Phenomenological Theory of small scales turbulence.

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Experimental measurements of spatial (Eulerian) velocity profiles

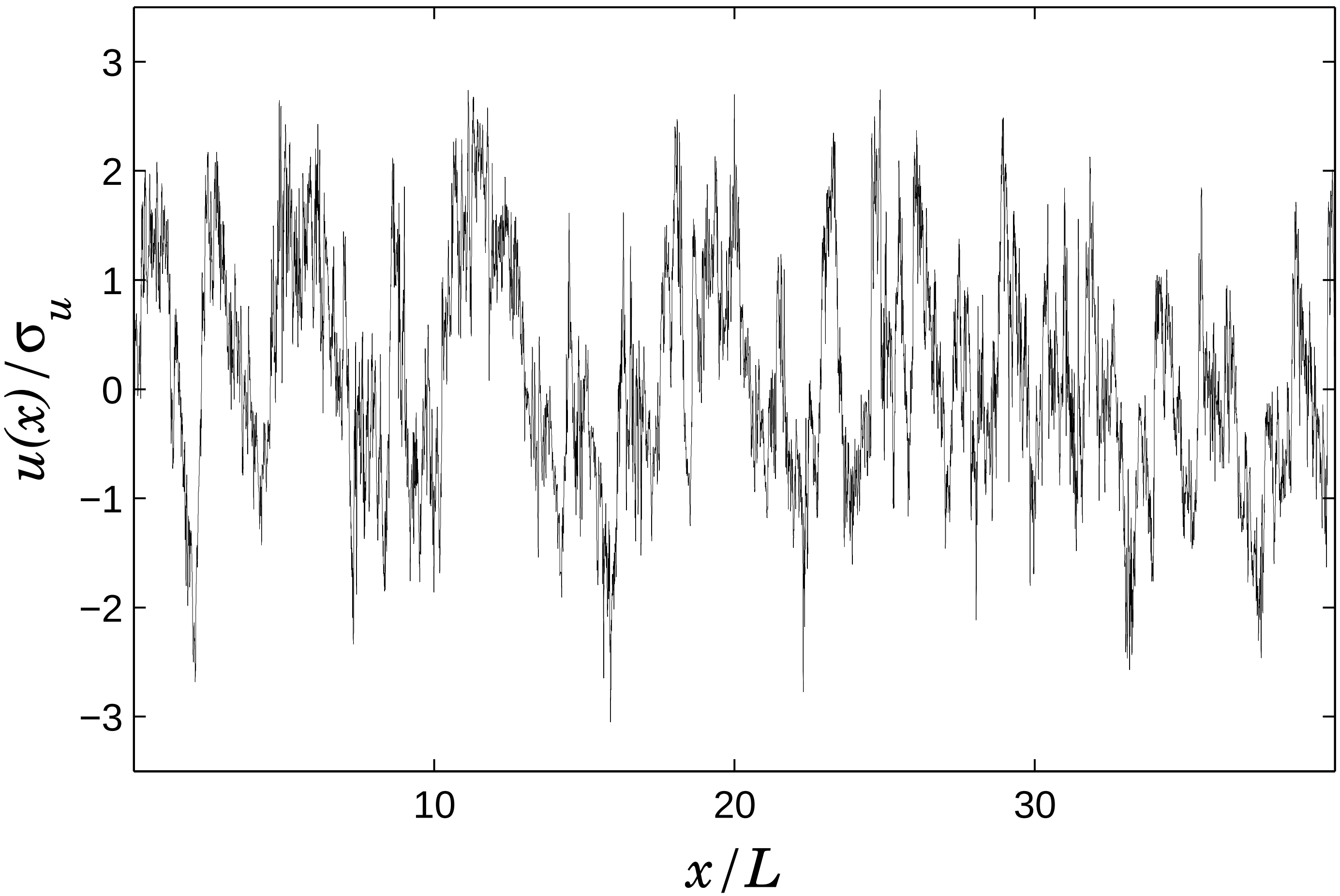
Taylor Hypothesis $\Rightarrow u_x(x)$

- Gagne, Castaing, Hopfinger *et al.* at Modane (1995) : wind tunnel $\mathcal{R}_\lambda \approx 2500$
- Chanal, Chabaud, Castaing, Hébral at Grenoble (2000)
  Helium jet $\mathcal{R}_\lambda \approx 208; 463; 703; 929$
- Baudet and Naert at Lyon (2000)
  Air jet $\mathcal{R}_\lambda \approx 380$

Experimental measurements and numerics of temporal (Lagrangian) velocity profiles

- Mordant-Crawford-Bodenshatz $\mathcal{R}_\lambda \approx 690$
- Mordant-Pinton $\mathcal{R}_\lambda \approx 740$
- A. Arneodo, J. Berg, R. Benzi, L. Biferale, E. Bodenschatz, A. Busse, E. Calzavarini, B. Castaing, M. Cencini, L. Chevillard, R. Fisher, R. Grauer, H. Homann, D. Lamb, A. S. Lanotte, E. Lévêque, B. Luthi, J. Mann, N. Mordant, W.-C. Muller, S. Ott, N. T. Ouellette, J.-F. Pinton, S. B. Pope, S. G. Roux, F. Toschi, H. Xu, and P. K. Yeung...

Longitudinal Velocity profile: Eulerian

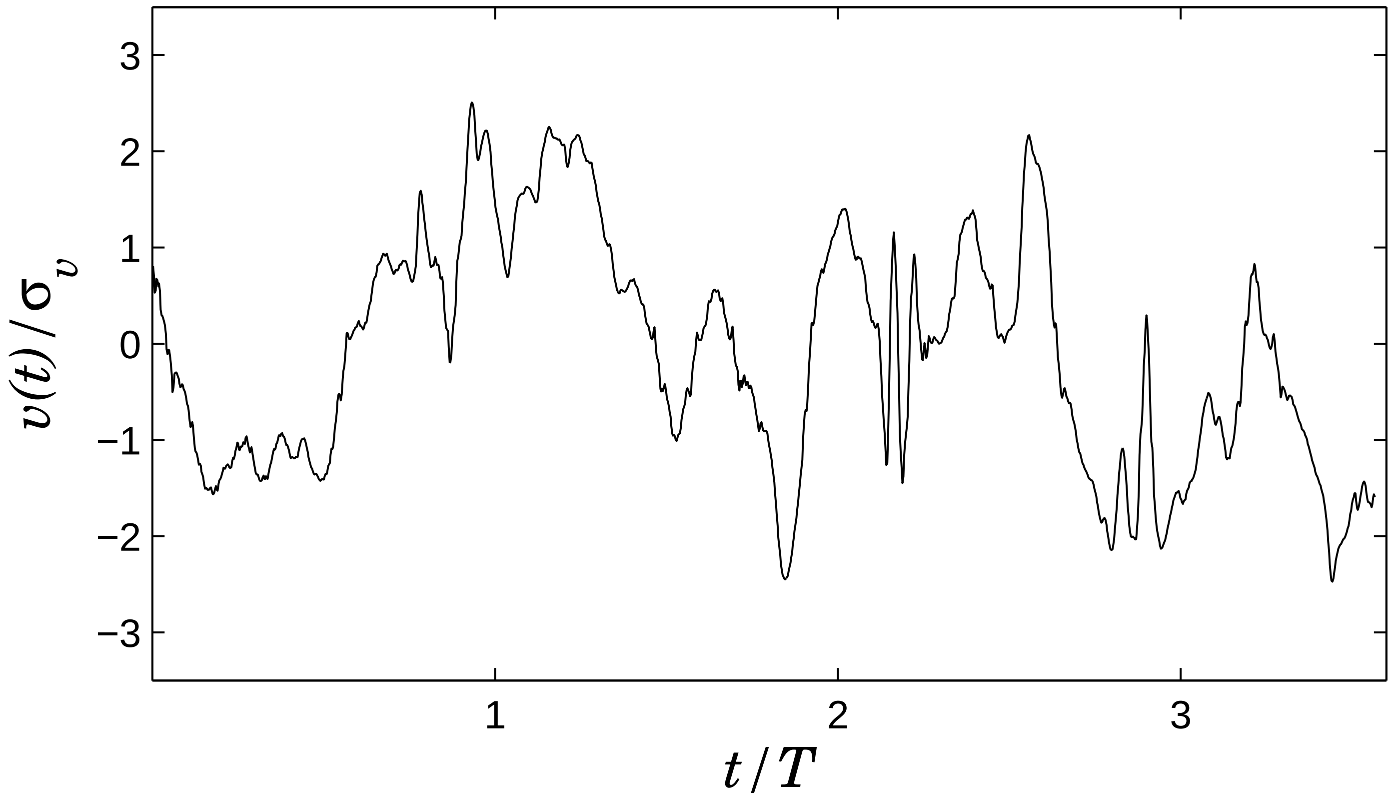


Noise $\leftrightarrow$ Signal

$\rightarrow$ Stochastic modelling

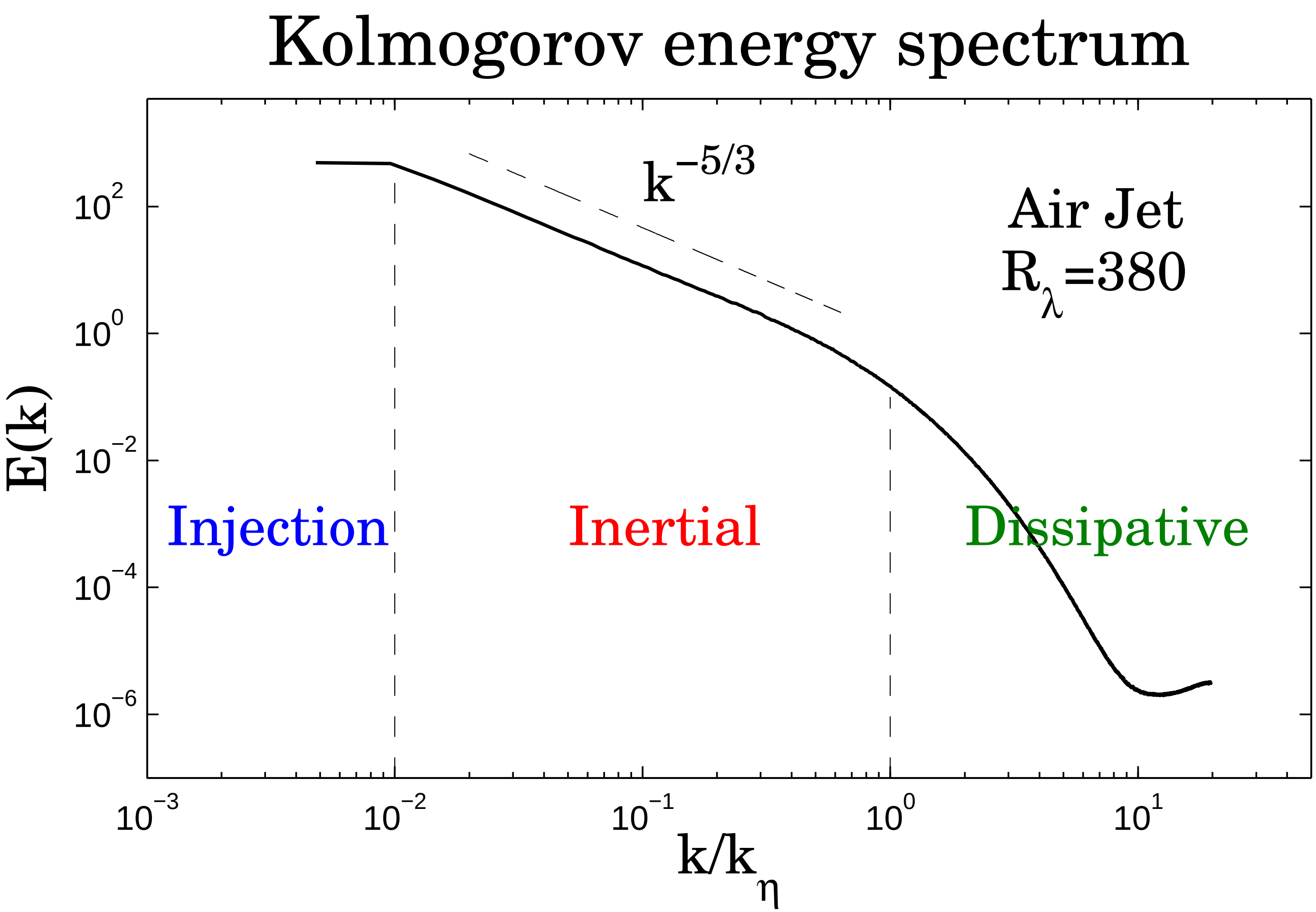
Relationships with Navier-Stokes equations?

Temporal Velocity profile: Lagrangian



$\mathcal{R}_\lambda = 740$ of Mordant et al.

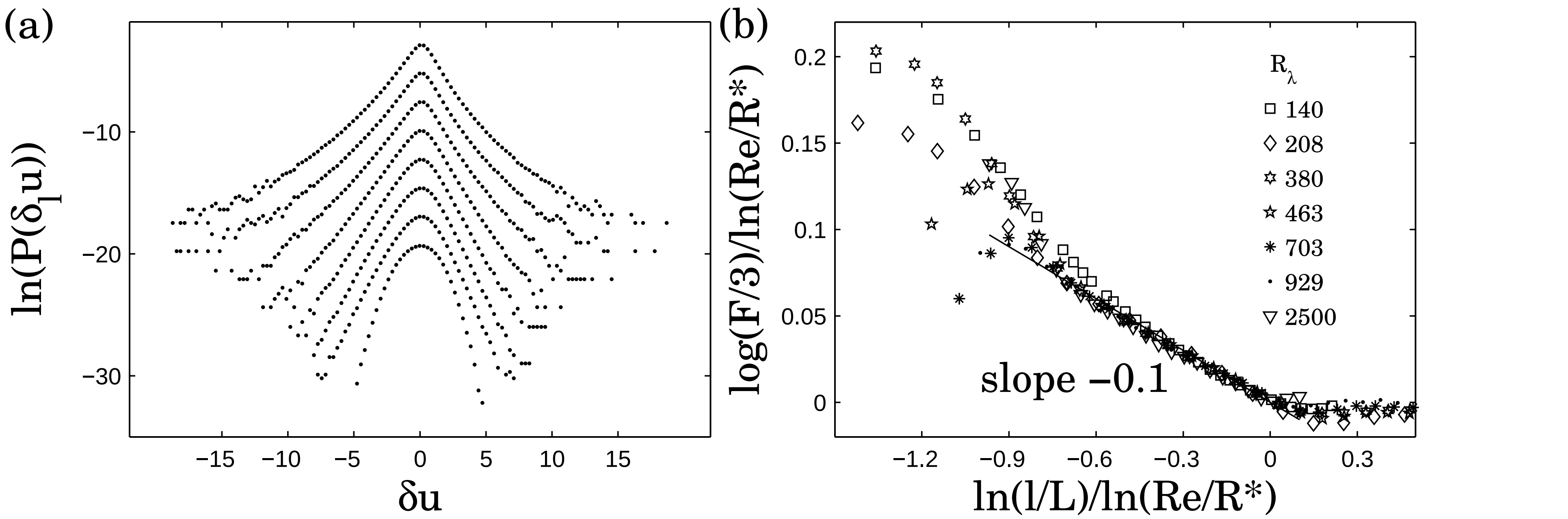
Small scales of turbulence and Kolmogorov phenomenology



Intermittency in Eulerian fluctuations

Eulerian longitudinal velocity increments: $\delta_\ell u(x) = u(x+\ell) - u(x)$

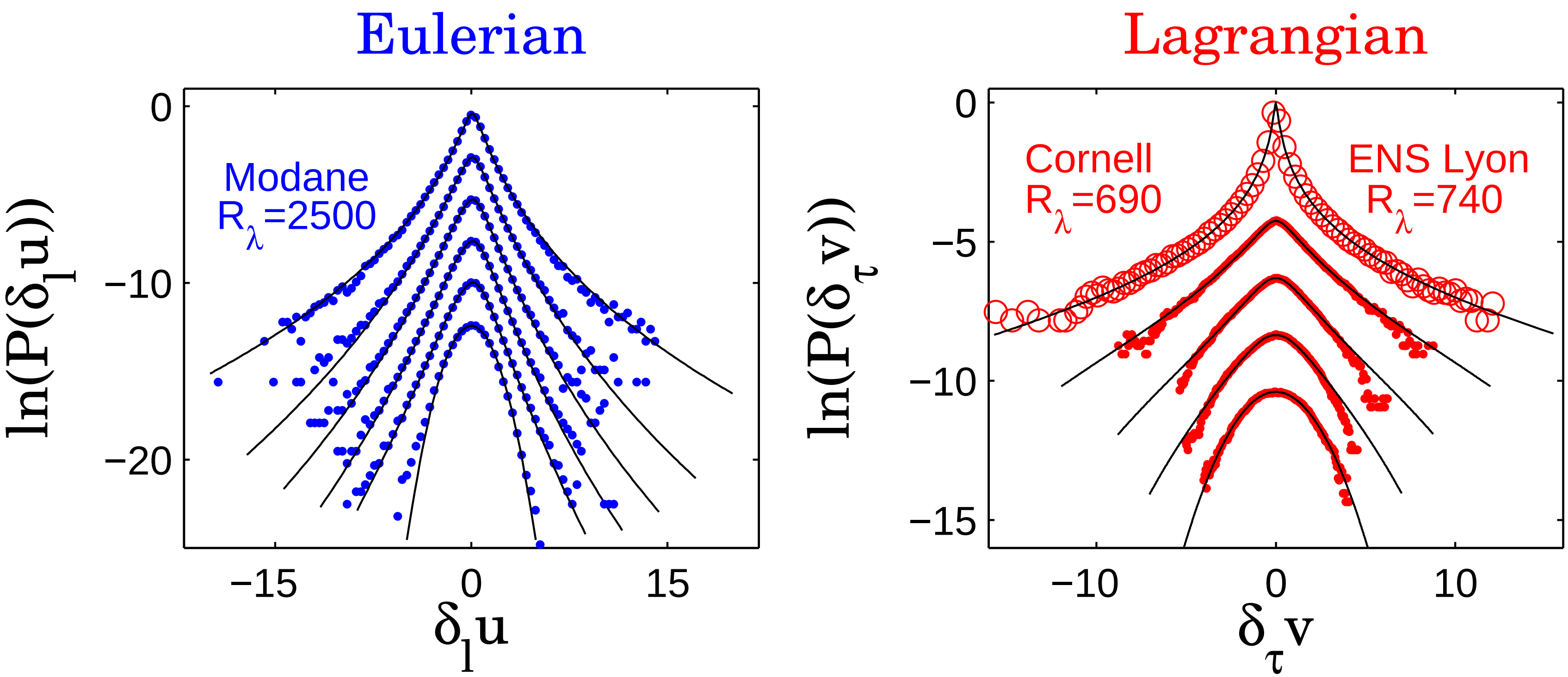
Flatness $F = \frac{\langle (\delta_\ell u)^4 \rangle}{\langle (\delta_\ell u)^2 \rangle^2}$



Universal ***description*** *of Eulerian and Lagrangian velocity fluctuations*

Eulerian longitudinal velocity increments: $\delta_\ell u(x) = u(x+\ell) - u(x)$

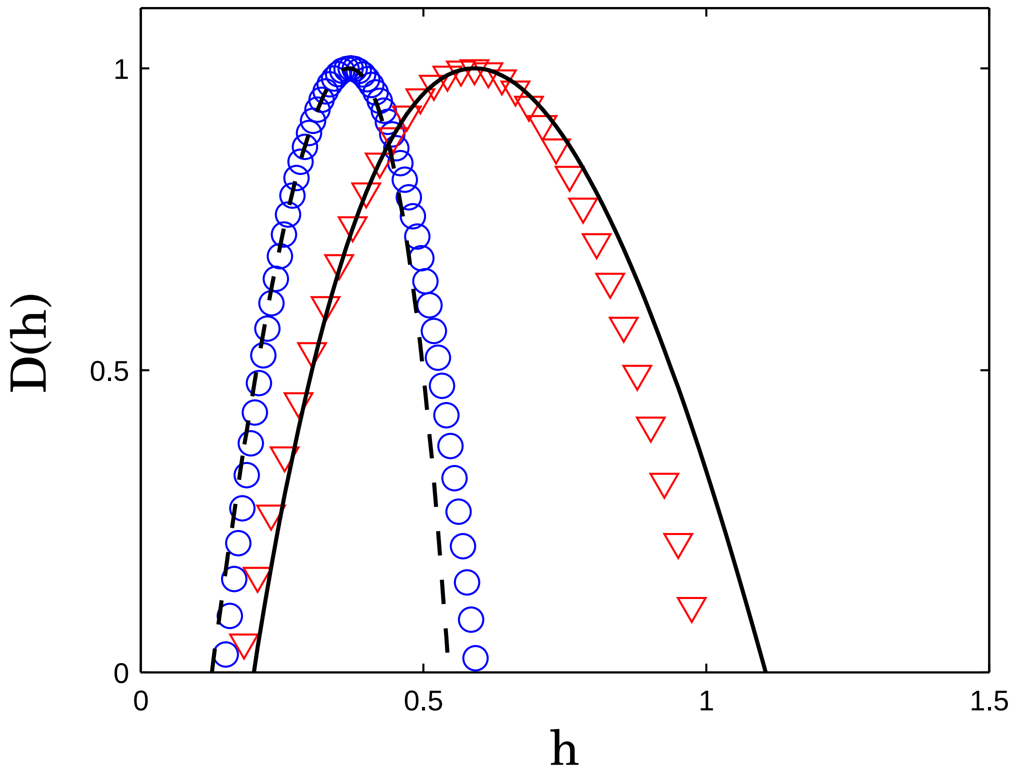
Lagrangian velocity increments: $\delta_\tau v(t) = v(t+\tau) - v(t)$



$\Rightarrow$ **Unique** Singularity Spectrum $\boxed{\mathcal{D}(h)}$

Singularity Spectra of Eulerian and Lagrangian velocity fluctuations

Eulerian longitudinal velocity increments: $\delta_\ell u(x) = u(x+\ell) - u(x)$
Lagrangian velocity increments: $\delta_\tau v(t) = v(t+\tau) - v(t)$



Borgas (93)

$$\Rightarrow \mathcal{D}^L(h) = -h + (1+h)\mathcal{D}^E(h/(1+h))$$

$$\Rightarrow \mathcal{D}^E(h) = h + (1-h)\mathcal{D}^L(h/(1-h))$$

Long-range correlations → ***Multifractals***

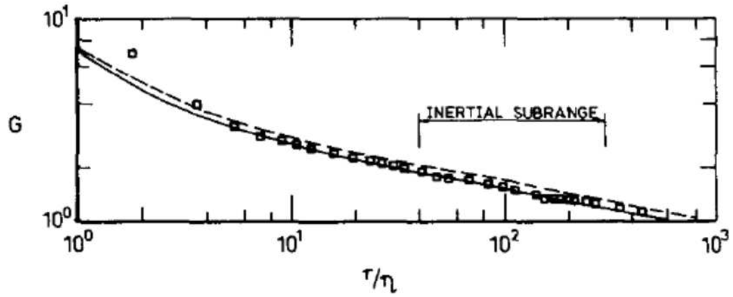
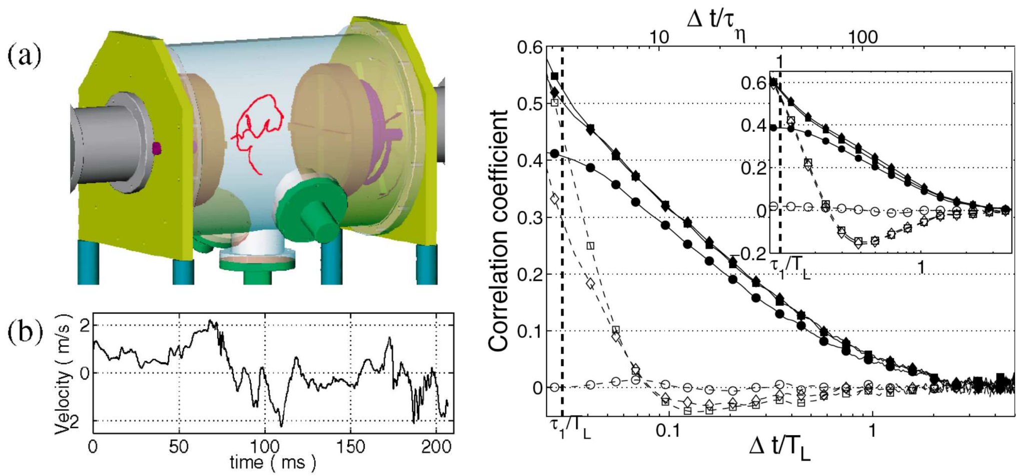


FIG. 1. Autocorrelation of dissipation fluctuations. □, experiment; —, Eq. (6) with $B = 4.92$, $D = 6.83$, and $\mu = 0.2$; ---, Eq. (4) with $C = 6.46$ and $\mu = 0.2$.

Monin-Yaglom (75), Gagne *et al.* (78), Antonia *et al.* (80)



Yeung (97), Mordant et al. (02), Mordant et al. (04)

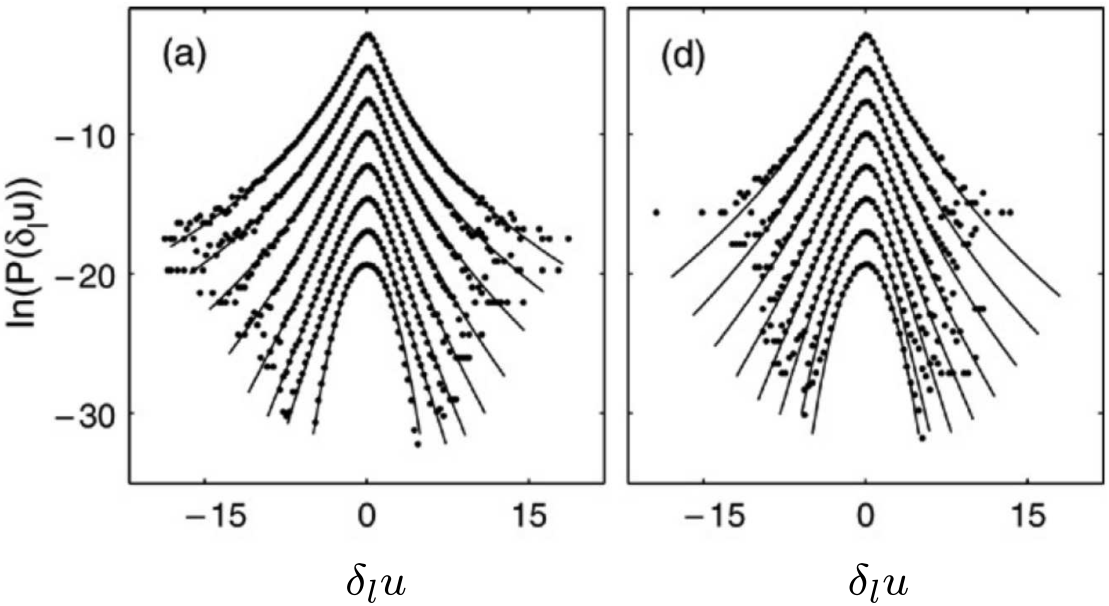
Optimal **description** *of longitudinal fluctuations*

1. **Singularity spectrum** $\mathcal{D}(h)$

- **Inertial** range
  - K41 scalings
  - Intermittent corrections
- **Dissipative** range
  - Reynolds number dependence of the **velocity gradients**
  - Consistent description of the **intermediate** dissipative range

2. a **universal** constant $\mathcal{R}^*$

- **Kolmogorov** constant
- Consistent description of the **Skewness** (Karman-Howarth relation)



(see Chevillard et al. 2006)

Probabilistic **description** *of* **Eulerian** *turbulence*

Chevillard et al. 03, Chevillard et al. 06, + many recent reviews (Benzi-Biferale 09, Boffetta et al. 08, Toschi et al. 09)

$$\text{Eulerian } \delta_\ell u = \overbrace{\underbrace{\beta_\ell}_{\text{stochastic weight}} \times \underbrace{\delta}_{\text{Gaussian noise}}}^{\textbf{independent}} \quad \text{Castaing et al. 90}$$

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Inertial range

$$\beta_\ell(h, \mathcal{R}_e/\mathcal{R}^*) = \left(\frac{\ell}{L}\right)^h$$

$$\mathcal{P}_\ell(h, \mathcal{R}_e/\mathcal{R}^*, \mathcal{D}) = \frac{1}{\mathcal{Z}(\ell)}\left(\frac{\ell}{L}\right)^{1-\mathcal{D}(h)}$$

Probabilistic ***description*** *of* ***Eulerian*** *turbulence*

Chevillard et al. 03, Chevillard et al. 06, + many recent reviews (Benzi-Biferale 09, Boffetta et al. 08, Toschi et al. 09)

Eulerian $\delta_\ell u = \overbrace{\underbrace{\beta_\ell}_{\text{stochastic weight}} \times \underbrace{\delta}_{\text{Gaussian noise}}}^{\textbf{independent}}$ Castaing et al. 90

Dissipative range

$$\beta_\ell(h, \mathcal{R}_e/\mathcal{R}^*) = \frac{\ell}{L}\left(\frac{\eta(h)}{L}\right)^{h-1} \equiv \ell\partial u$$

$$\mathcal{P}_\ell(h, \mathcal{R}_e/\mathcal{R}^*, \mathcal{D}) = \frac{1}{\mathcal{Z}(0)}\left(\frac{\eta(h)}{L}\right)^{1-\mathcal{D}(h)} \text{ and } \eta(h) = L\left(\frac{\mathcal{R}_e}{\mathcal{R}^*}\right)^{-\frac{1}{1+h}}$$

Probabilistic **description** *of* **Eulerian** *turbulence*

Chevillard et al. 03, Chevillard et al. 06, + many recent reviews (Benzi-Biferale 09, Boffetta et al. 08, Toschi et al. 09)

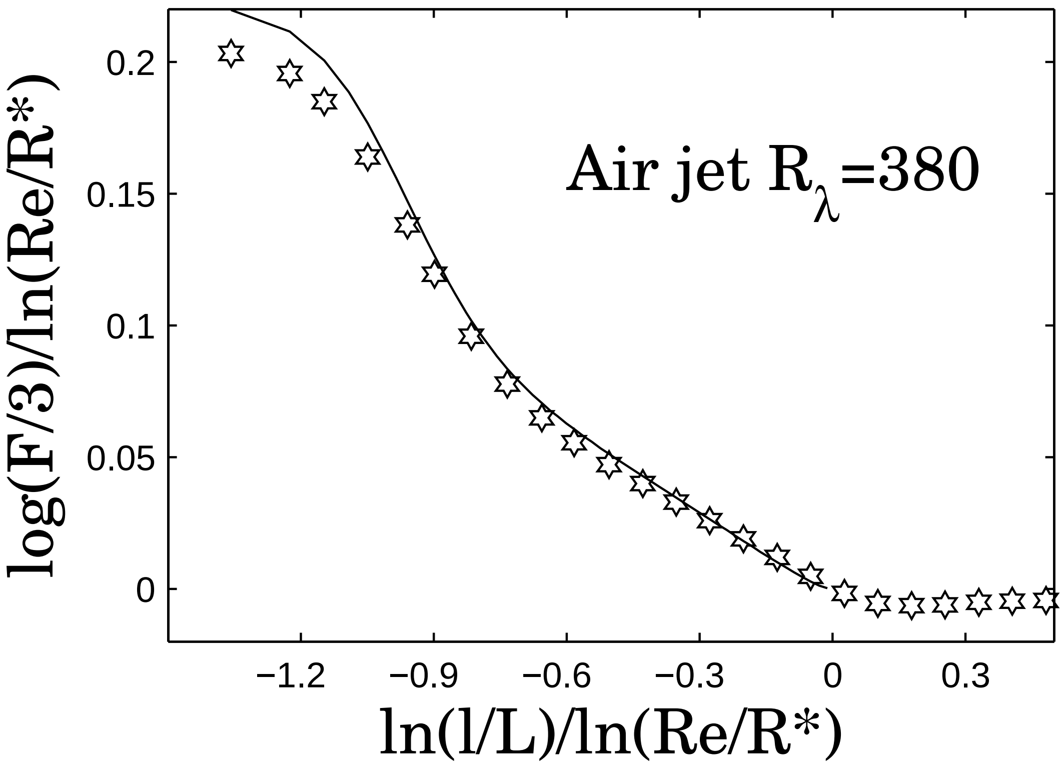
$$\text{Eulerian } \delta_\ell u = \overbrace{\underbrace{\beta_\ell}_{\text{stochastic weight}} \times \underbrace{\delta}_{\text{Gaussian noise}}}^{\textbf{independent}} \quad \text{Castaing et al. 90}$$

Batchelor's interpolation for both Inertial and Dissipative ranges

$$\beta_\ell(h, \mathcal{R}_e/\mathcal{R}^*) = \frac{\left(\frac{\ell}{L}\right)^h}{\left[1+\left(\frac{\ell}{\eta(h)}\right)^{-2}\right]^{(1-h)/2}}$$

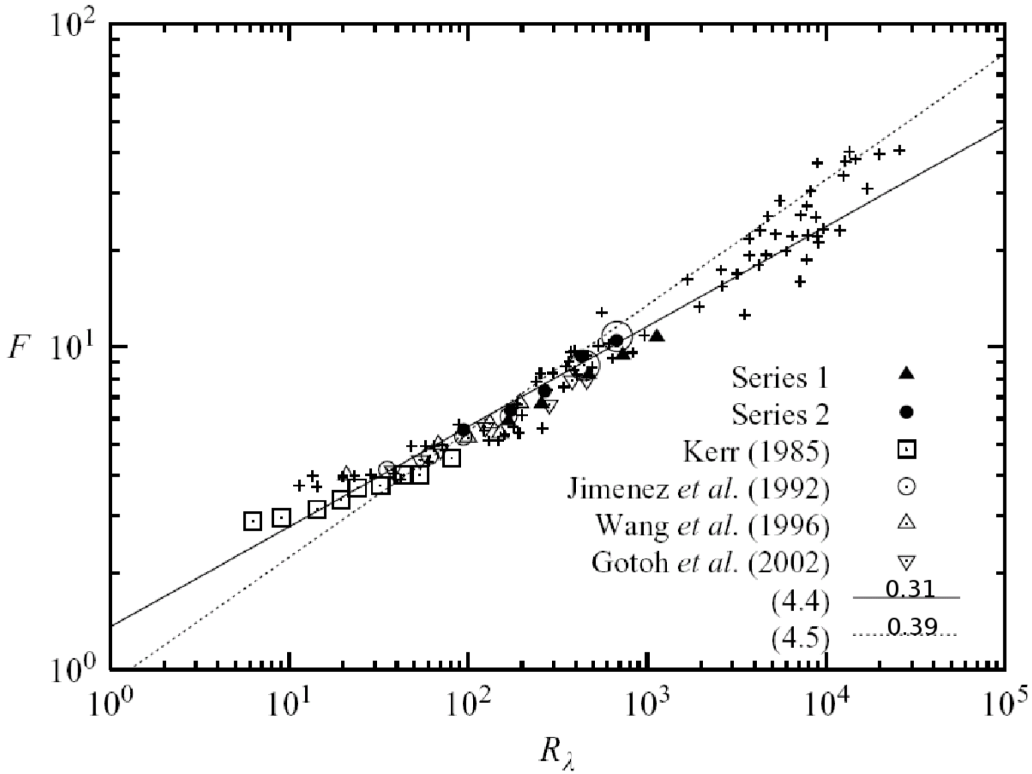
$$\mathcal{P}_\ell(h, \mathcal{R}_e/\mathcal{R}^*, \mathcal{D}) = \frac{1}{\mathcal{Z}(\ell)} \frac{\left(\frac{\ell}{L}\right)^{1-\mathcal{D}(h)}}{\left[1+\left(\frac{\ell}{\eta(h)}\right)^{-2}\right]^{(\mathcal{D}(h)-1)/2}} \text{ and } \eta(h) = L\left(\frac{\mathcal{R}_e}{\mathcal{R}^*}\right)^{-\frac{1}{1+h}}$$

*Prediction of **Eulerian** intermediate dissipative range*



Prediction of Eulerian gradients

(plot from Ishihara et al. 07)



Paladin-Vulpiani (87) + Nelkin (90) $\Rightarrow F \sim \mathcal{R}_\lambda^{\chi_4 - 2\chi_2}$ and $\chi_4 - 2\chi_2 \approx 0.37$

with $\chi_n = \min_h -\frac{n(h-1)+1-\mathcal{D}^E(h)}{1+h}$

Probabilistic **description** *of* ***Lagrangian*** *turbulence*

Chevillard et al. 03, Arneodo et al. (ICTR et al.) 09 + many recent reviews
(Benzi-Biferale 09, Boffetta et al. 08, Toschi et al. 09)

$$\text{Lagrangian } \delta_\tau v = \overbrace{\underbrace{\beta_\tau}_{\text{stochastic weight}} \times \underbrace{\delta}_{\text{Gaussian noise}}}^{\textbf{independent}} \quad \text{Castaing et al. 90}$$

$$\beta_\tau(h, \mathcal{R}_e/\mathcal{R}^*) = \frac{\left(\frac{\tau}{T}\right)^h}{\left[1 + \left(\frac{\tau}{\tau_\eta(h)}\right)^{-\delta}\right]^{(1-h)/2}}$$

$$\mathcal{P}_\tau(h, \mathcal{R}_e/\mathcal{R}^*, \mathcal{D}^L) = \frac{1}{\mathcal{Z}(\tau)} \frac{\left(\frac{\tau}{T}\right)^{1-\mathcal{D}^L(h)}}{\left[1 + \left(\frac{\tau}{\tau_\eta(h)}\right)^{-\delta}\right]^{(\mathcal{D}^L(h)-1)/2}} \text{ and } \tau_\eta(h) = T\left(\frac{\mathcal{R}_e}{\mathcal{R}^*}\right)^{-\frac{1}{1+2h}}$$

Prediction of Lagrangian intermediate dissipative range

From Chevillard et al. 03 and Arneodo et al. (ICTR) 09:

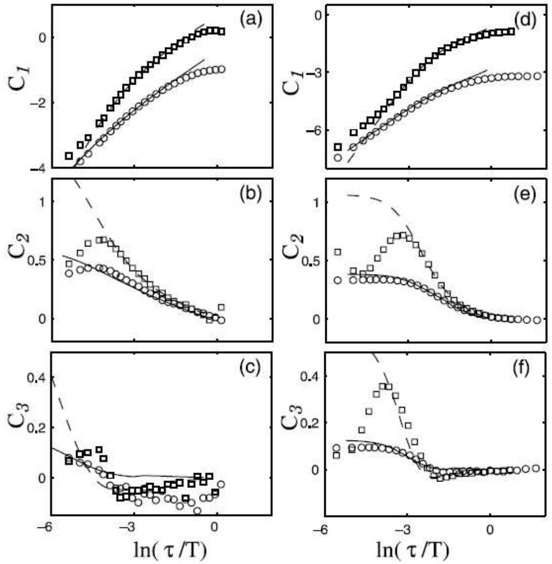
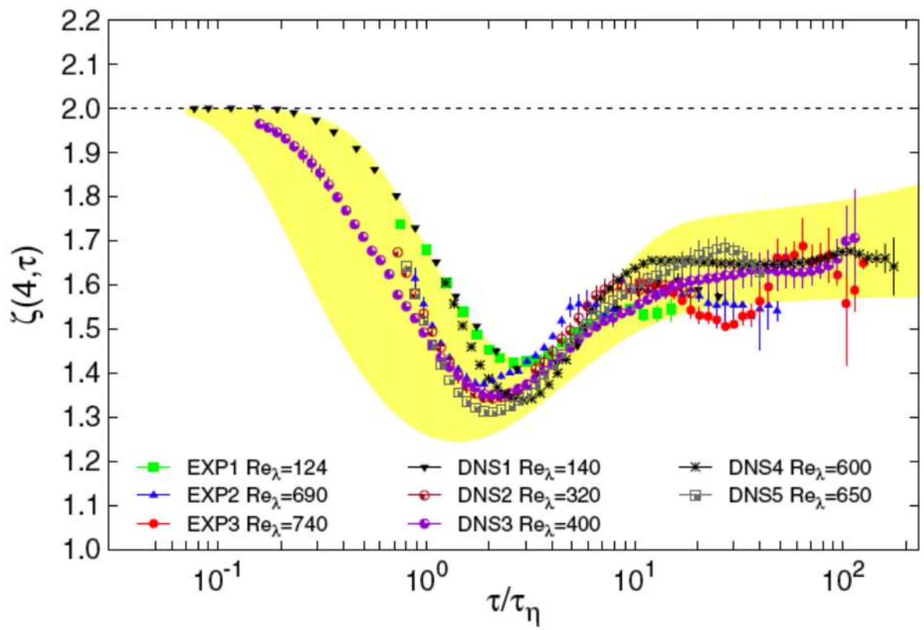


FIG. 3. Cumulant analysis $C_n(\tau/T)$ vs $\ln\tau/T$ for (a)–(c) ENS-Lyon experimental data and (d)–(f) DNS data. (○) first-order velocity increments ($N = 1$); (□) second-order velocity increments ($N = 2$). The associated multifractal descriptions for $N = 1$ (solid lines) and $N = 2$ (dashed lines) correspond to the same parameter values as in Fig. 1. Curves in (a),(d) are vertically shifted for clarity.

$C_2 = Var[\ln|\delta_\tau v|]$ and $\zeta(4,\tau) = \frac{d\ln\langle(\delta_\tau v)^4\rangle}{d\ln\langle(\delta_\tau v)^2\rangle}$



Prediction of Lagrangian gradients ≡ acceleration

(plot from Ishihara et al. 07)

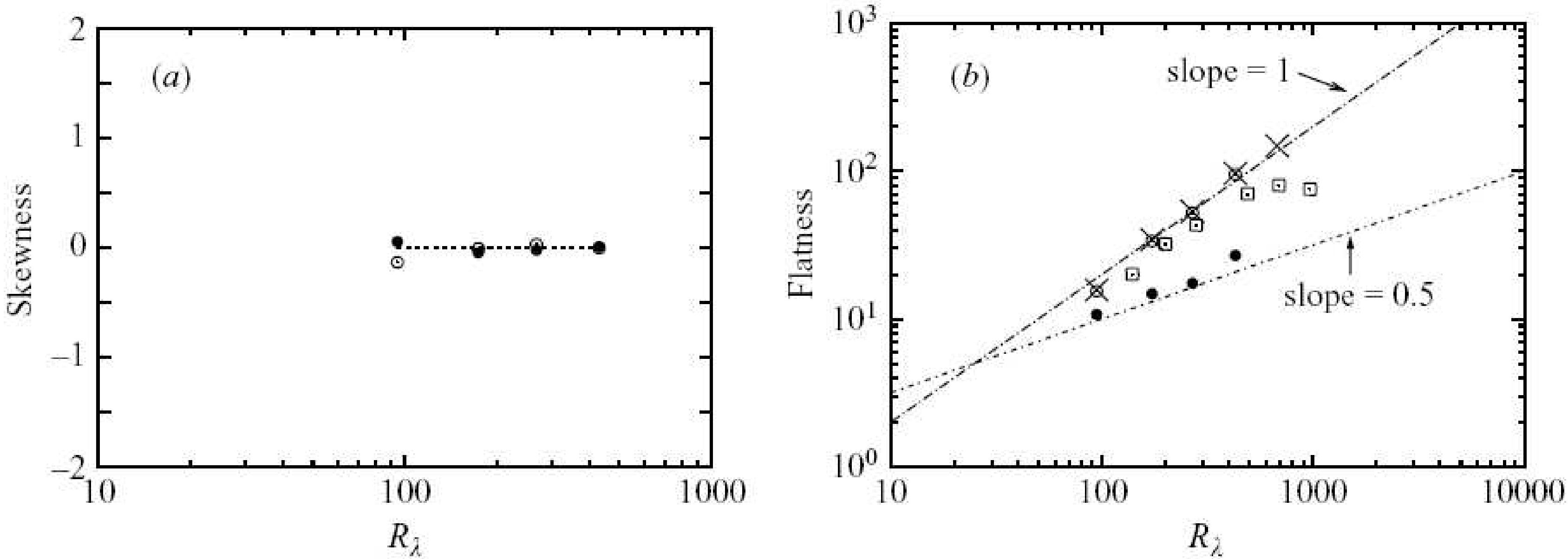


FIGURE 11. (a) Skewness of $a_i = \partial u_i/\partial t$ (●) and $A_i = \mathrm{D}u_i/\mathrm{D}t$ (○), and (b) flatness factors of a_i (●), A_i (○) and $\partial p/\partial x_i$ (×). Squares (□) are experimental data for A_i by La Porta *et al.* (2001).

$F \sim \mathcal{R}_\lambda^{\chi_4 - 2\chi_2}$ with $\chi_n = \min_h - \frac{n(h-1)+1-\mathcal{D}^L(h)}{1+2h}$

- if $\mathcal{D}^L(h)$ quadratic ⇒ $\chi_4 - 2\chi_2 \approx 1.8$
- if $\mathcal{D}^E(h)$ quadratic ⇒ (Borgas) $\chi_4 - 2\chi_2 \approx 0.92$
- if $\mathcal{D}^E(h)$ She-Leveque ⇒ (Borgas) $\chi_4 - 2\chi_2 \approx 0.7$

The critical Reynolds number $\mathcal{R}^*$

Definition (through the fluctuating dissipative length scale):

$$\frac{\eta(h)}{L} = \left(\frac{\mathcal{R}_e}{\mathcal{R}^*}\right)^{-\frac{1}{h+1}}$$

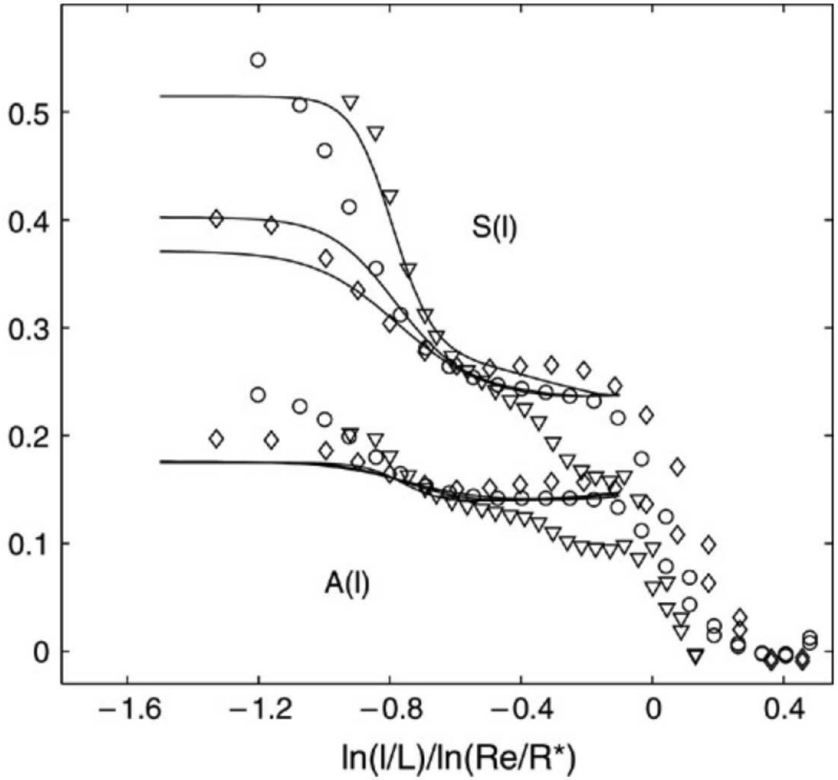
- $\mathcal{R}_e = \frac{\sigma L}{\nu}$ Reynolds number
- $\sigma^2 = \langle (\delta_L u)^2 \rangle$ the large-scale variance
- h holder exponent, $h = 1/3$ in K41

$\overset{\text{EXP}}{\Rightarrow}$ $\boxed{\mathcal{R}^* = 52}$ **Universal** (see Gagne et al. 2004)

- First implication: mean dissipation (neglecting intermittency)
  $\langle \epsilon \rangle = \frac{15}{\mathcal{R}^*} \frac{\sigma^3}{L}$
- Second implication: Kolmogorov constant c_K (neglecting intermittency)
  $c_K = \left(\frac{\mathcal{R}^*}{15}\right)^{2/3} \approx 2.3$

Skewness and $\mathcal{R}^*$

$$\mathcal{S}(\ell) = -\frac{\langle (\delta_\ell u)^3 \rangle}{\langle (\delta_\ell u)^2 \rangle^{3/2}} \text{ and } \mathcal{A}(\ell) = -\frac{\langle (\delta_\ell u)^3 \rangle}{\langle |\delta_\ell u|^3 \rangle}$$

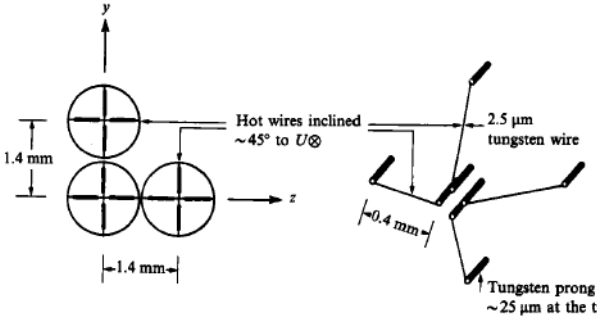


$\diamond$ ($\mathcal{R}_\lambda = 208$)
$\circ$ ($\mathcal{R}_\lambda = 380$)
$\triangledown$ ($\mathcal{R}_\lambda = 2500$)

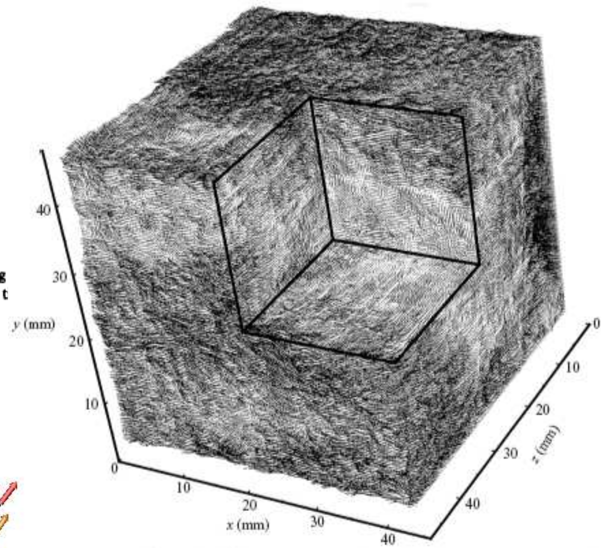
- Inertial range (using KH) $\langle (\delta_\ell u)^3 \rangle = -\frac{4}{5}\langle \epsilon \rangle \ell$
  - $\mathcal{S}(\ell) = \frac{12}{\mathcal{R}^*} \approx 0.23$
  - $\mathcal{A}(\ell) = \frac{3\sqrt{2\pi}}{\mathcal{R}^*} \approx 0.145$
- Full-line $\rightarrow$ MF predictions
- EDQNM predictions on the way...

$\mathcal{S}(\ell) = \frac{12}{\mathcal{R}^*} \leftarrow$ Phenomenological $\rightarrow c_K = \left(\frac{\mathcal{R}^*}{15}\right)^{2/3}$

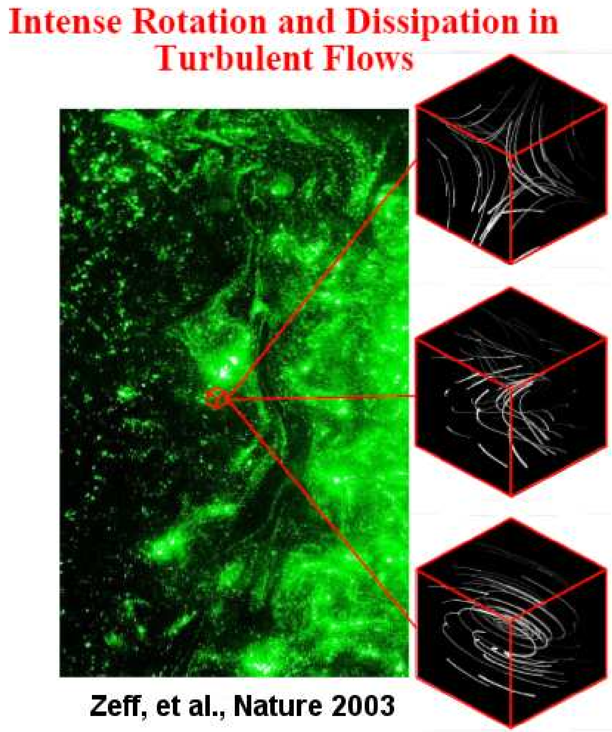
3D Fluid **Turbulence**: Full velocity gradients



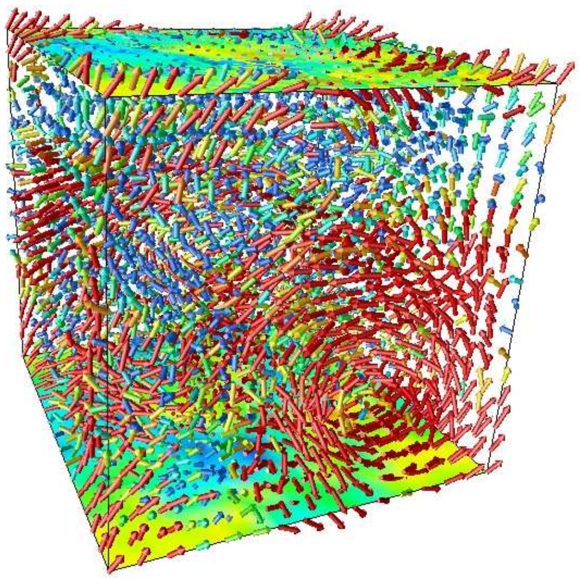
Tsinober, Kit, Dracos J.F.M. 1992



Tao, Katz, Meneveau J.F.M. 2002



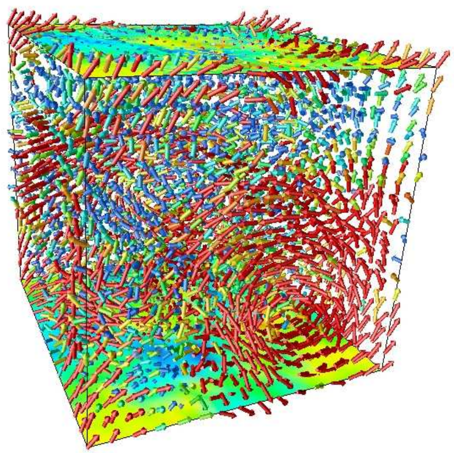
Zeff, et al., Nature 2003



Direct Numerical Simulations
(picture by Toschi)

$$\mathbf{A} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

Tracking Velocity Gradients along Lagrangian trajectories



DNS

- Yeung & Pope (89).
- Girimaji & Pope, J.F.M. (90).
- Pope & Chen , Phys. Fluids (90).

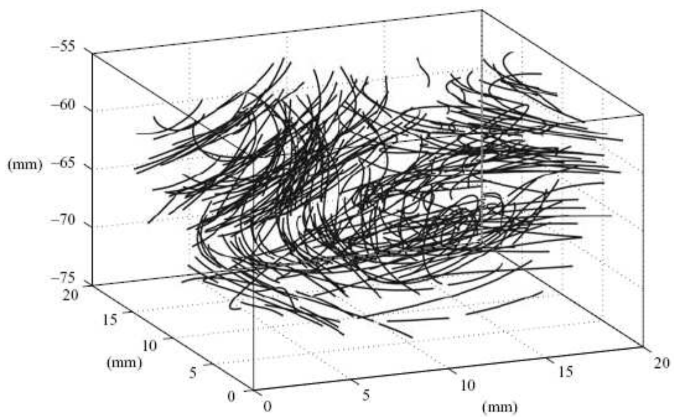


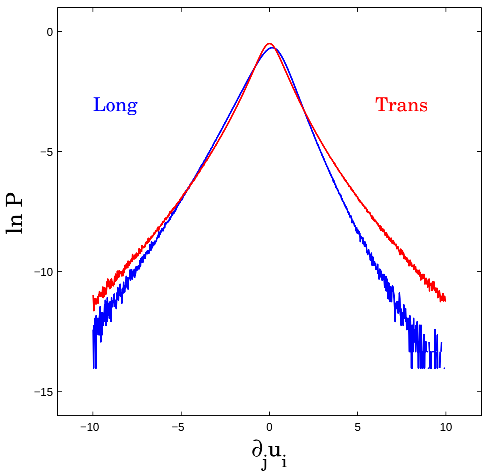
FIGURE 2. Selected particle trajectories as obtained from 3D-PTV.

Experimental

- Zeff *et al.*, Nature (2003).
- Lüthi, Tsinober & Kinzelbach, J.F.M. (2005).

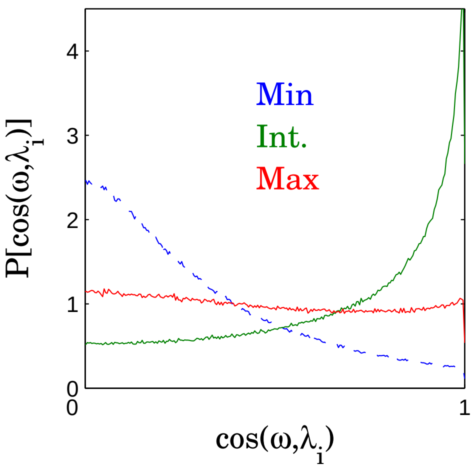
Statistical ***Intermittency*** *and* ***Geometry*** *in turbulence*

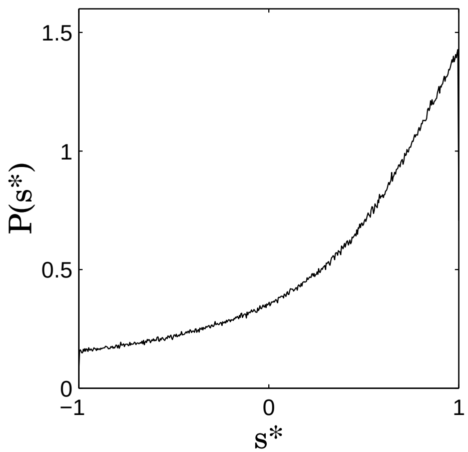
DNS Results $\mathcal{R}_\lambda = 150$



Intermittency

- Non-Gaussianity
- Skewness
- Anomalous scaling with Reynolds number



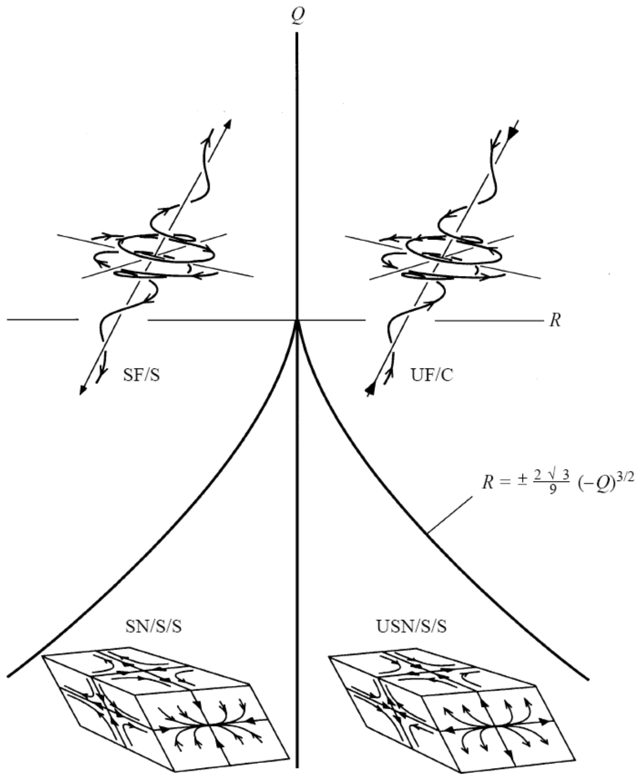


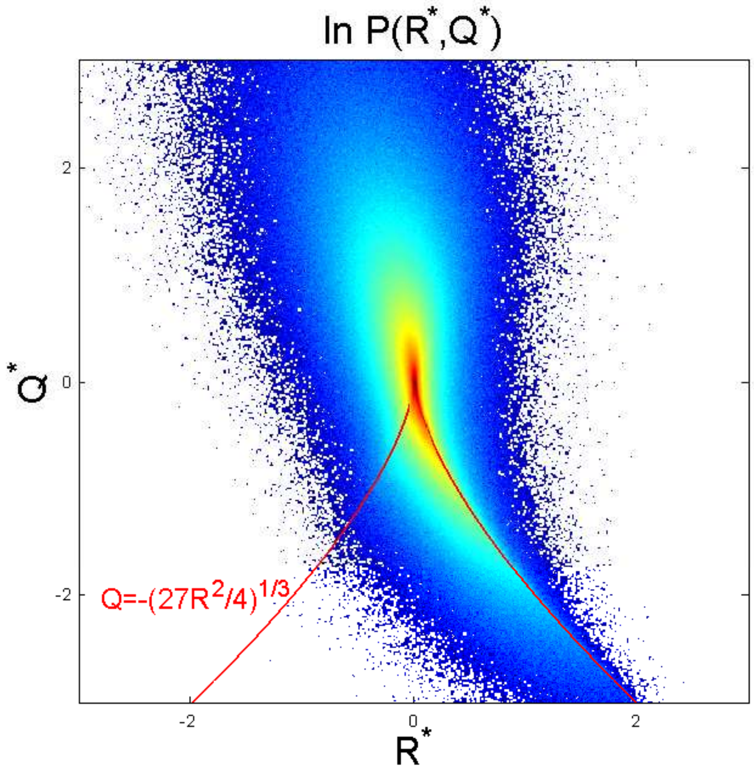
Geometry

- Preferential alignment of vorticity
- Preferential axisymmetric expansion

The RQ plane - Local Topology

See Chong, Perry and Cantwell (90) and Cantwell (93)





$\lambda_i = f(R,Q) \in \mathbb{C}$: Eigenvalues of $\mathbf{A}$

- Second invariant: $Q = -\frac{1}{2}\mathrm{Tr}(\mathbf{A}^2) = \frac{1}{4} \overbrace{|\omega|^2}^{\text{Enstrophy}} - \frac{1}{2} \overbrace{\mathrm{Tr}(\mathbf{S}^2)}^{\text{Dissipation}}$
- Third invariant: $R = -\frac{1}{3}\mathrm{Tr}(\mathbf{A}^3) = -\frac{1}{4} \underbrace{\omega_i S_{ij} \omega_j}_{\text{Enstrophy Production}} - \frac{1}{3} \underbrace{\mathrm{Tr}(\mathbf{S}^3)}_{\text{Strain Skewness}}$