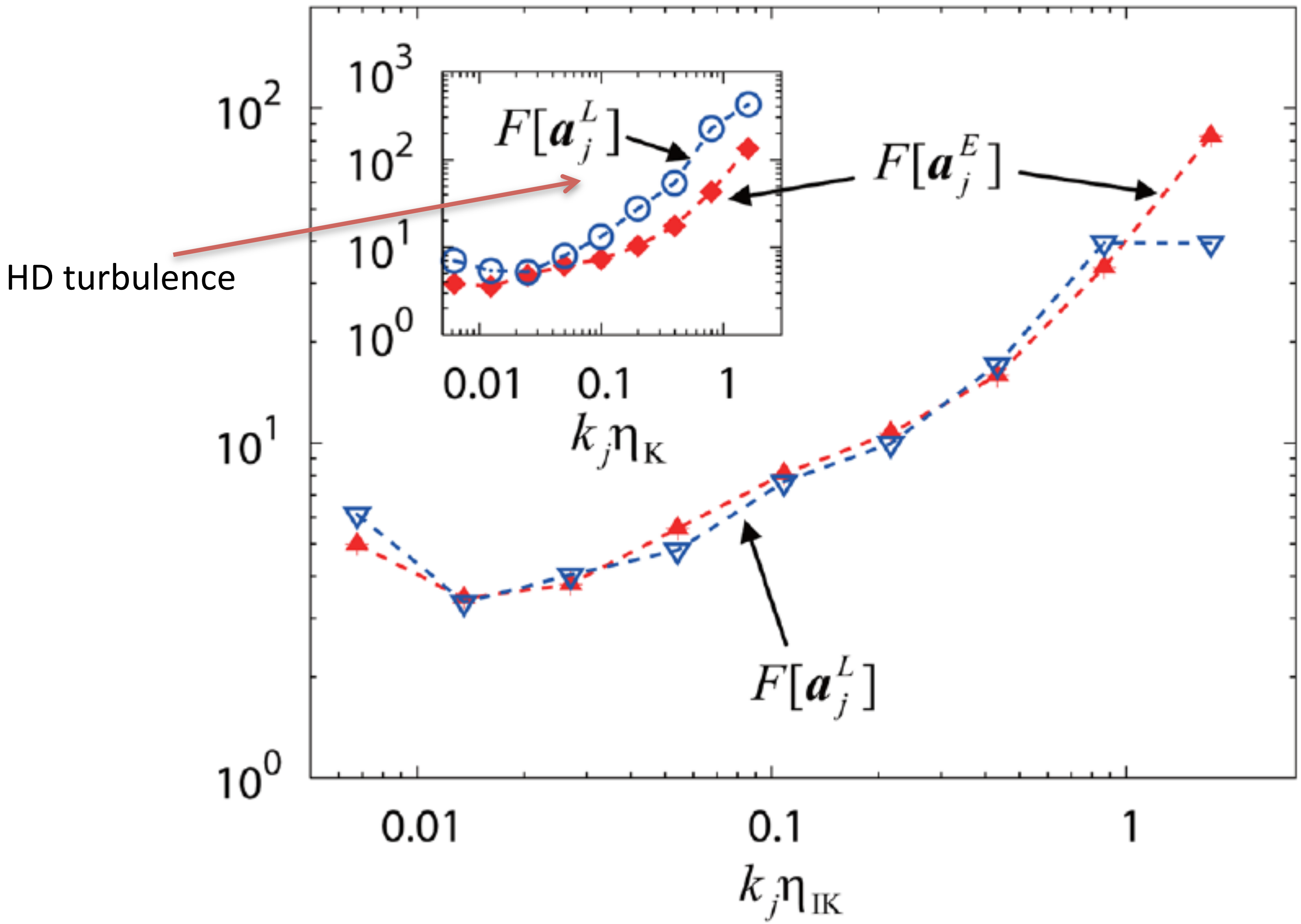
Scale dependent PDFs of Eulerian acceleration



Scale dependent PDFs of Lagrangian acceleration



Scale dependent flatness of Eulerian and Lagrangian acceleration

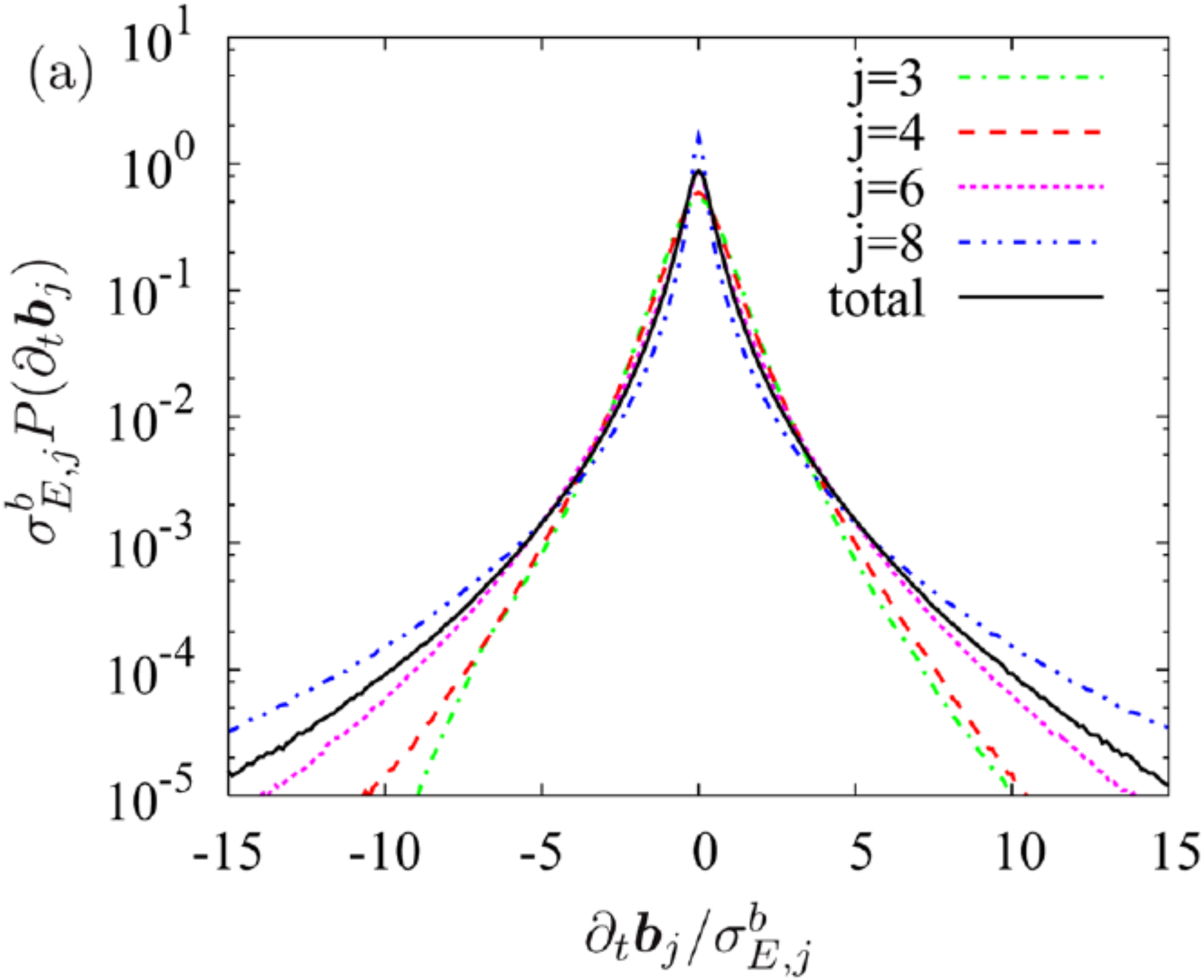


Eulerian and Lagrangian time derivatives of the magnetic field

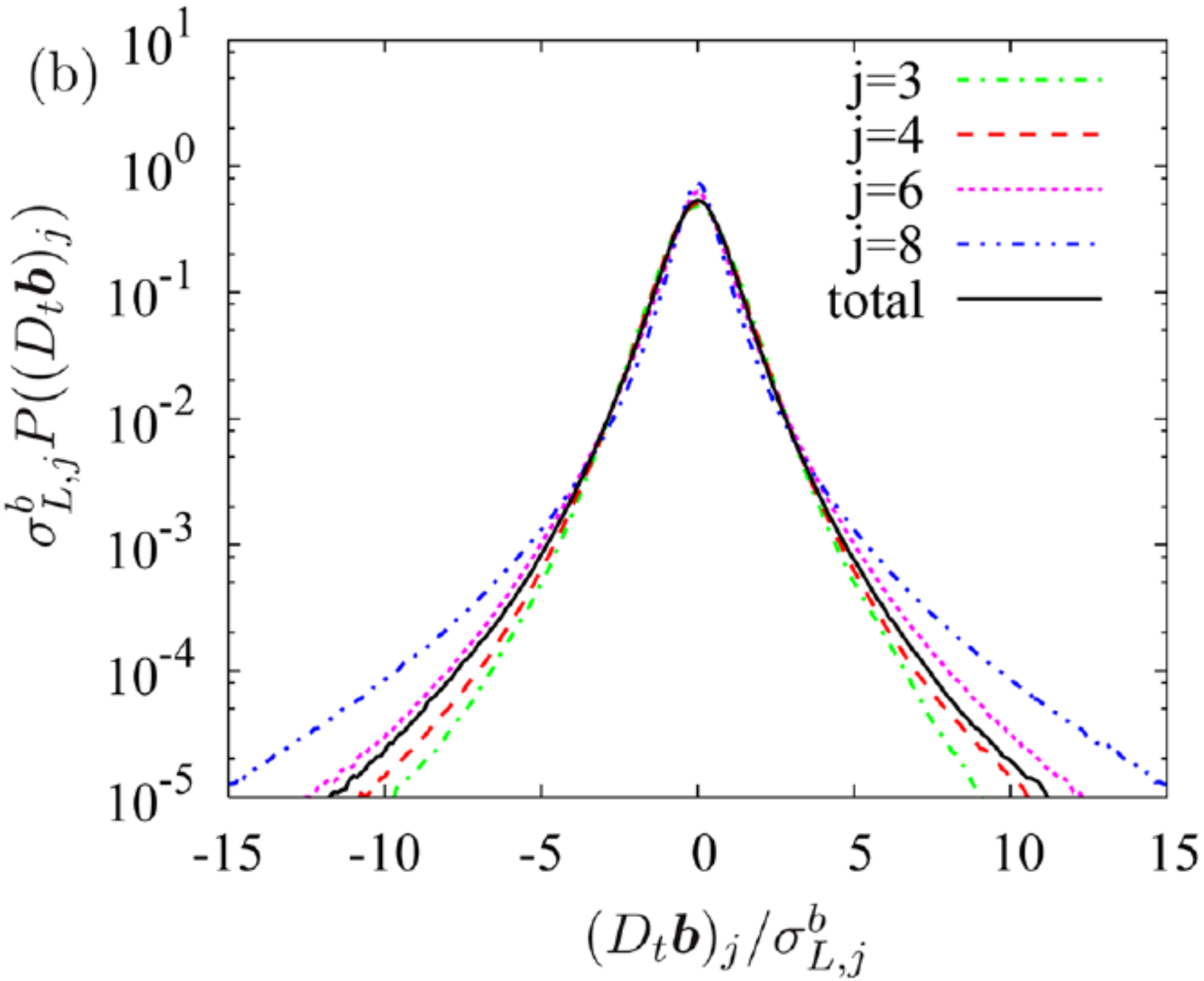
$$\partial_t \boldsymbol{b} \text{ and } D_t \boldsymbol{b}$$

$$D_t \boldsymbol{b} = \partial_t \boldsymbol{b} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{b}$$

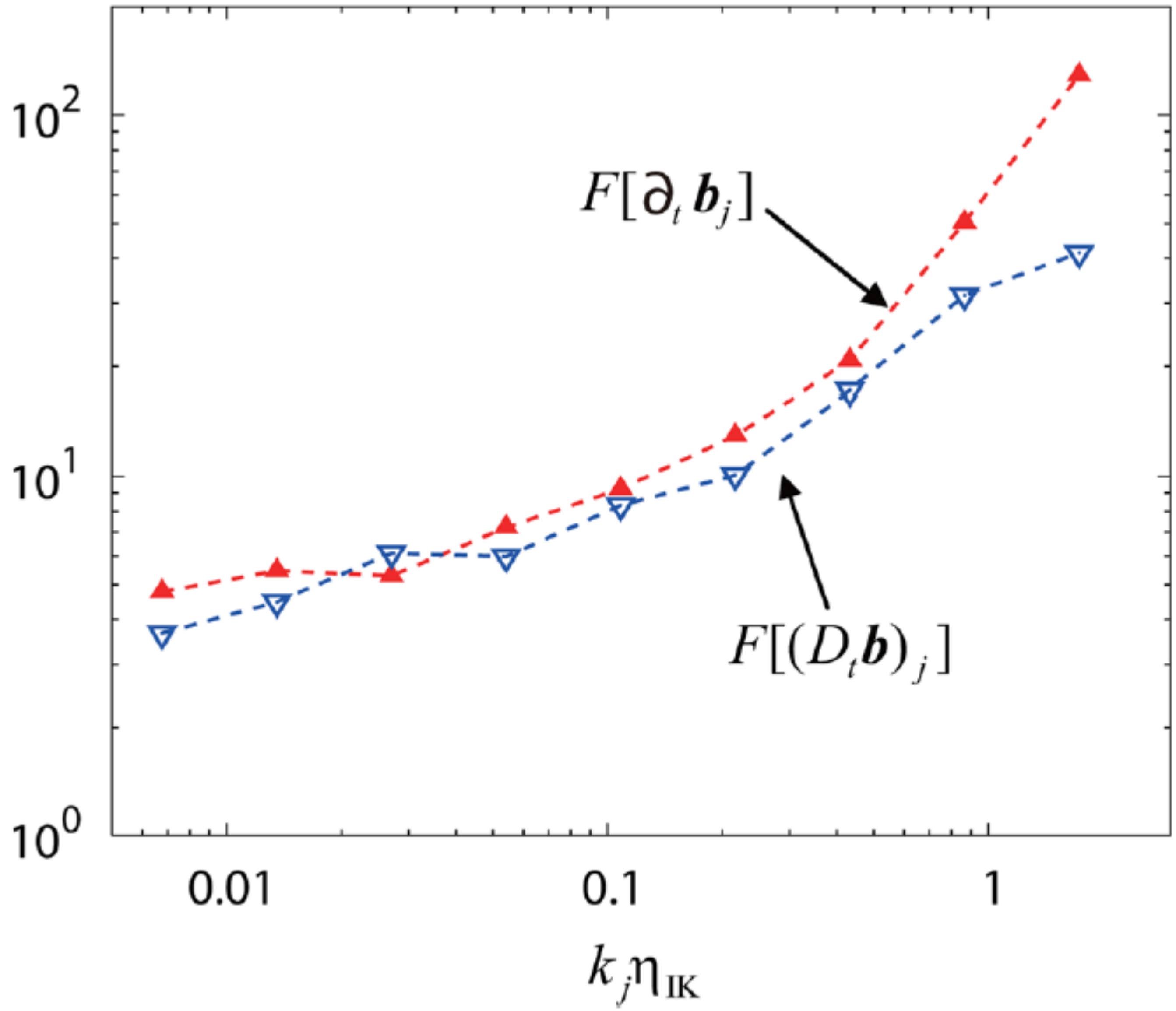
Scale dependent PDFs of Eulerian time derivative



Scale dependent PDFs of Lagrangian time derivative



Scale dependent flatness of Eulerian and Lagrangian time derivatives



Conclusions (I)

- Geometrical and scale-dependent statistics 3D MHD turbulence using orthogonal wavelets

- Wavelet decomposition yields clear scale separation spatial localization to quantify the intermittency of the flow

- Appplication to DNS data of stat. stat. MHD turbulence at $R^u_\lambda = 150$

Results:

- Magnetic field more intermittent than velocity (faster increase of flatness with scale)

- Multiscale measures to study geometrical statistics:

  - Relative scale-dependent kinetic, cross and magnetic helicities.
  - Higher probability for velocity and vorticity vectors to be aligned or anti-aligned, i.e., helical flow, at small scales for MHD turbulence.

Conclusions (II)

Analysis of scale-dependent statistics of Eulerian and Lagrangian accelerations, and corresponding time-derivatives of the magnetic field.

Different dynamics of MHD compared to HD turbulent flows.
In MHD turbulence, intermittency of the Lagrangian acc. is comparable to that of the Eulerian acc..

In HD turbulence, the Lagrangian acc. exhibits substantially stronger intermittency than the Eulerian one.

The Eulerian time-derivative of the magnetic field is more intermittent than the corresponding Lagrangian time derivative.

Intermittency in MHD turbulence is different from HD turbulence.

Ref.:K. Yoshimatsu, K. Schneider, N. Okamoto, Y. Kawahura and M. Farge. Intermittency and geometrical statistics of three-dimensional homogeneous magnetohydrodynamic turbulence: A wavelet viewpoint. *Phys. Plasmas*, 18, 092304, 2011.

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http://wavelets.ens.fr

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