2-D hybrid simulations of kinetic plasma turbulence

Lorenzo Matteini
Imperial College London

and
Luca Franci, Simone Landi, Andrea Verdini, University of Florence
and Petr Hellinger, Astronomical Inst. Prague

Franci et al., ApJ Letters 2015 (arXiv:1503.05457)
Franci et al., to be submitted

Several numerical approaches:

- Hall MHD: Matthaeus 2003,...
- Electron MHD: Meyrand&Galtier 2013
- Gyrokinetics: Howes et al 2008, 2011
- Landau Fluid: Passot et al 2014
- Hybrid-PIC: Markoski&Vasquez 2011
- Hybrid-Vlasov: Servidio 2012, Perrone 2013, Valentini 2014
- Full PIC: Camporeale&Burgess 2011
- ...

Hybrid approximation

Kinetic ions (PIC) and fluid electrons

- MHD ✔
- ion scales ✔
- electron scales ✗

isothermal electrons:
$P_e = n k_B T_e$
physical limit:
$kd_e = 1$ or $kd_p \sim 40$

Hybrid approximation

Kinetic ions (PIC) and fluid electrons

- MHD ✔
- ion scales ✔
- electron scales ✗

isothermal electrons:
$P_e = n k_B T_e$
physical limit:
$kd_e = 1$ or $kd_p \sim 40$

Ohm's law for the electric field:

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{n} - \frac{\nabla P_e}{n}$$

Hybrid approximation

Kinetic ions (PIC) and fluid electrons

- MHD ✔
- ion scales ✔
- electron scales ✗

isothermal electrons:
$P_e = nk_B T_e$
physical limit:
$kd_e = 1$ or $kd_p \sim 40$

Ohm's law for the electric field:

$$\mathbf{E} = \boxed{-\mathbf{V} \times \mathbf{B}} + \frac{\mathbf{J} \times \mathbf{B}}{n} - \frac{\nabla P_e}{n}$$

ideal MHD

Hybrid approximation

Kinetic ions (PIC) and fluid electrons

- MHD ✔
- ion scales ✔
- electron scales ✗

isothermal electrons:
$P_e = nk_B T_e$
physical limit:
$kd_e = 1$ or $kd_p \sim 40$

Ohm's law for the electric field:

$$\mathbf{E} = \boxed{-\mathbf{V} \times \mathbf{B}} + \boxed{\frac{\mathbf{J} \times \mathbf{B}}{n}} - \frac{\nabla P_e}{n}$$

ideal MHD

Hall-MHD

Hybrid approximation

Kinetic ions (PIC) and fluid electrons

- MHD ✔
- ion scales ✔
- electron scales ✗

isothermal electrons:
$P_e = nk_B T_e$
physical limit:
$kd_e = 1$ or $kd_p \sim 40$

Ohm's law for the electric field:

$$\mathbf{E} = \boxed{-\mathbf{V} \times \mathbf{B}} + \boxed{\frac{\mathbf{J} \times \mathbf{B}}{n}} - \boxed{\frac{\nabla P_e}{n}}$$

ideal MHD

Hall-MHD

density gradient

Hybrid approximation

Kinetic ions (PIC) and fluid electrons

- MHD ✔
- ion scales ✔
- electron scales ✗

isothermal electrons:
$P_e = n k_B T_e$
physical limit:
$kd_e = 1$ or $kd_p \sim 40$

Ohm's law for the electric field:

$$\mathbf{E} = \boxed{-\mathbf{V} \times \mathbf{B}} + \boxed{\frac{\mathbf{J} \times \mathbf{B}}{n}} - \boxed{\frac{\nabla P_e}{n}} + \eta \mathbf{J}$$

resistivity (small)

ideal MHD

Hall-MHD

density gradient

Simulation Setup

Units and normalizations:

space: $d_p = c/\omega_p = V_A/\Omega_p$
time: Ω_p^{-1}
magnetic field: B_0
velocity: V_A

2048x2048 grid
$\Delta t = 0.025$, $\Delta x = \Delta y = 0.125$
($0.02 < k < 20$ in Fourier space)
$\beta_p = \beta_e = 0.5$
$T_\perp / T_{||} = 1$
$\mathbf{B}_0$ out of simulation plane

Run	$\Delta x/d_p$	L_{box}/d_p	η	ppc
A	0.125	256	5×10^{-4}	8000
B	0.125	256	5×10^{-4}	4000
C	0.125	256	5×10^{-4}	2000
D	0.125	256	5×10^{-4}	1000
E	0.125	256	5×10^{-4}	500
F	0.125	256	1×10^{-4}	8000
G	0.125	256	1×10^{-3}	8000
H	0.250	512	1×10^{-3}	8000
I	0.500	1024	2×10^{-3}	8000

Simulation Setup

Units and normalizations:

space: $d_p = c/\omega_p = V_A/\Omega_p$
time: Ω_p^{-1}
magnetic field: B_0
velocity: V_A

2048x2048 grid
$\Delta t = 0.025$, $\Delta x = \Delta y = 0.125$
($0.02 < k < 20$ in Fourier space)
$\beta_p = \beta_e = 0.5$
$T_\perp / T_{||} = 1$
$\mathbf{B}_0$ out of simulation plane

Run	$\Delta x/d_p$	$L_{\rm box}/d_p$	η	ppc
A	0.125	256	5×10^{-4}	8000
B	0.125	256	5×10^{-4}	4000
C	0.125	256	5×10^{-4}	2000
D	0.125	256	5×10^{-4}	1000
E	0.125	256	5×10^{-4}	500
F	0.125	256	1×10^{-4}	8000
G	0.125	256	1×10^{-3}	8000
H	0.250	512	1×10^{-3}	8000
I	0.500	1024	2×10^{-3}	8000

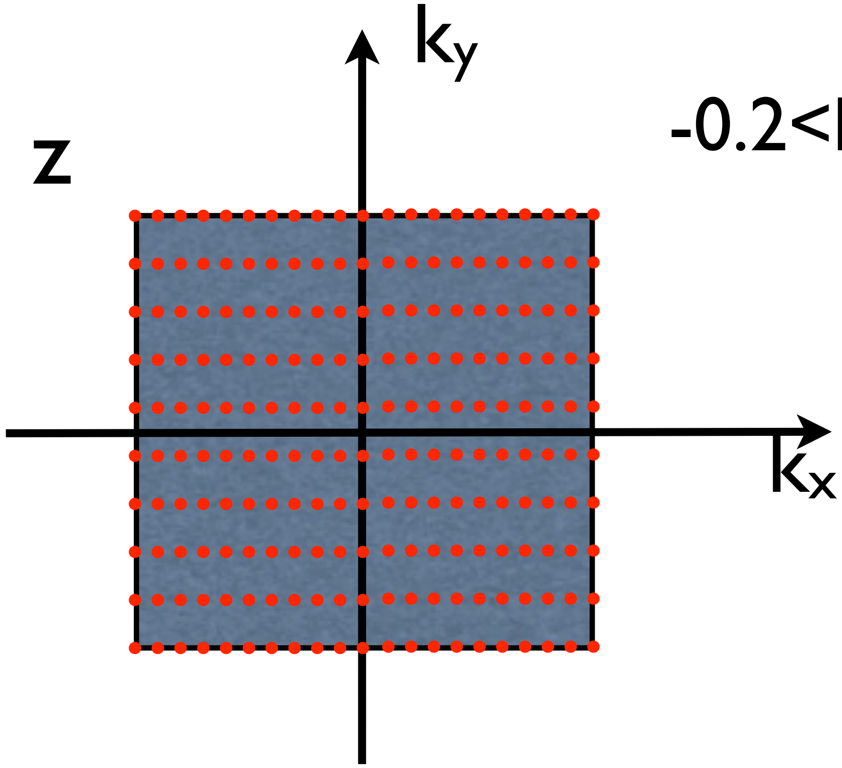
~3×10^{10} in total!

Initialization of in plane fluctuations

k-vector orthogonal to the main field

$B_0 \parallel z$

$-0.2 < k_{x,y} < 0.2$

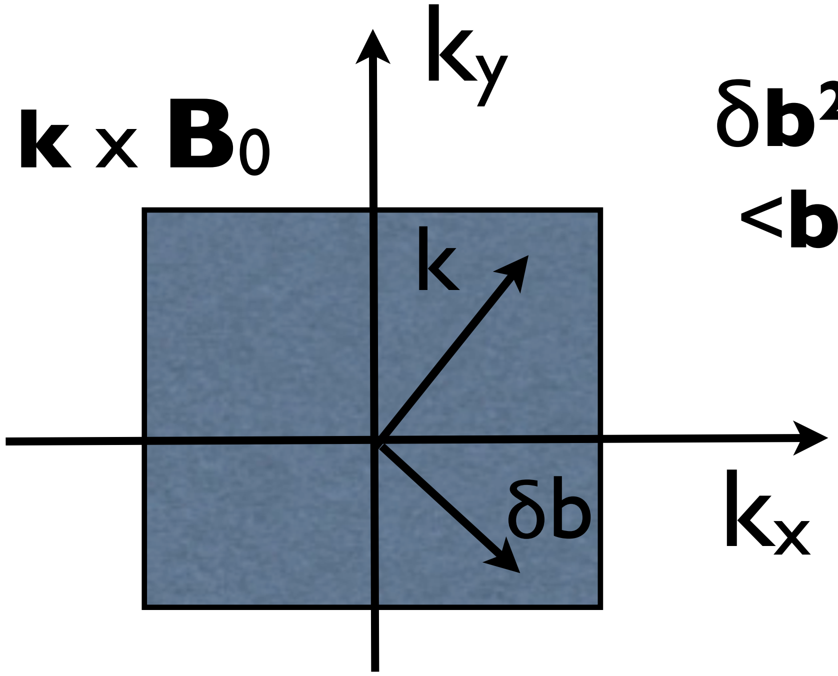


linear polarization

$\delta \mathbf{b} = \mathbf{k} \times \mathbf{B}_0$

$\delta \mathbf{b}^2 = \delta \mathbf{u}^2$

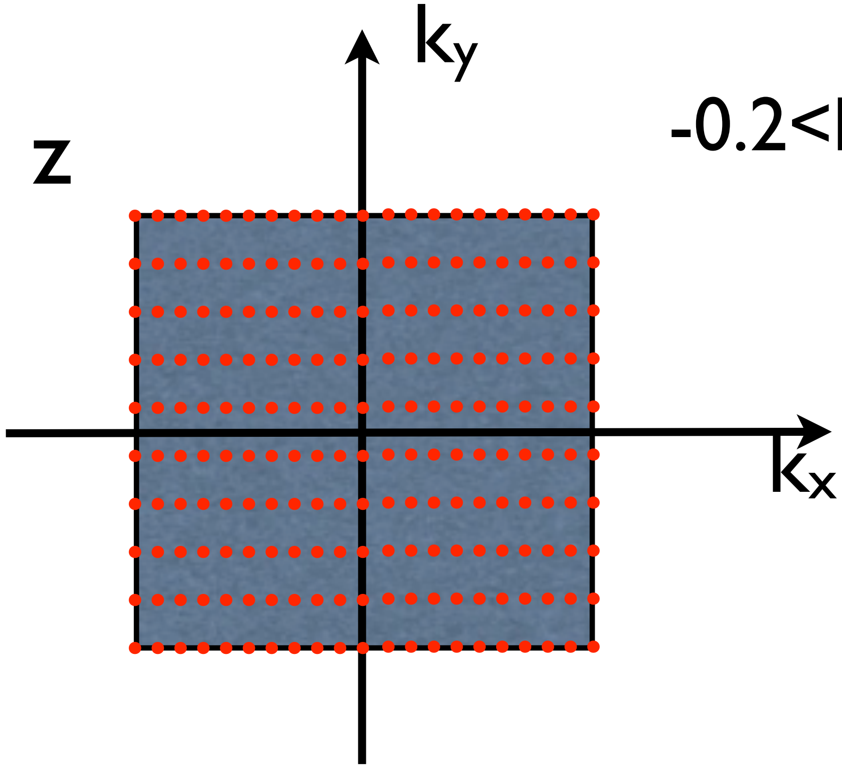
$\langle \mathbf{b}.\mathbf{u} \rangle = 0$



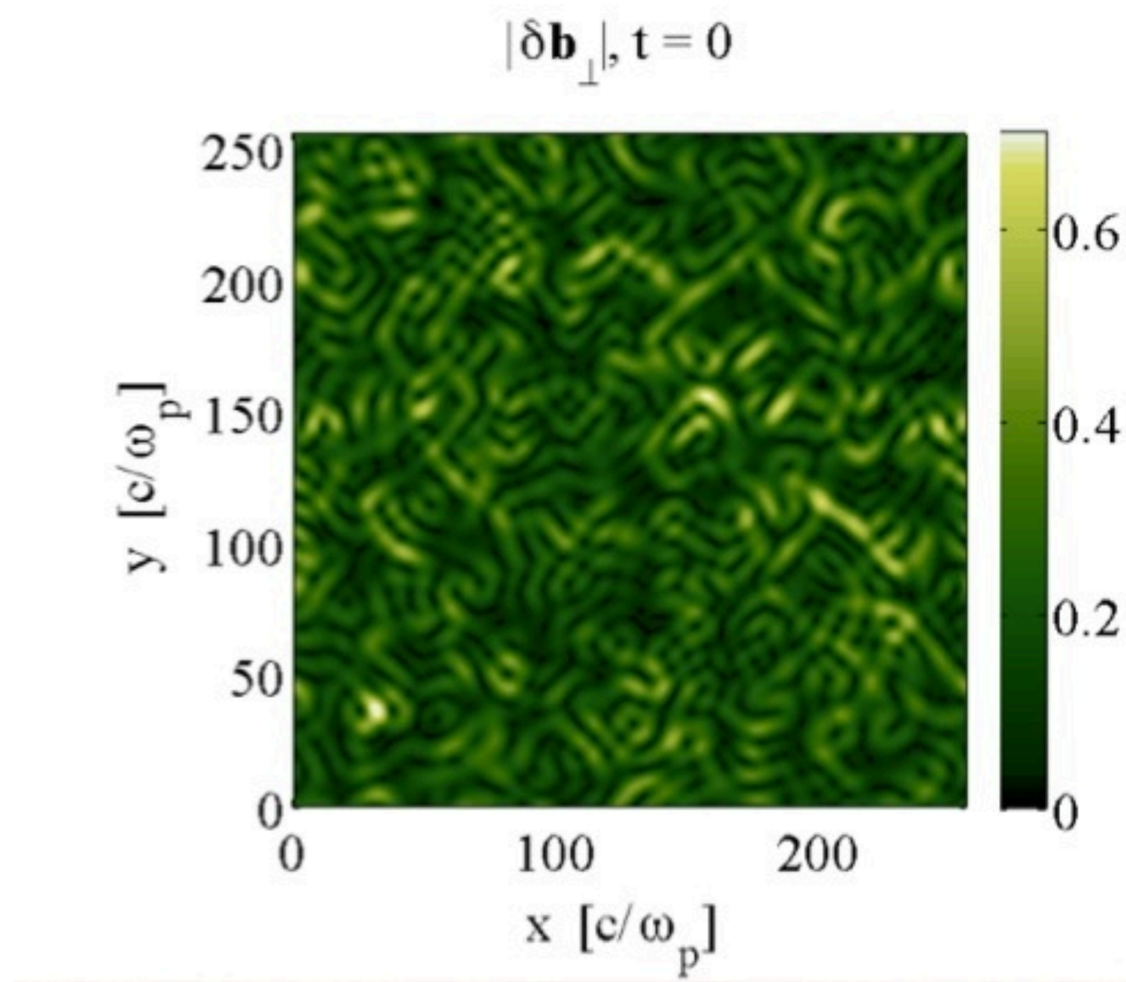
Initialization of in plane fluctuations

k-vector orthogonal to the main field

$B_0 \parallel z$

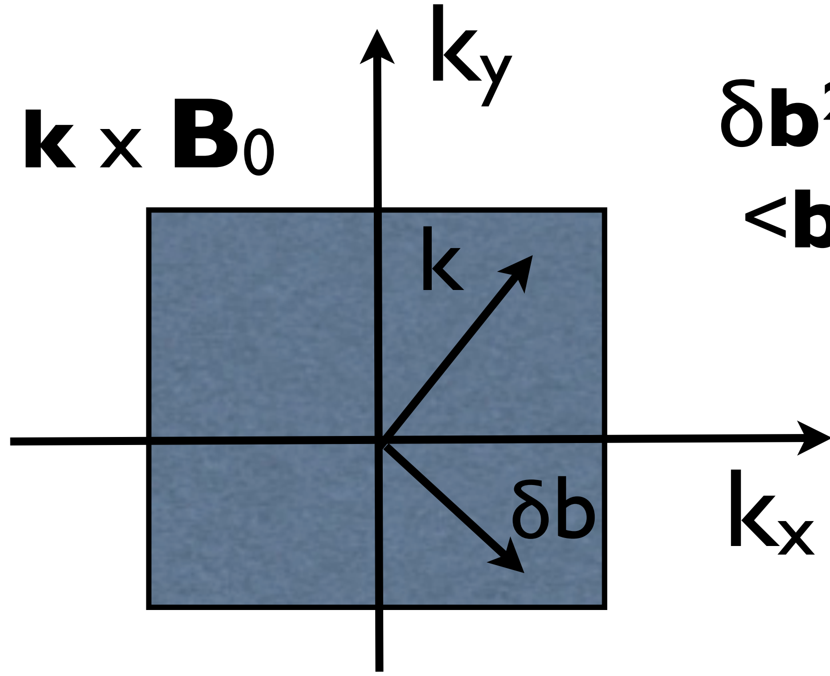


$-0.2<k_{x,y}<0.2$



linear polarization

$\delta \mathbf{b} = \mathbf{k} \times \mathbf{B}_0$



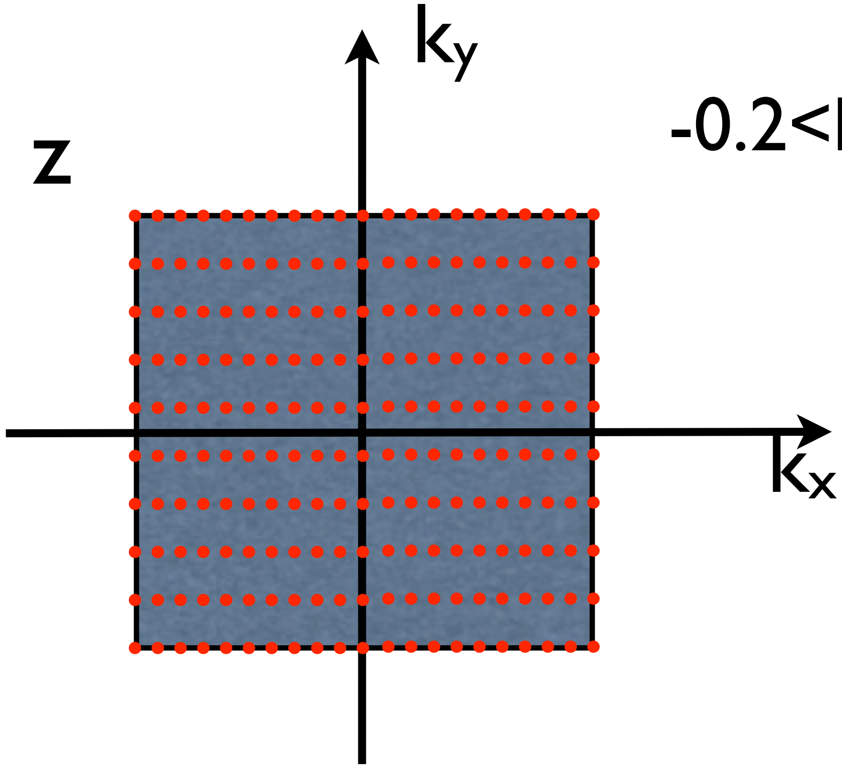
$\delta \mathbf{b}^2 = \delta \mathbf{u}^2$

$\langle \mathbf{b} . \mathbf{u} \rangle = 0$

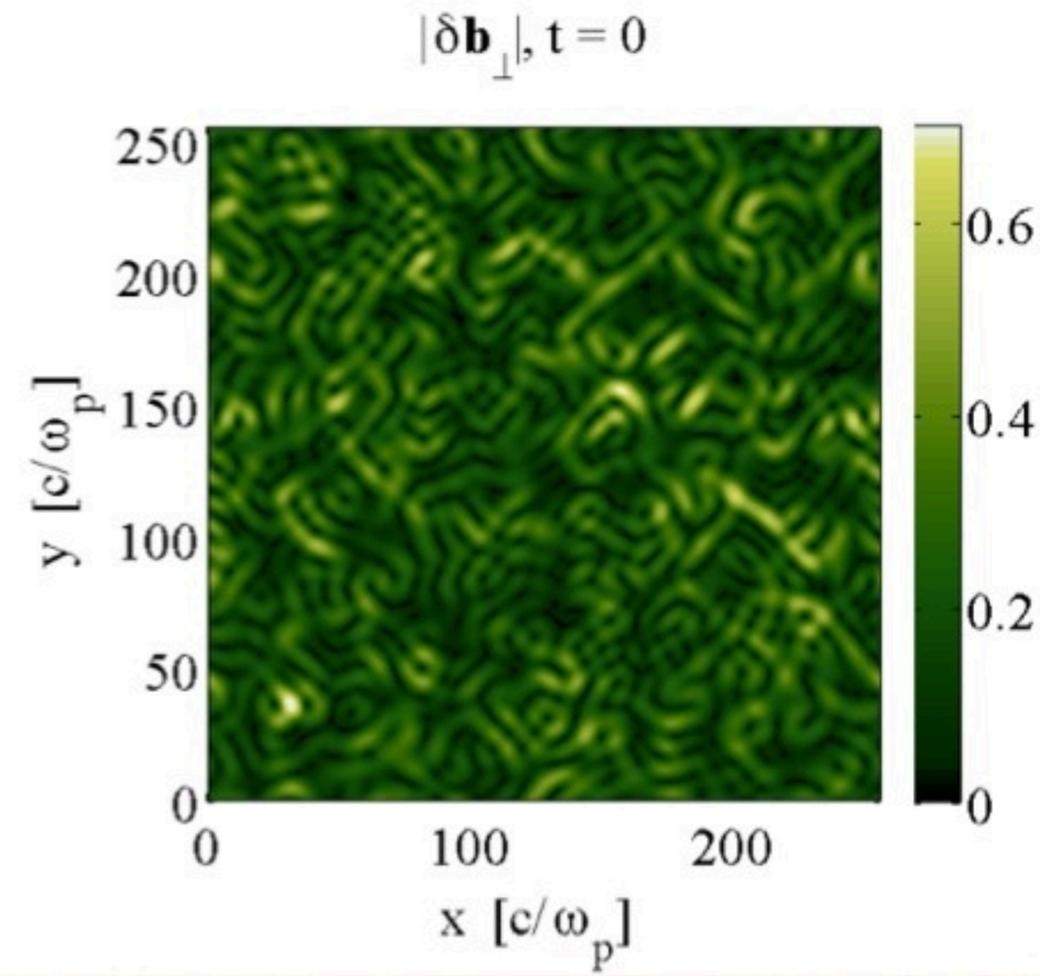
Initialization of in plane fluctuations

k-vector orthogonal to the main field

$B_0 \parallel z$

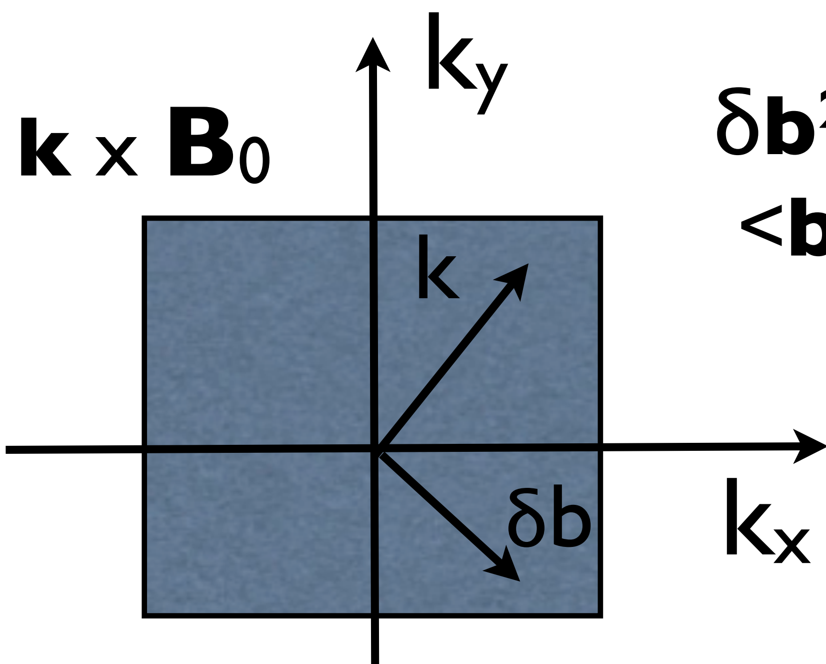


$-0.2 < k_{x,y} < 0.2$



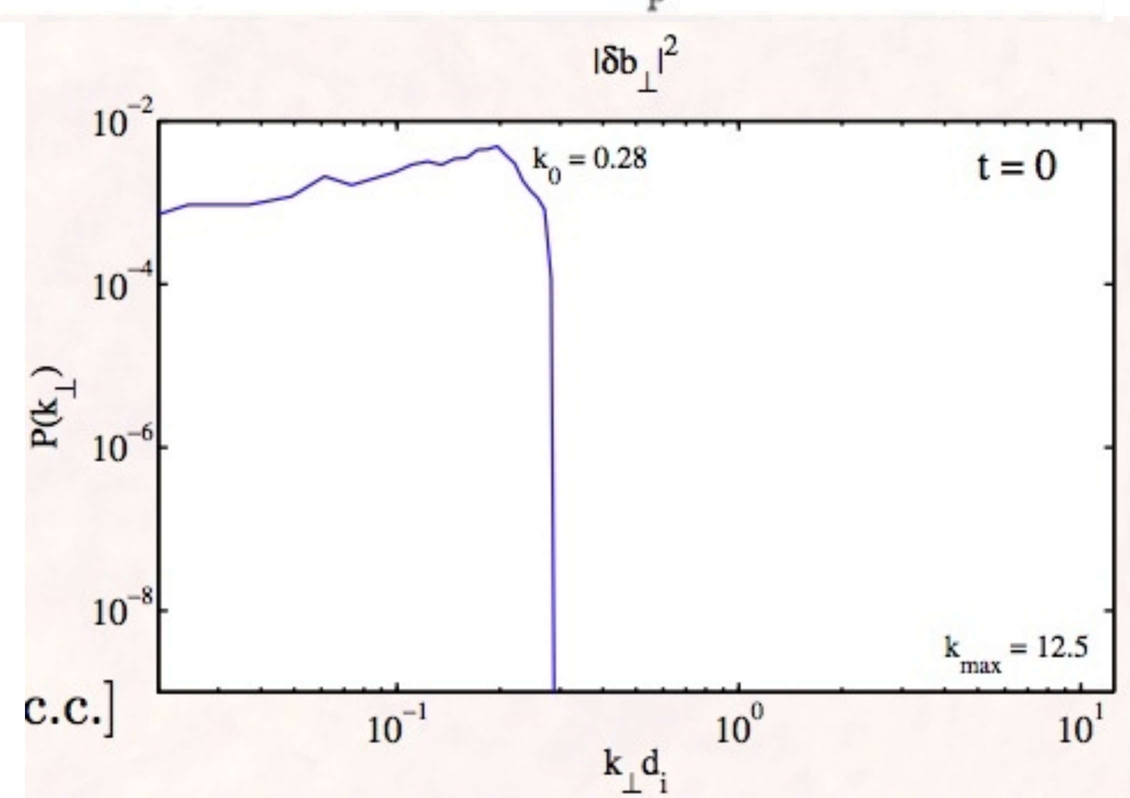
linear polarization

$\delta \mathbf{b} = \mathbf{k} \times \mathbf{B}_0$



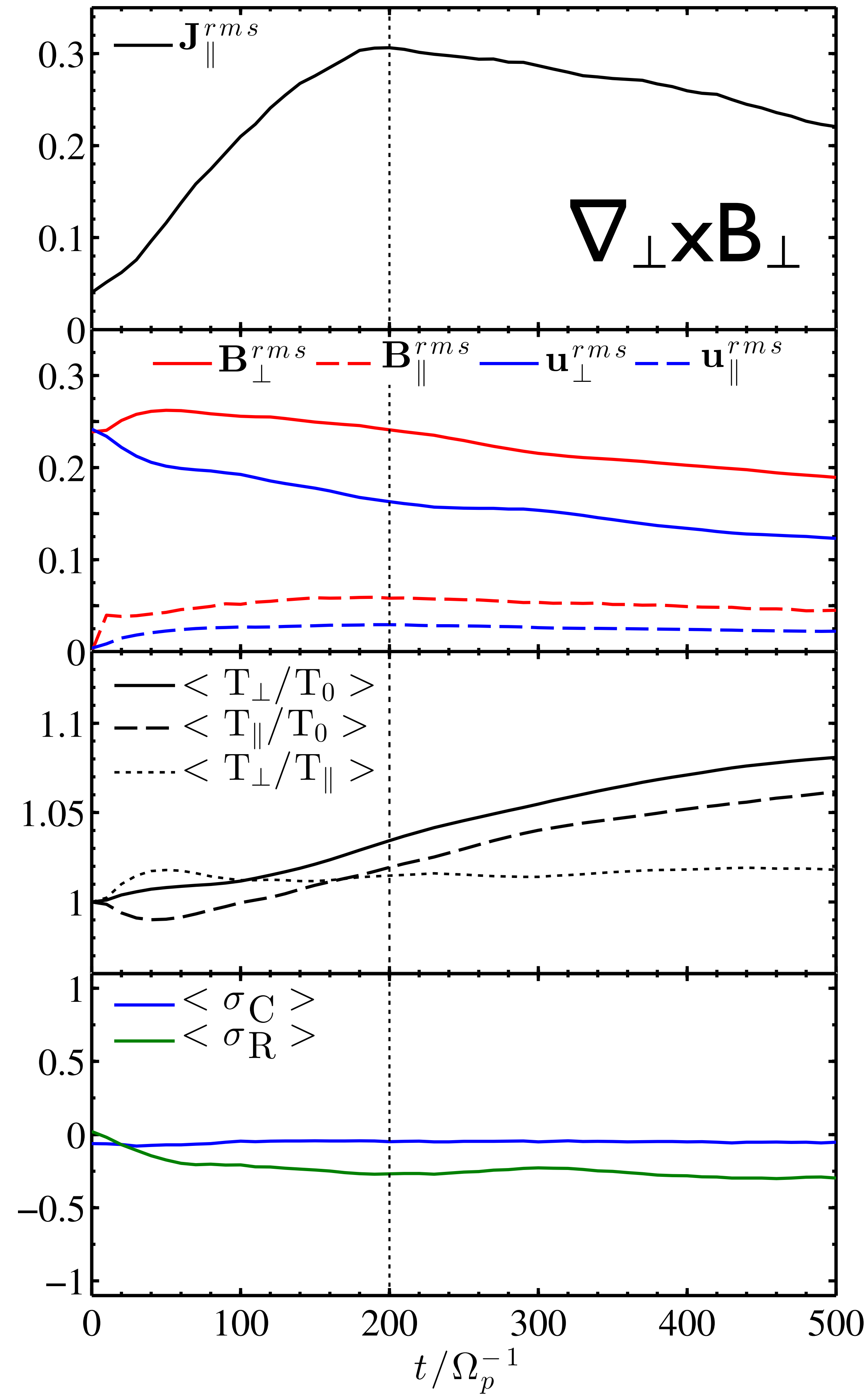
$\delta \mathbf{b}^2 = \delta \mathbf{u}^2$

$\langle \mathbf{b}.\mathbf{u} \rangle = 0$



Temporal evolution

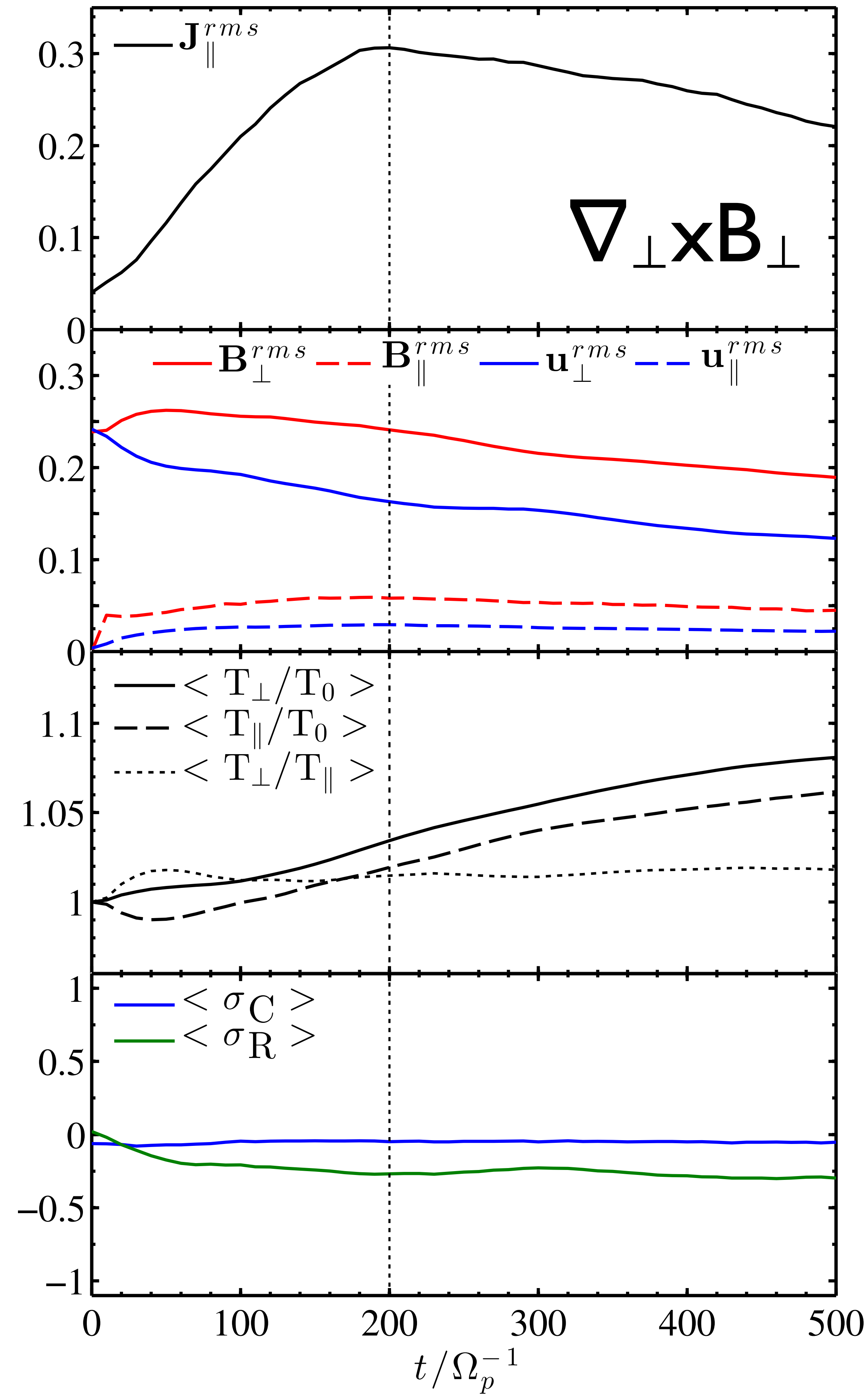
Peak in out-of-plane current identifies maximum of turbulent activity (*Servidio et al 2012*)



Temporal evolution

Peak in out-of-plane current identifies maximum of turbulent activity (*Servidio et al 2012*)

Large scale dominated by $B_\perp$ and $u_\perp$

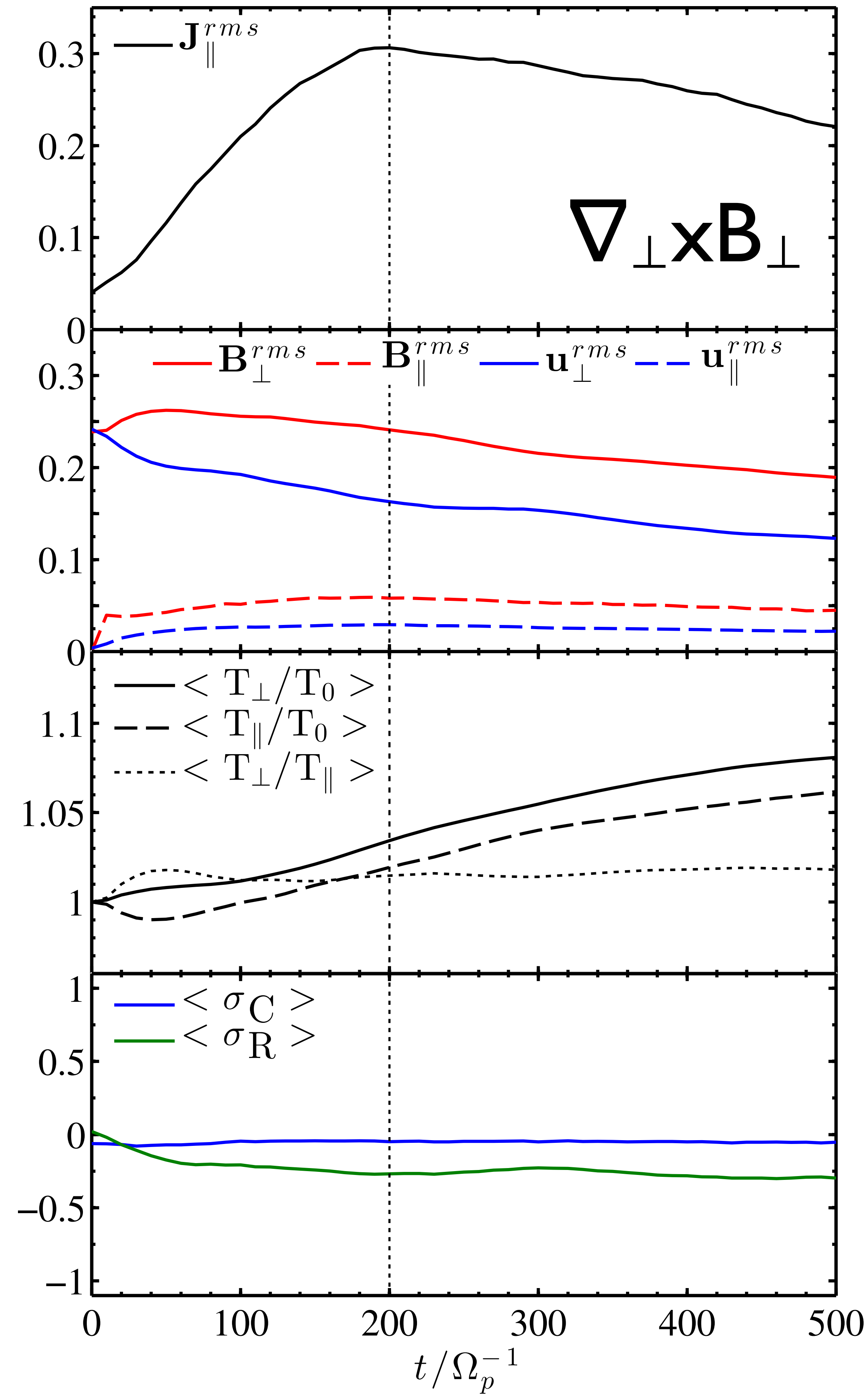


Temporal evolution

Peak in out-of-plane current identifies maximum of turbulent activity (*Servidio et al 2012*)

Large scale dominated by $B_\perp$ and $u_\perp$

Anisotropy and parallel/perp temperatures: heating! but no global anisotropy



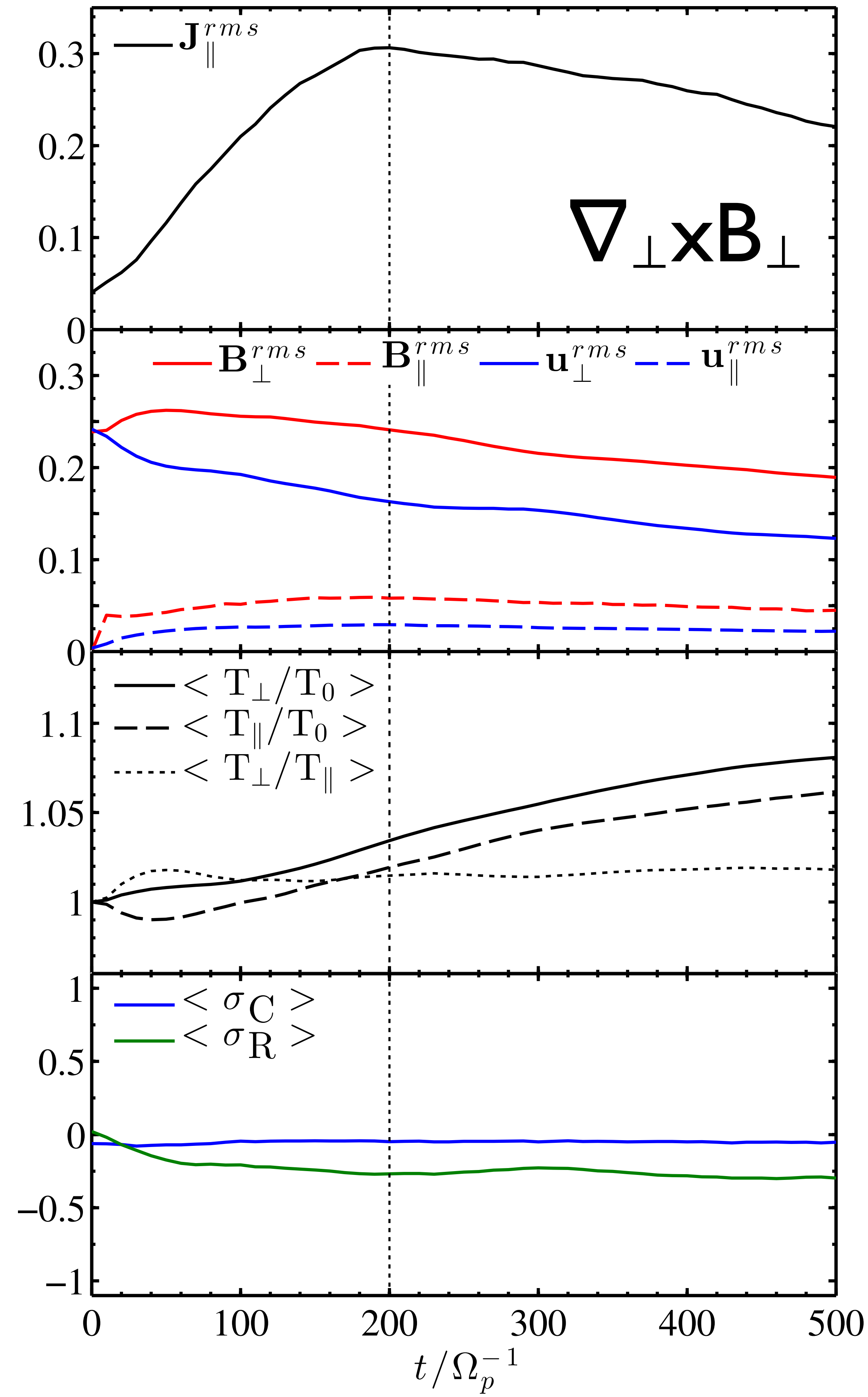
Temporal evolution

Peak in out-of-plane current identifies maximum of turbulent activity (*Servidio et al 2012*)

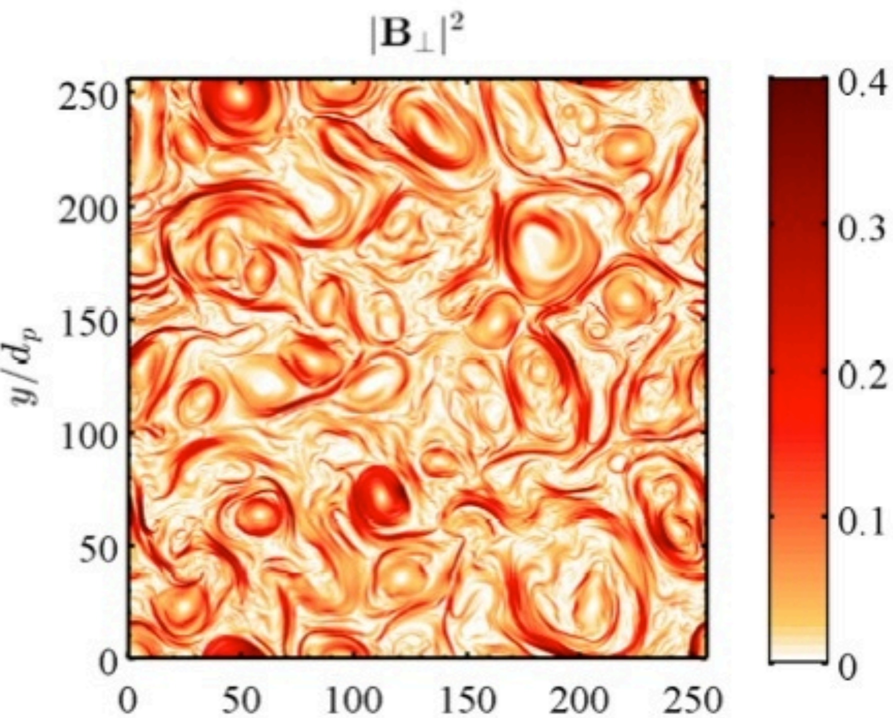
Large scale dominated by $B_\perp$ and $u_\perp$

Anisotropy and parallel/perp temperatures: heating! but no global anisotropy

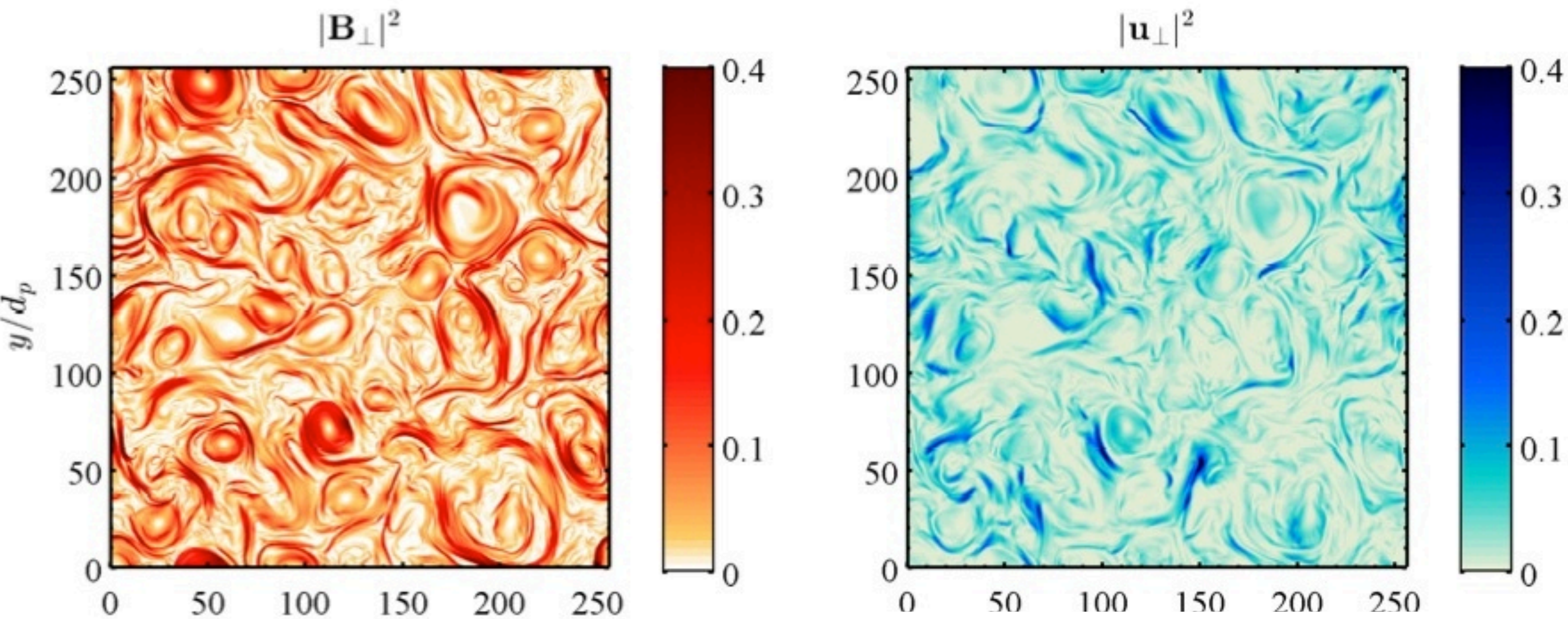
z^+ and z^- remain balanced in time while small excess $b^2>u^2$



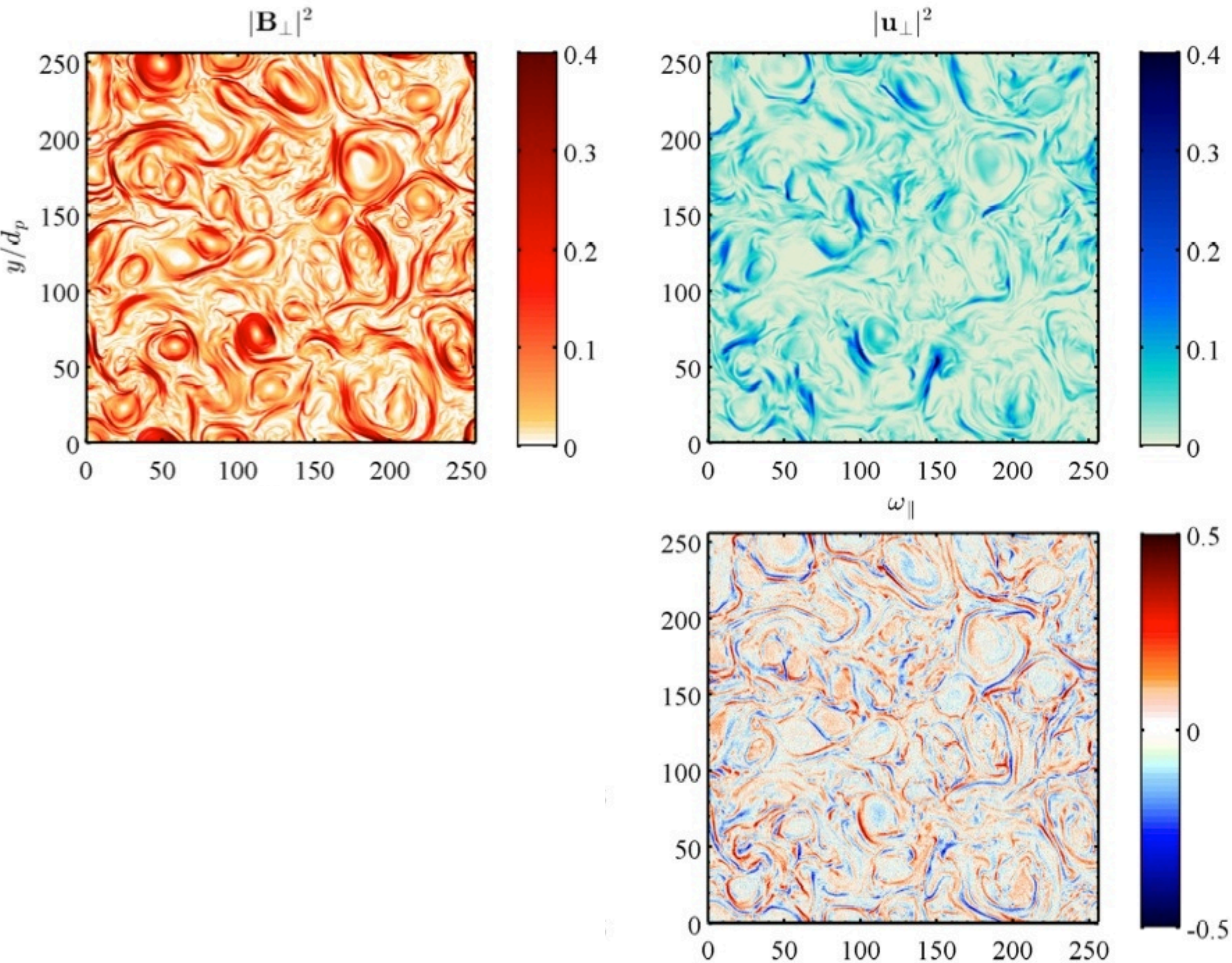
2D maps of fields



2D maps of fields

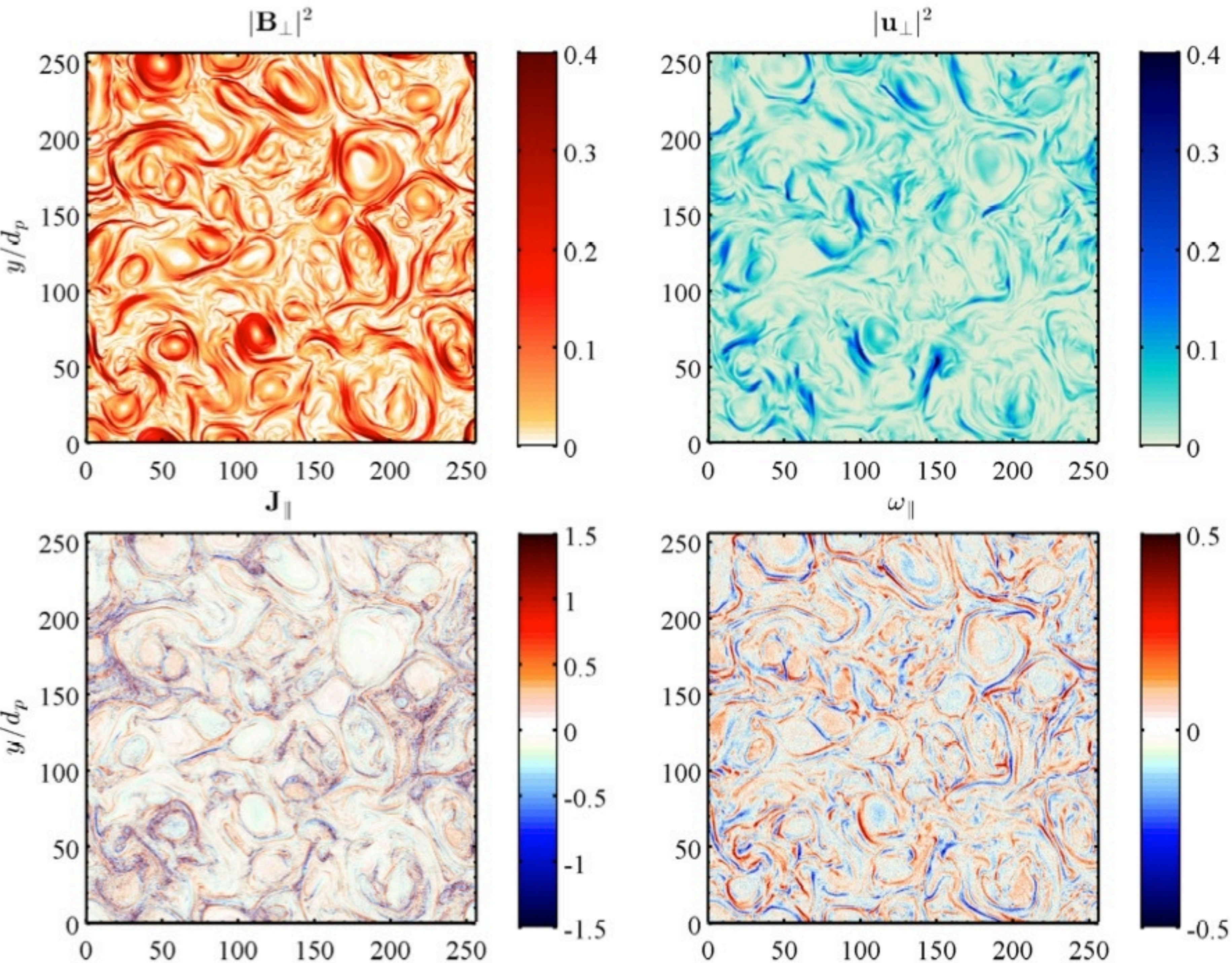


2D maps of fields



vorticity ∇×V

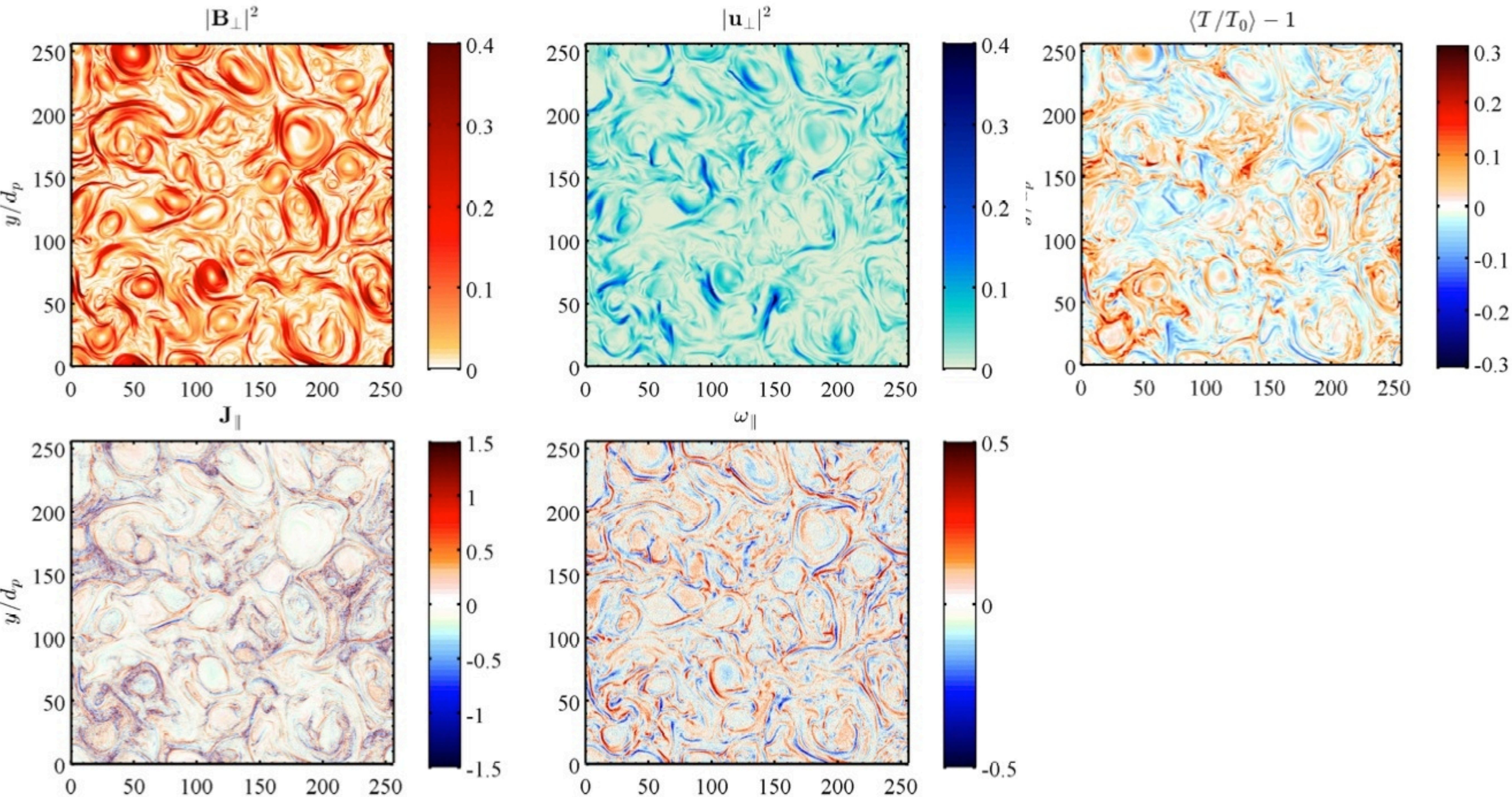
2D maps of fields



current ∇×B

vorticity ∇×V

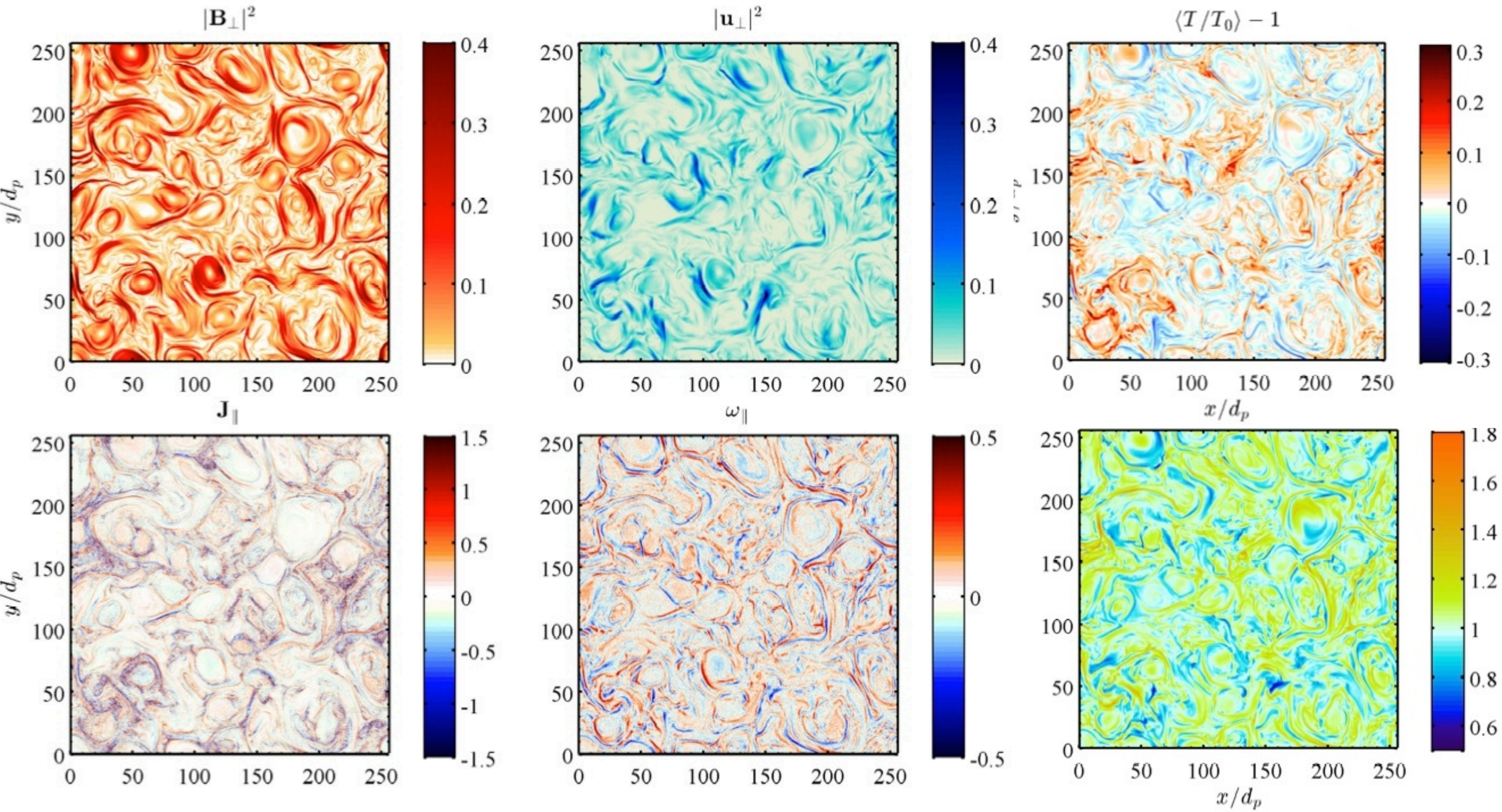
2D maps of fields



current ∇×B

vorticity ∇×V

2D maps of fields

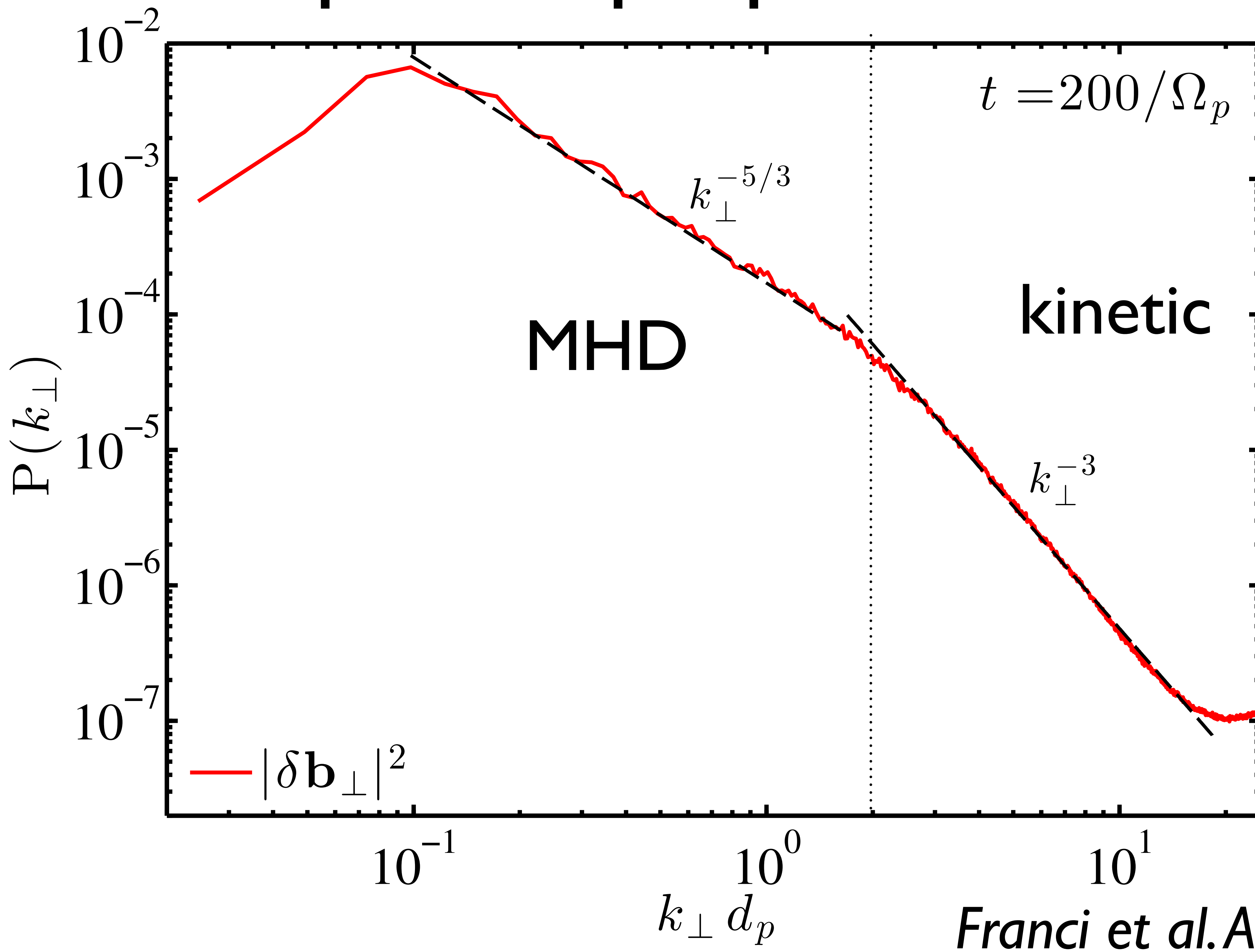


current ∇×B

vorticity ∇×V

Temperature anisotropy
(Servidio, Perrone, ...)

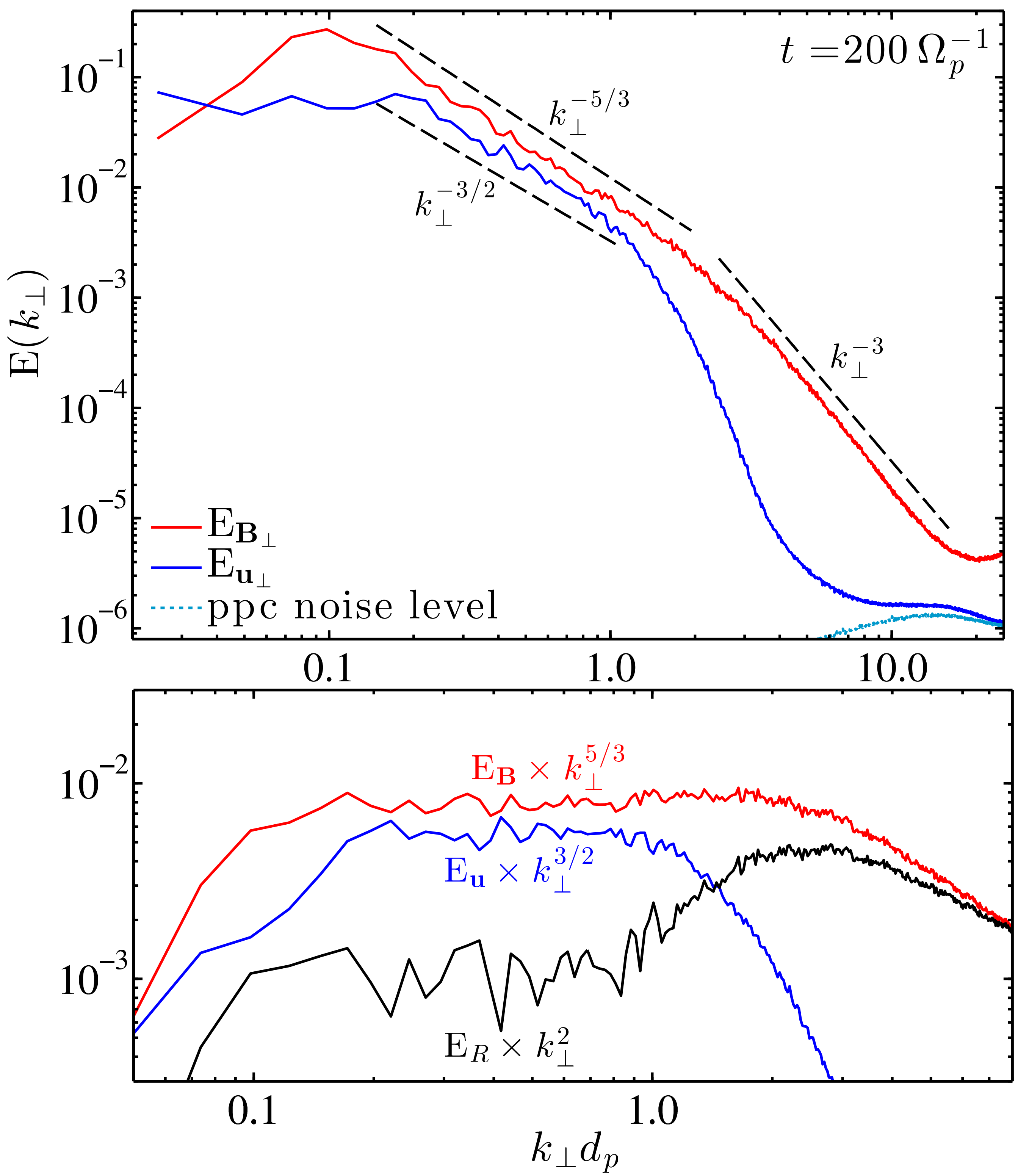
Spectral properties



Franci et al. ApJL 2015

Self-consistent transition between two turbulent regimes: Kolmogorov-like at large MHD scales and kinetic at sub-ion scales

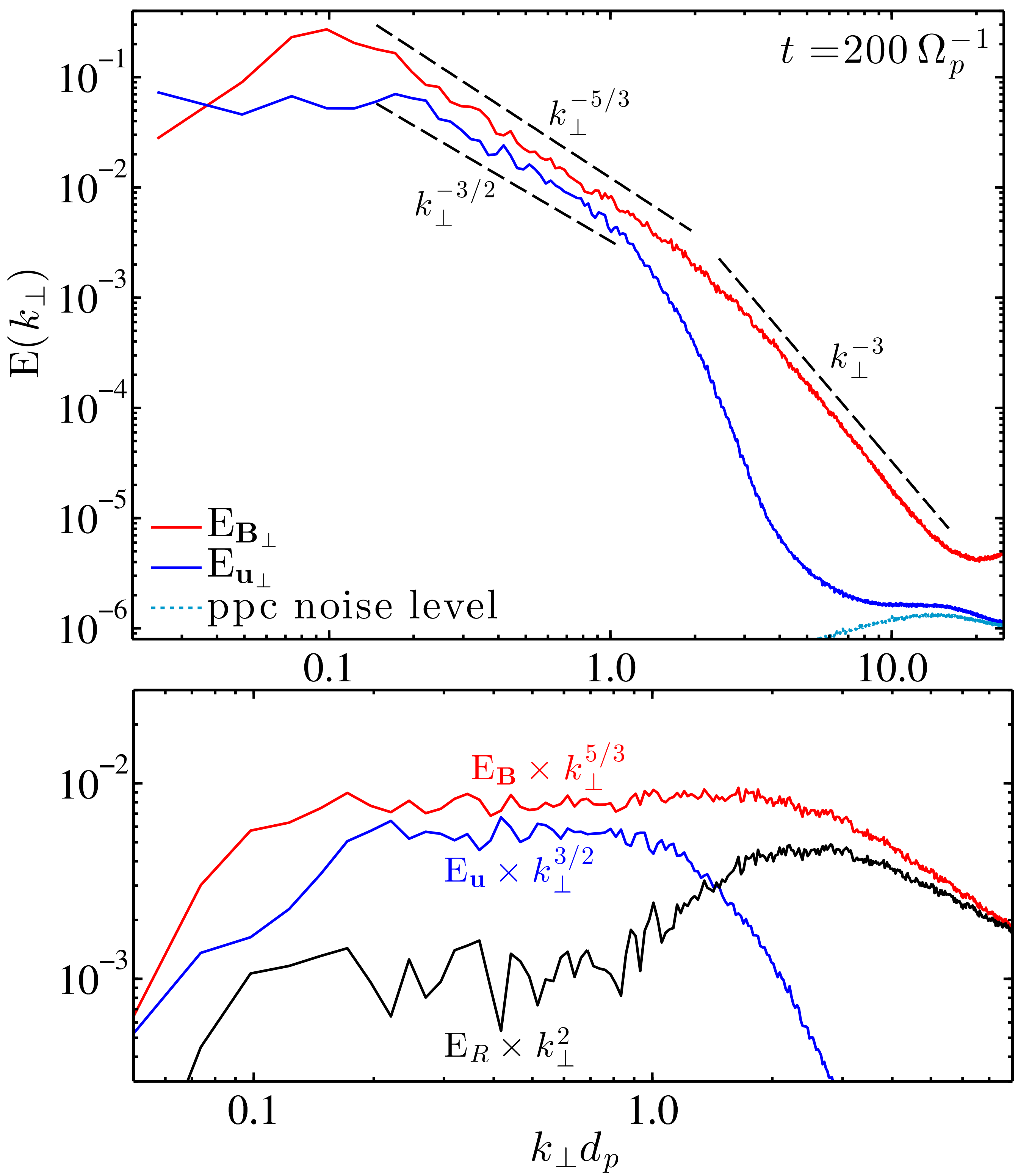
Spectrum of V and residual energy



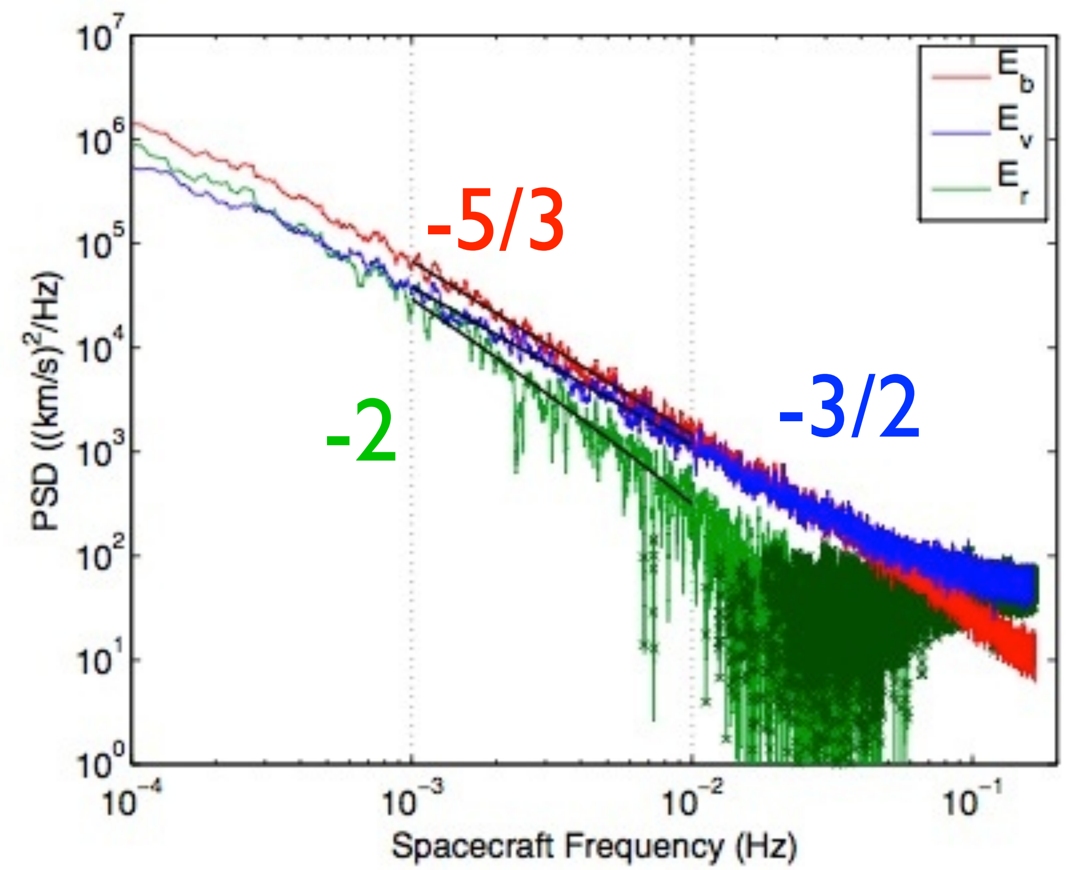
Different scaling for B and V in the MHD inertial range.
Their difference (residual energy) is also a power law!
(Roland's talk)

-5/3, -3/2, -2

Spectrum of V and residual energy



Different scaling for B and V in the MHD inertial range. Their difference (residual energy) is also a power law! (Roland's talk)

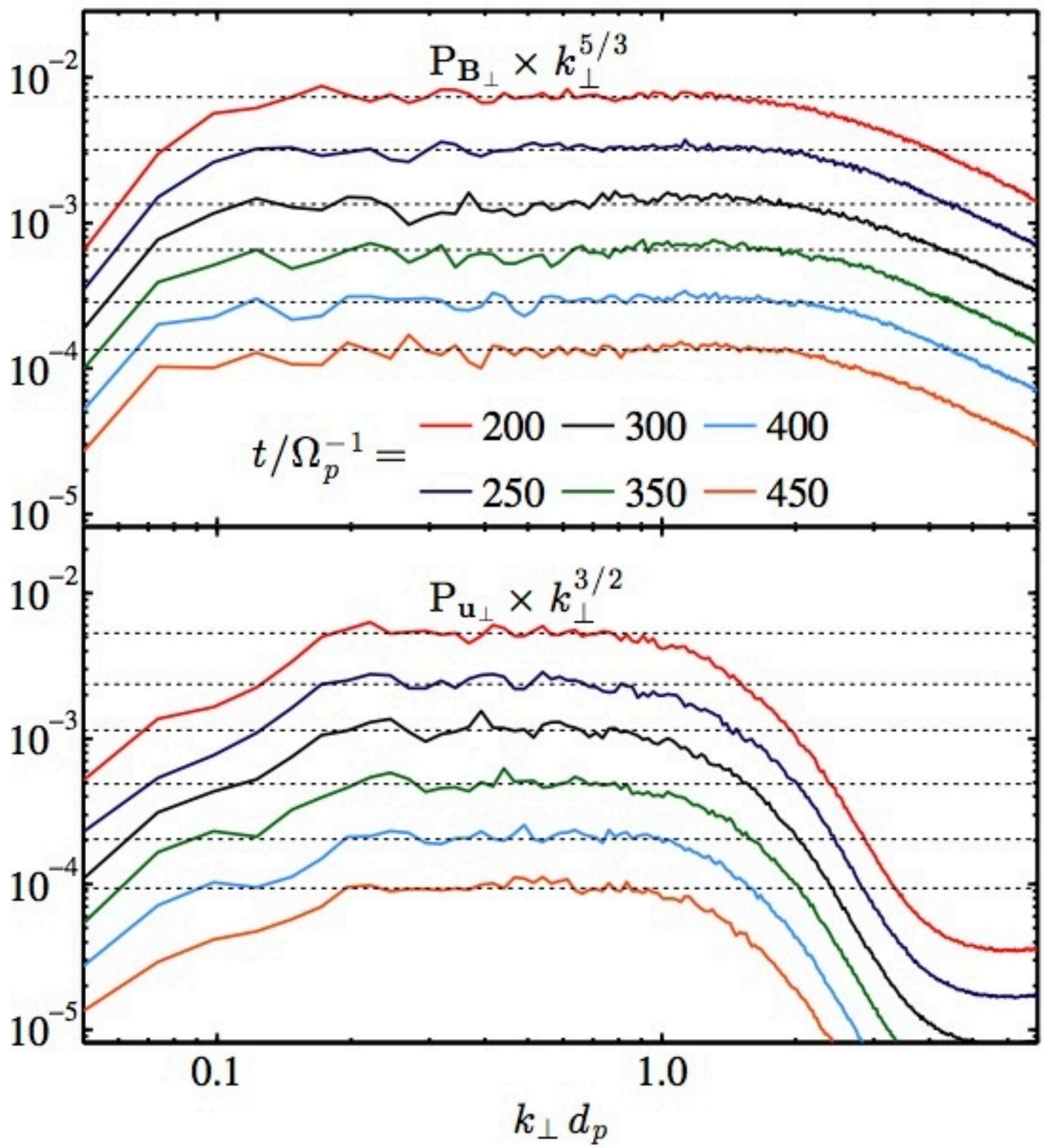


-5/3, -3/2, -2

Chen et al. 2013

Stability of the turbulent cascade

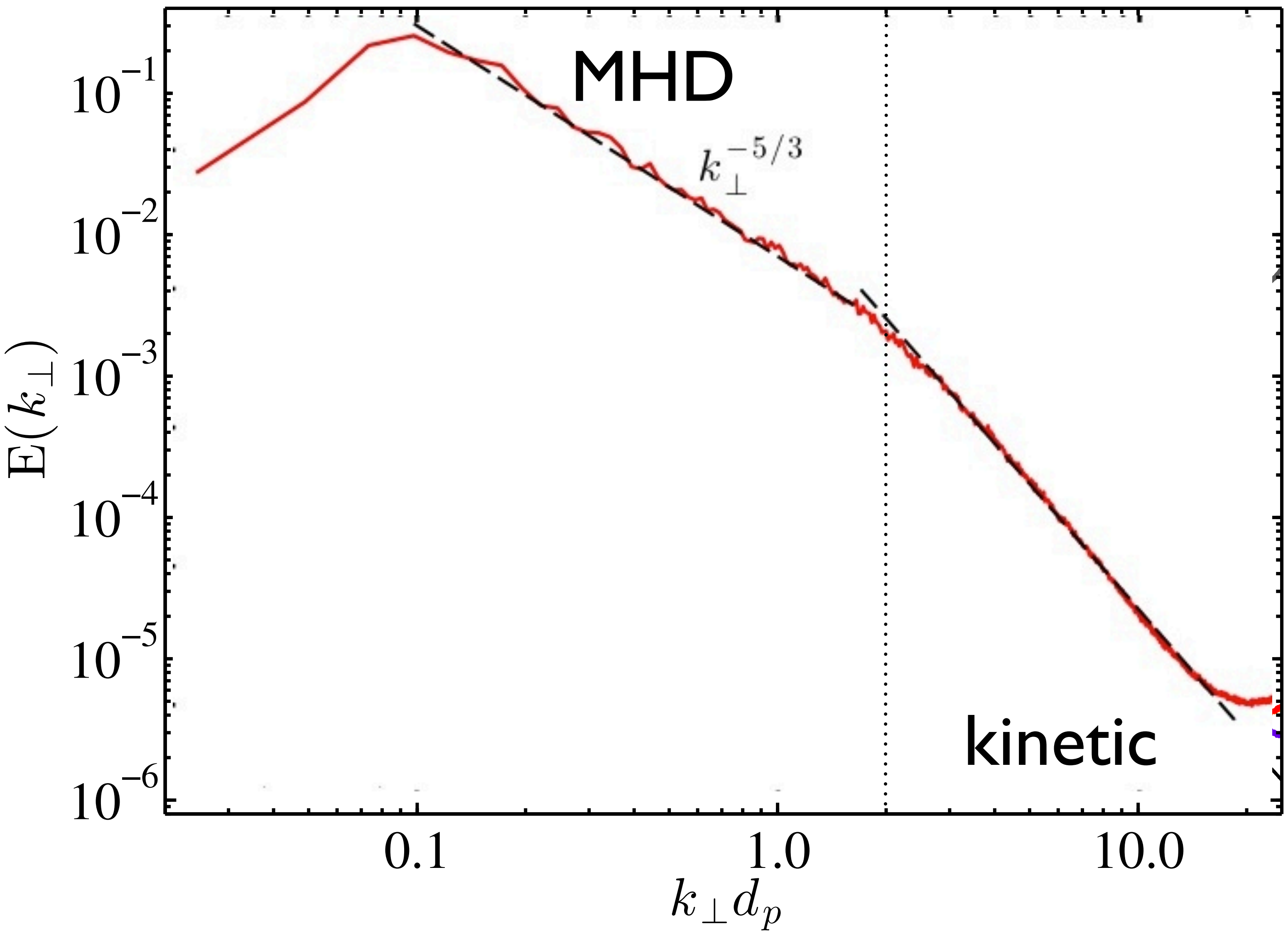
Compensated spectra at different simulation times



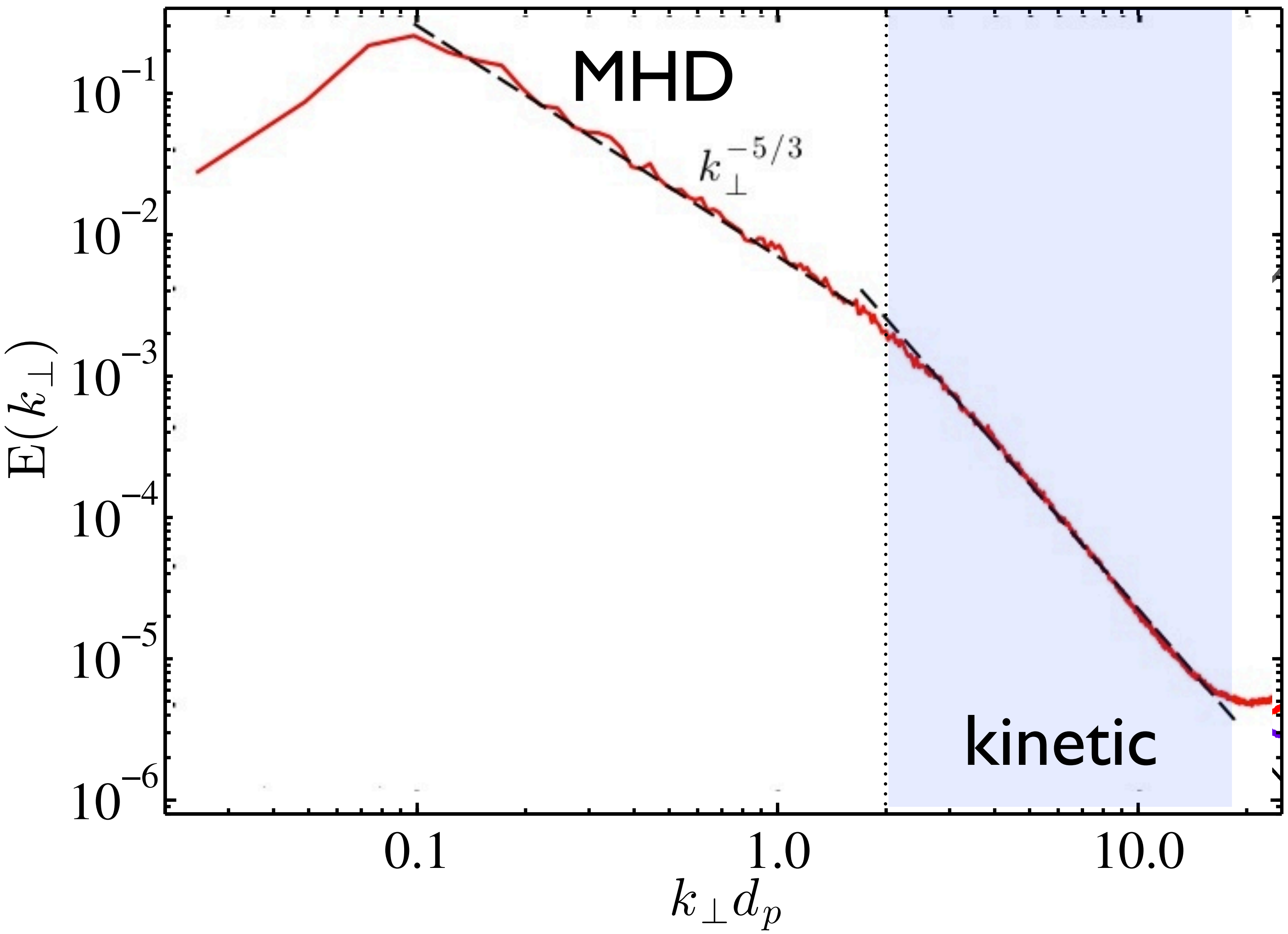
Magnetic field compensated by 5/3

Proton velocity compensated by 3/2

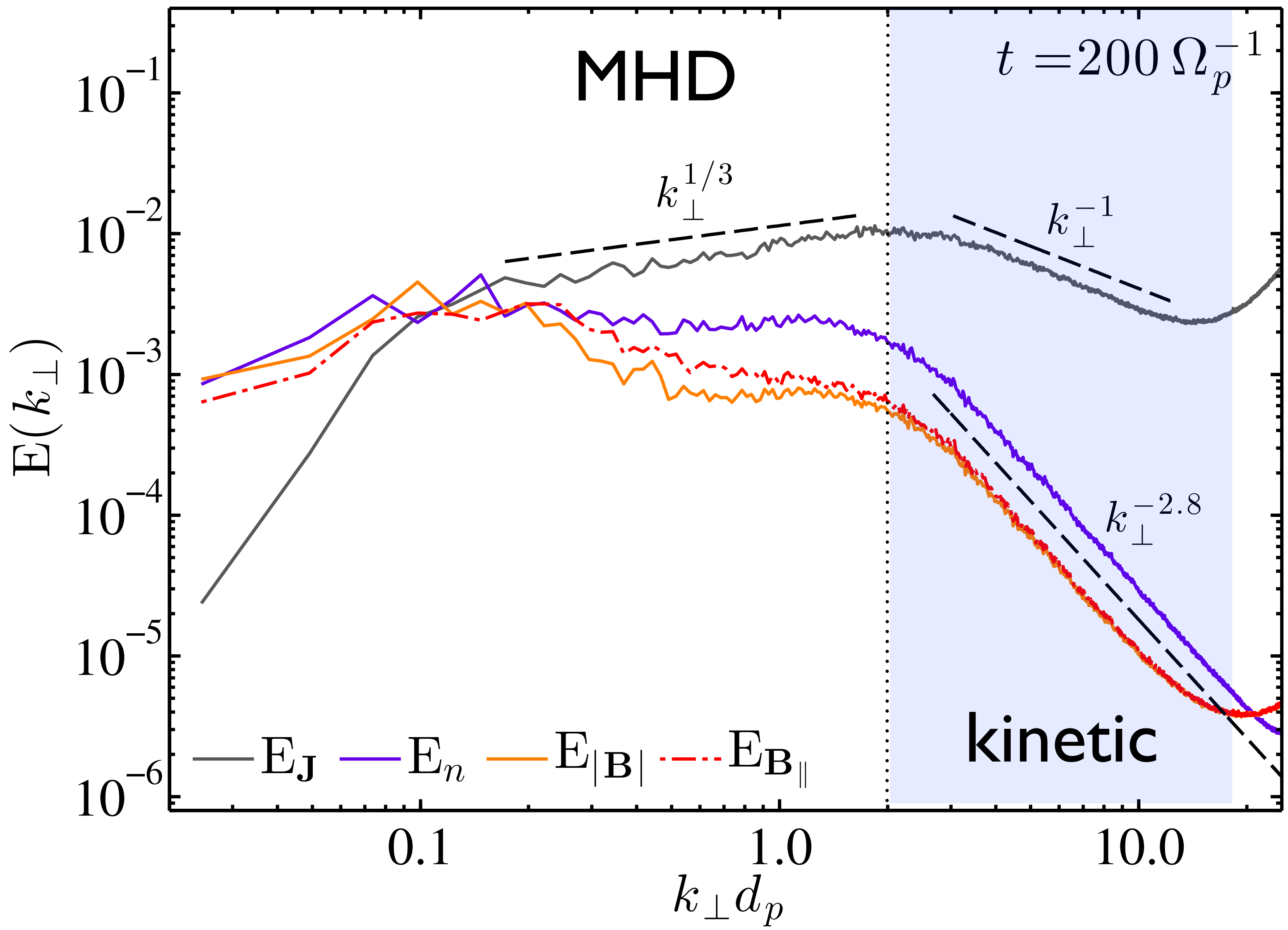
Turbulence at kinetic scales



Turbulence at kinetic scales

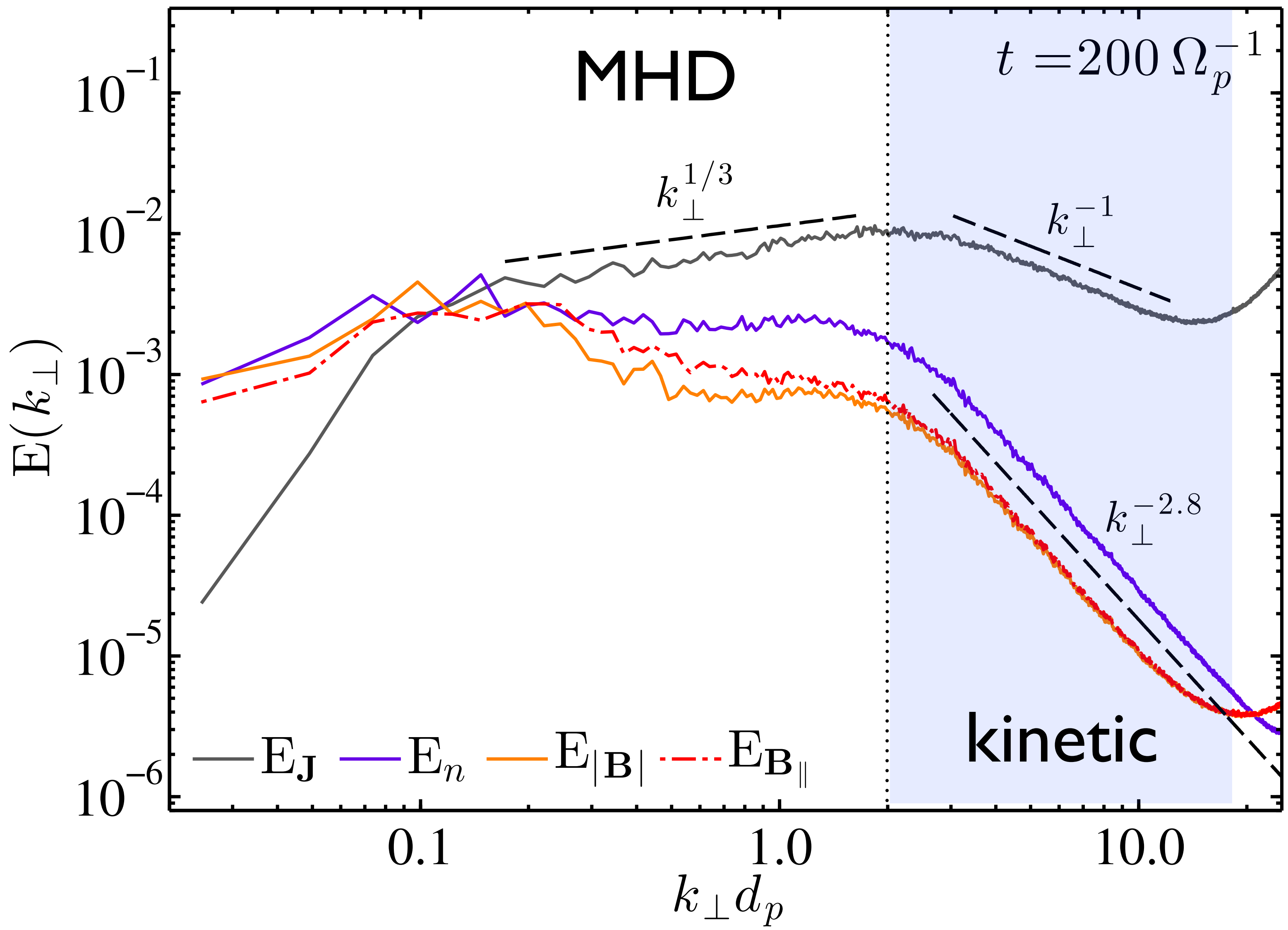


Turbulence at kinetic scales

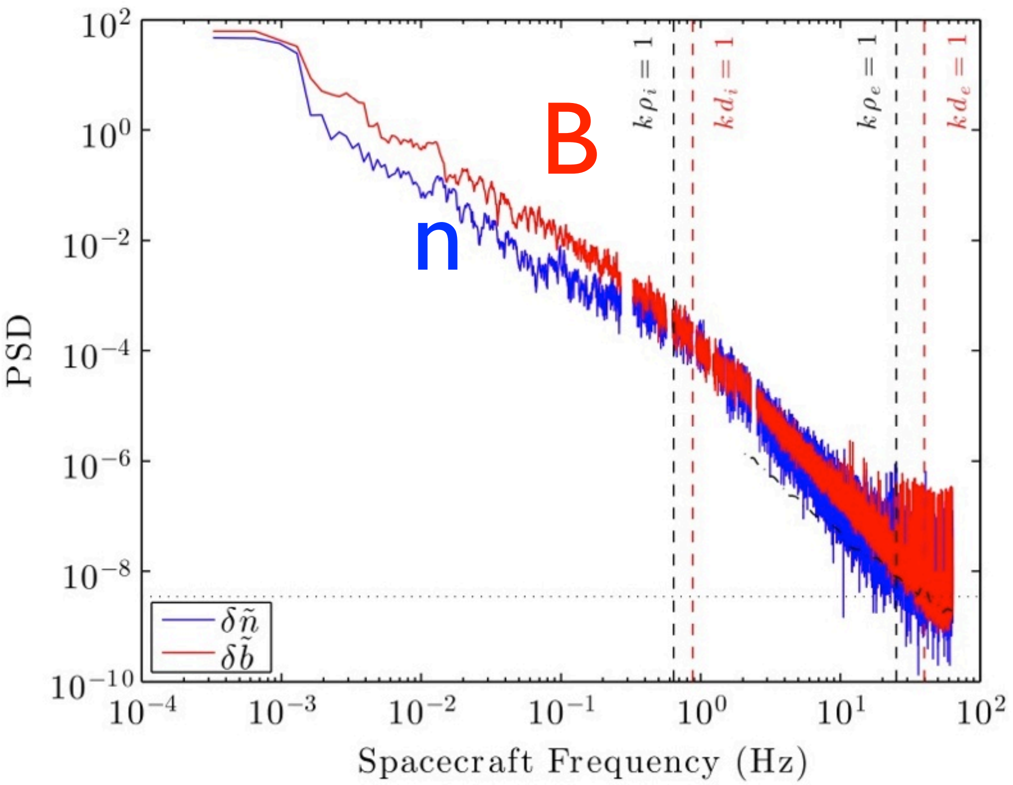


- Current has a peak:
  $J=\nabla \times B \sim k\delta b \rightarrow \delta J^2 \sim k^2 \delta b^2$
- $\delta b_{||} \sim \delta n \propto k^{-2.8}$
- $\delta b_{||} \sim \delta b_\perp$

Turbulence at kinetic scales

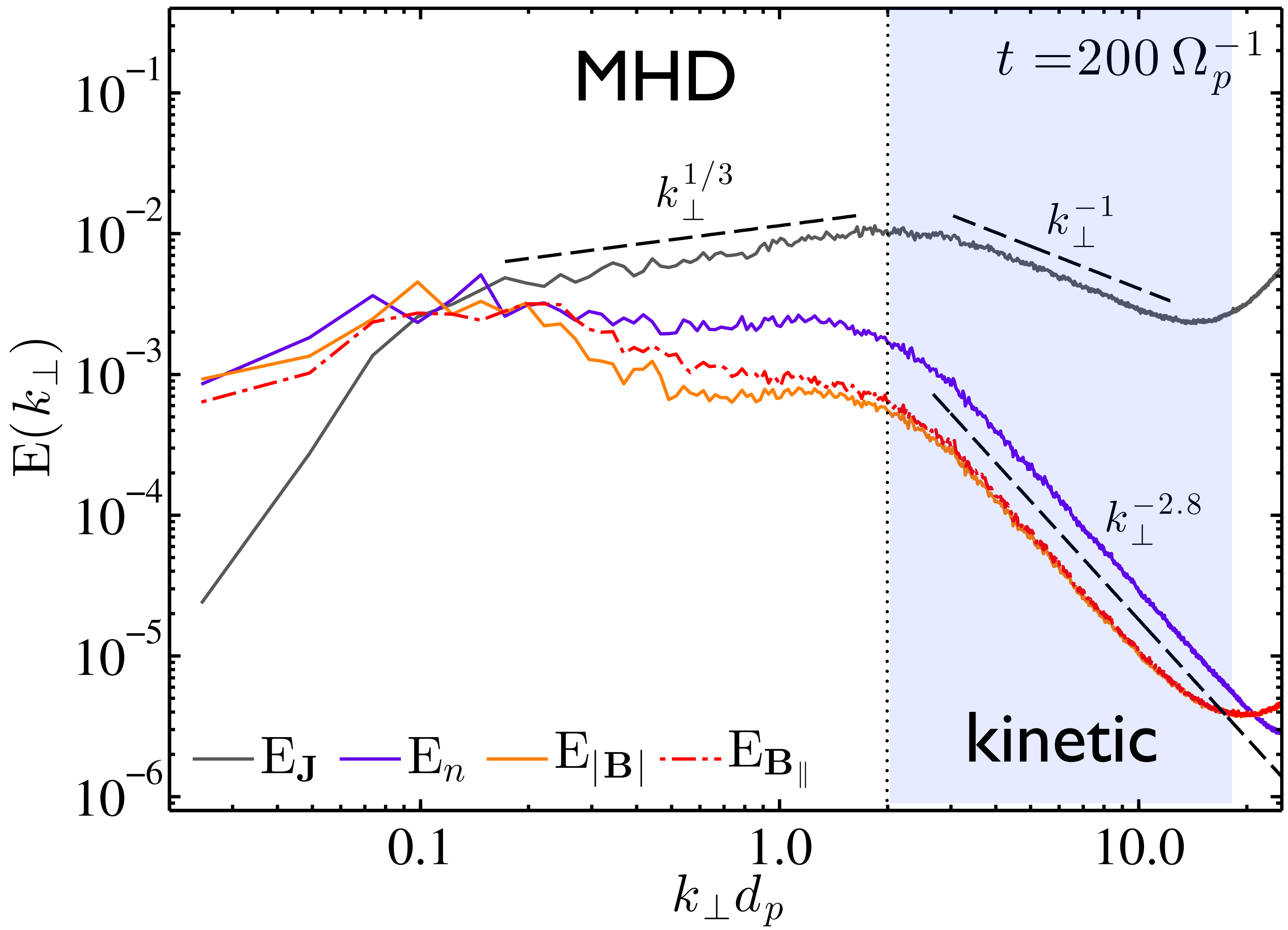


Chen et al. PRL 2013

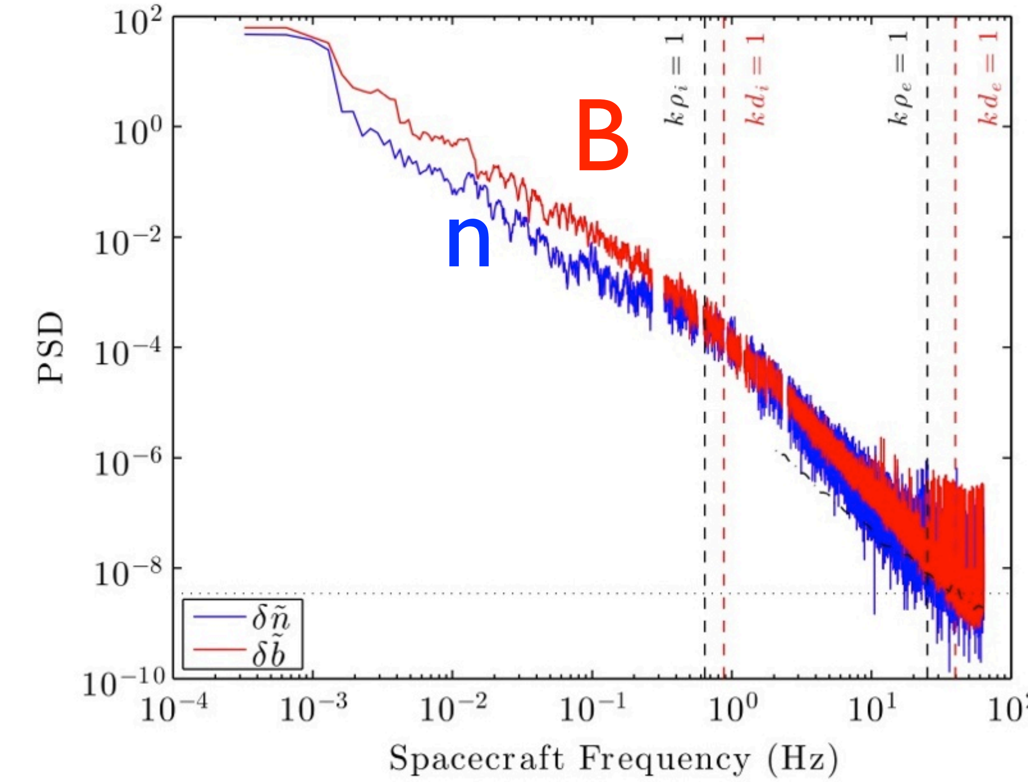


- Current has a peak: $J = \nabla \times B \sim k\delta b \rightarrow \delta J^2 \sim k^2 \delta b^2$
- $\delta b_{||} \sim \delta n \propto k^{-2.8}$
- $\delta b_{||} \sim \delta b_\perp$

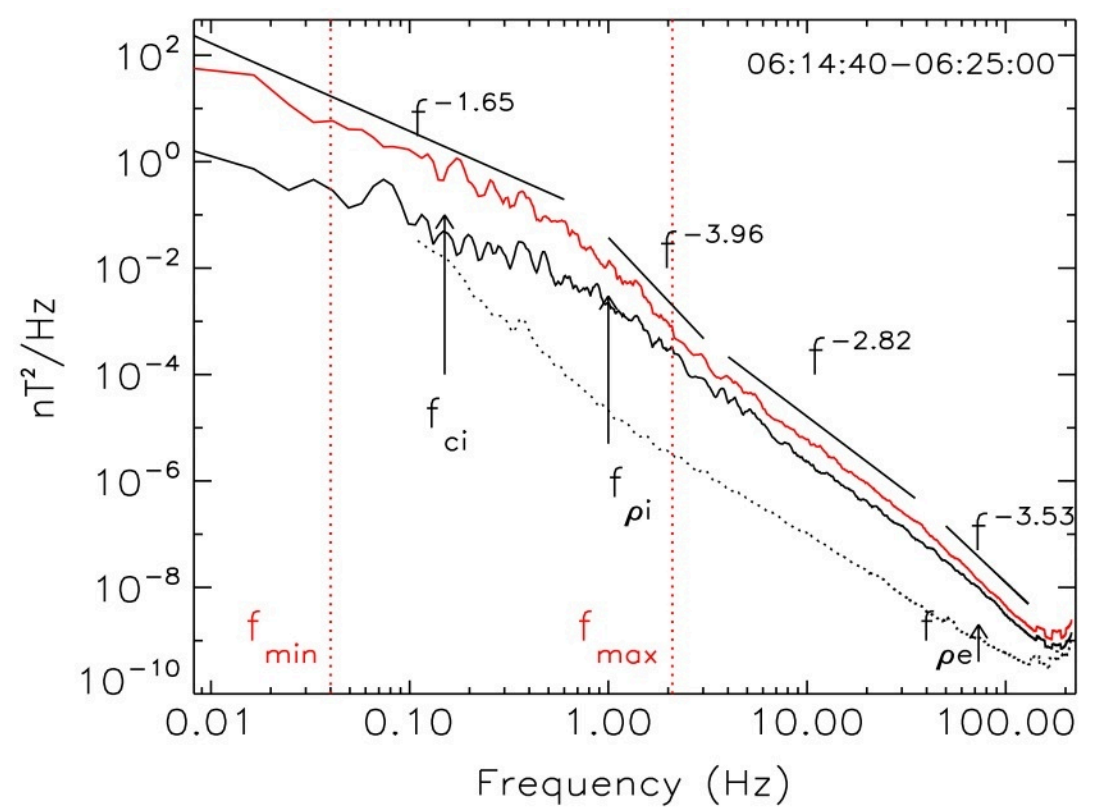
Turbulence at kinetic scales



Chen et al. PRL 2013

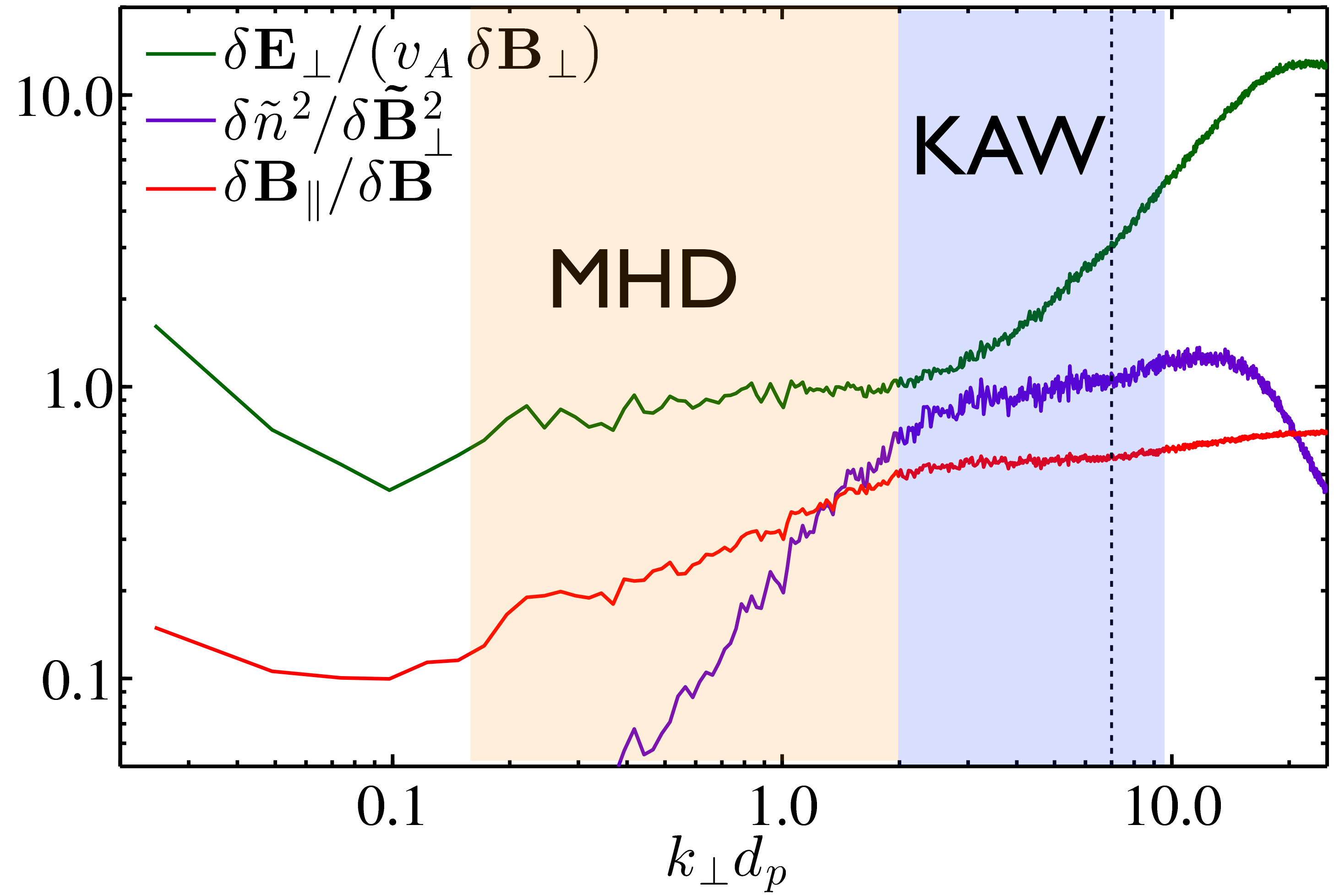


Sahraoui et al. PRL 2010



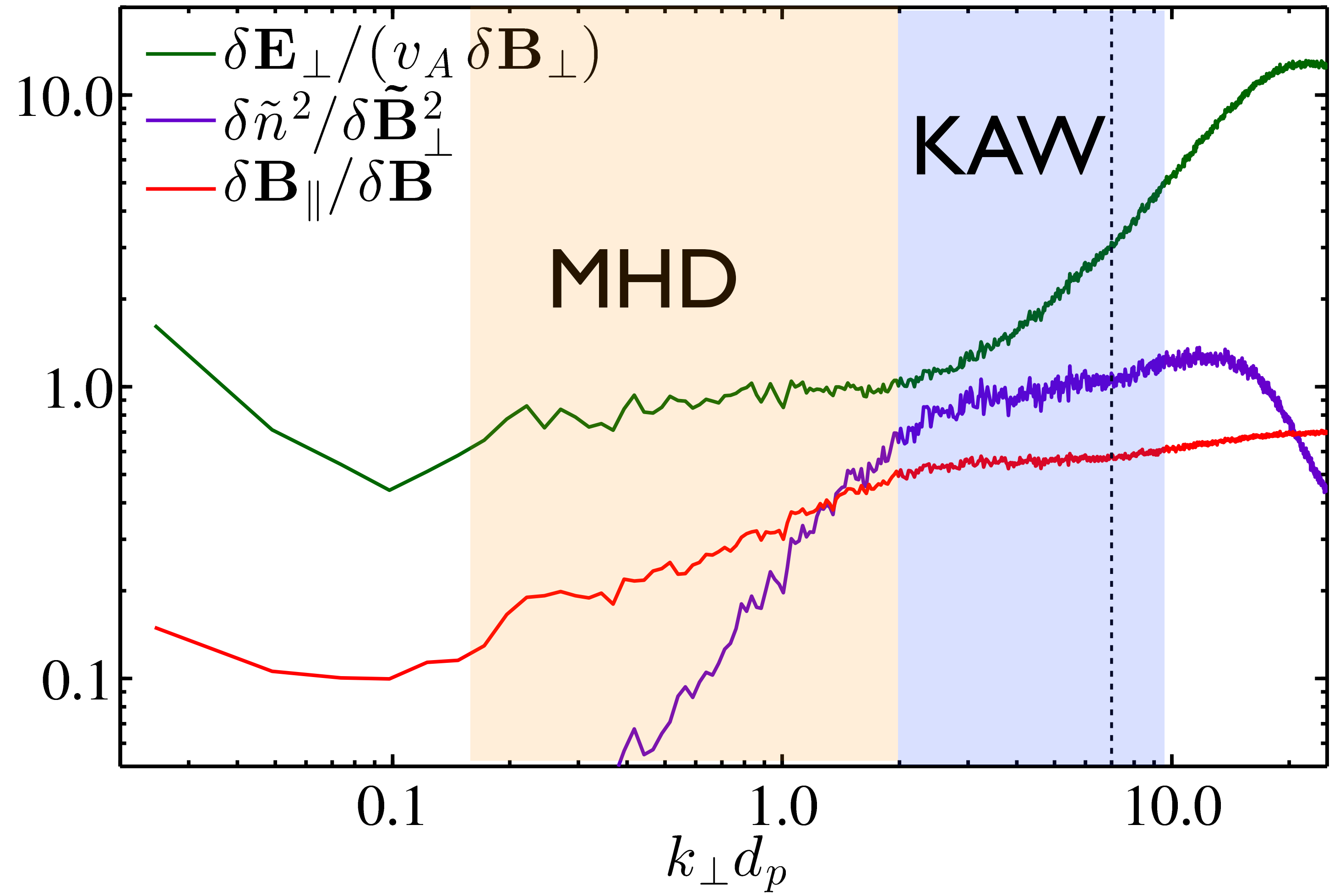
- Current has a peak:
  $J=\nabla \times B \sim k\delta b \;\rightarrow\; \delta J^2 \sim k^2 \delta b^2$
- $\delta b_{||} \sim \delta n \propto k^{-2.8}$
- $\delta b_{||} \sim \delta b_{\perp}$

Nature of sub-ion scale fluctuations?

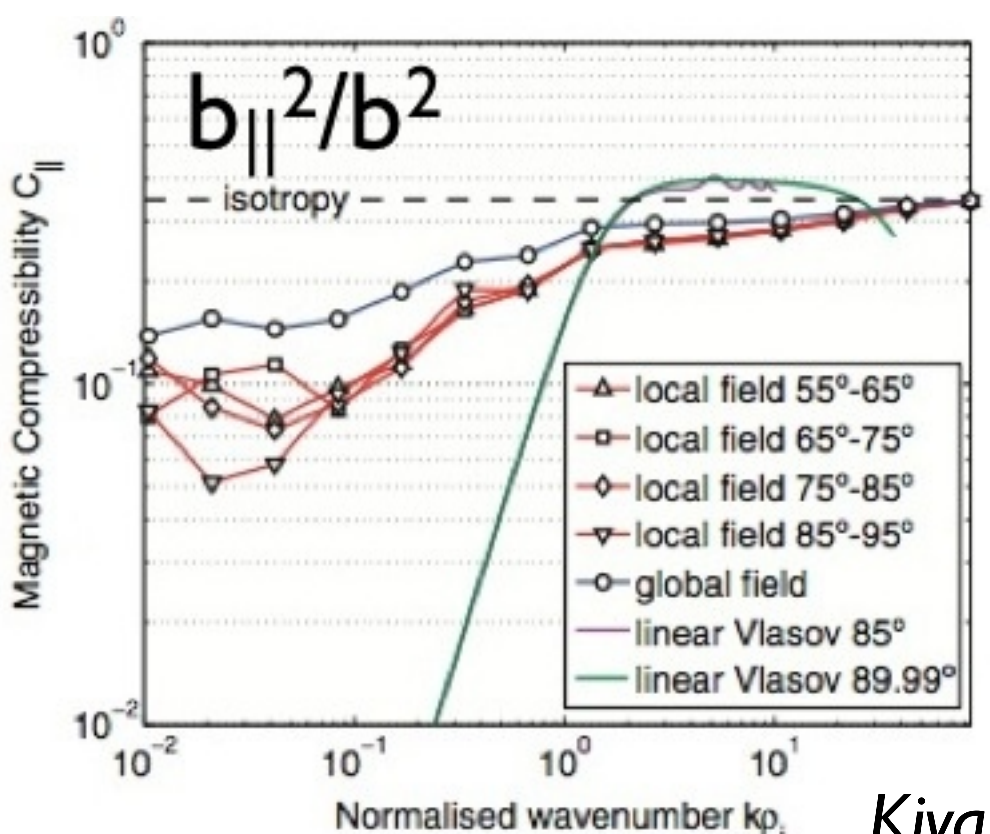


predictions from low frequency kinetic Alfvén wave (KAW) turbulence are very well verified! (which depend on β_p and T_p/T_e)

Nature of sub-ion scale fluctuations?

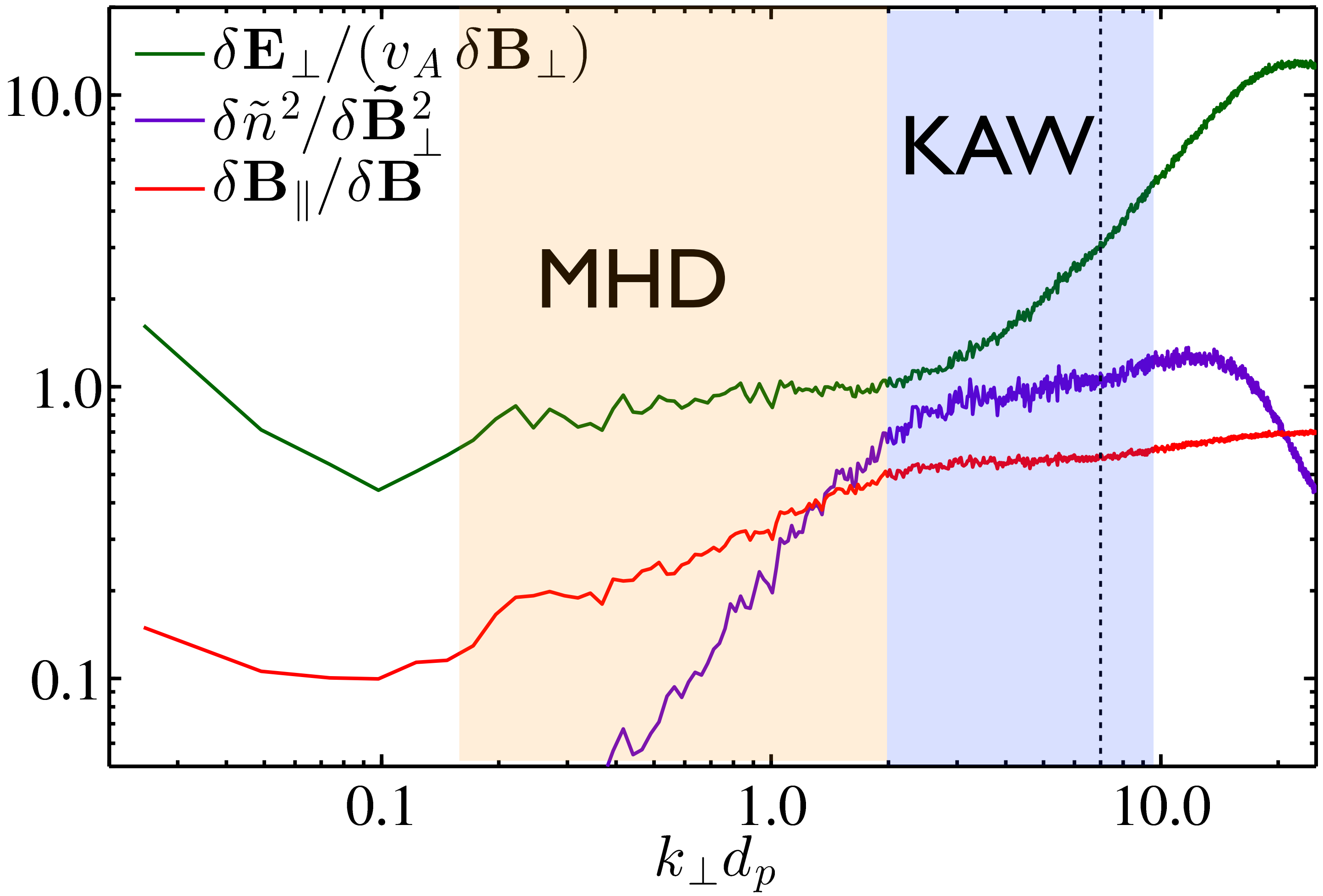


predictions from low frequency kinetic Alfvén wave (KAW) turbulence are very well verified! (which depend on β_p and T_p/T_e)

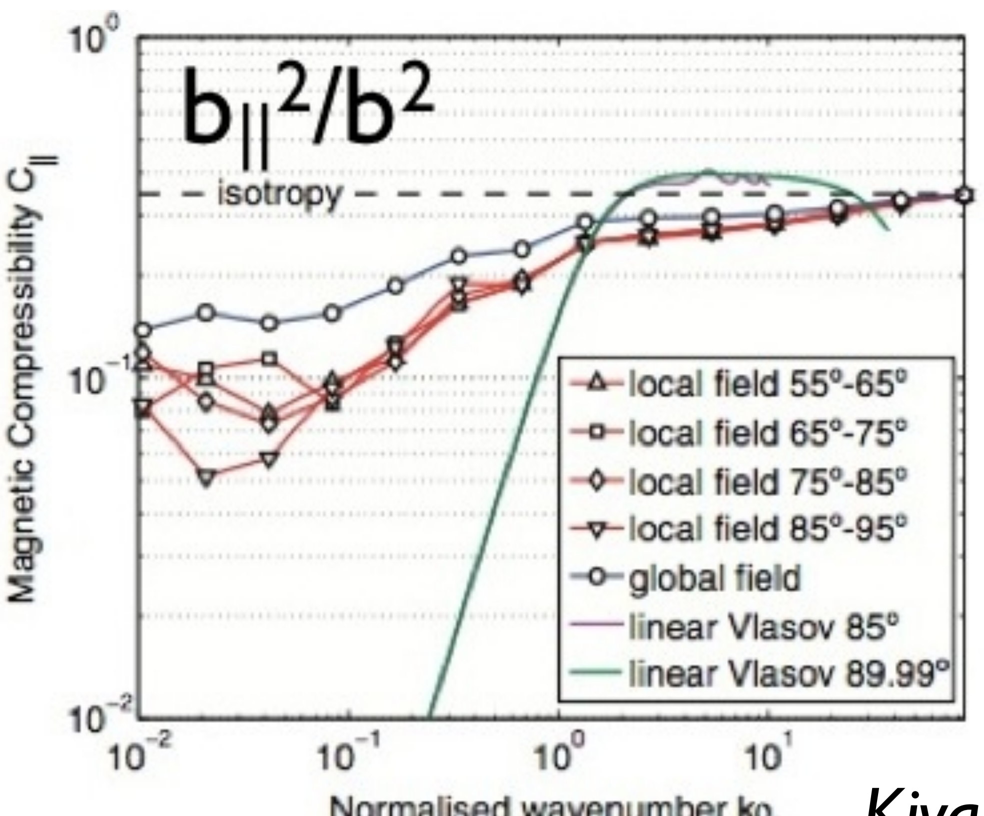


Kiyani et al. ApJ 2013

Nature of sub-ion scale fluctuations?

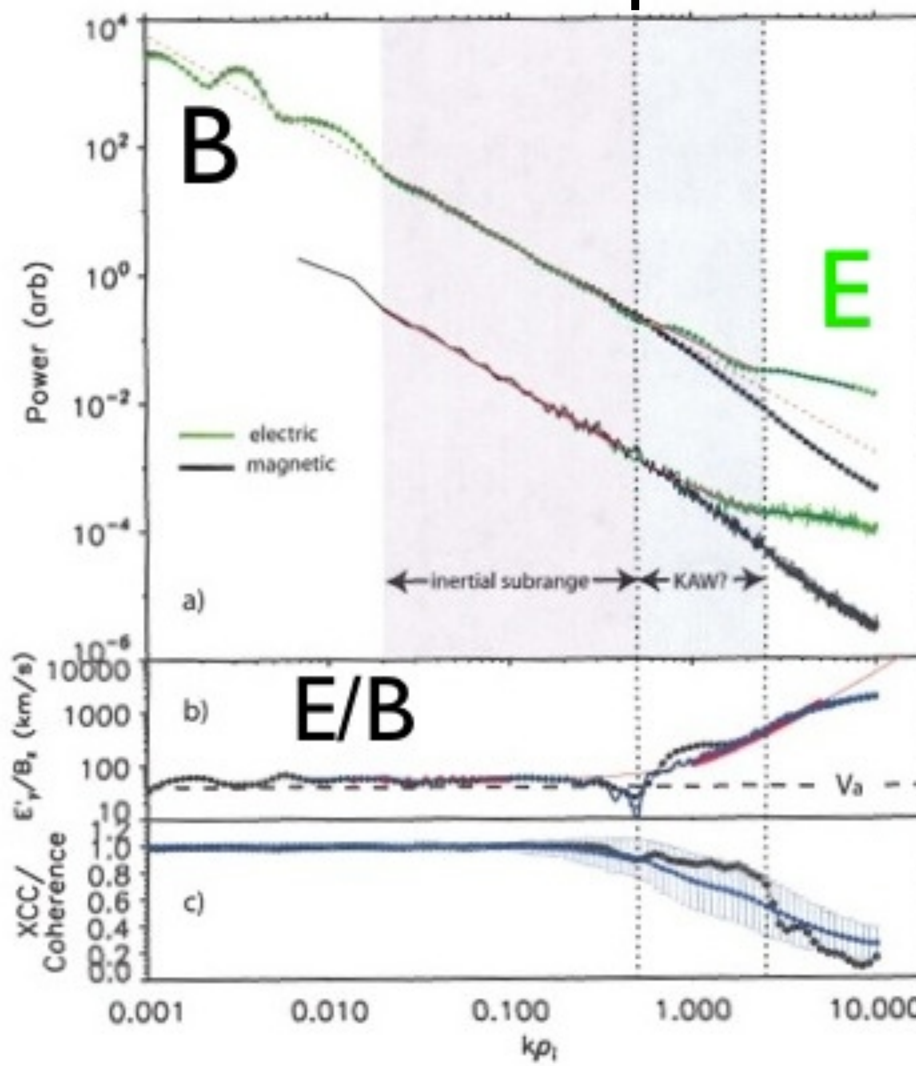


predictions from low frequency kinetic Alfvén wave (KAW) turbulence are very well verified! (which depend on β_p and T_p/T_e)



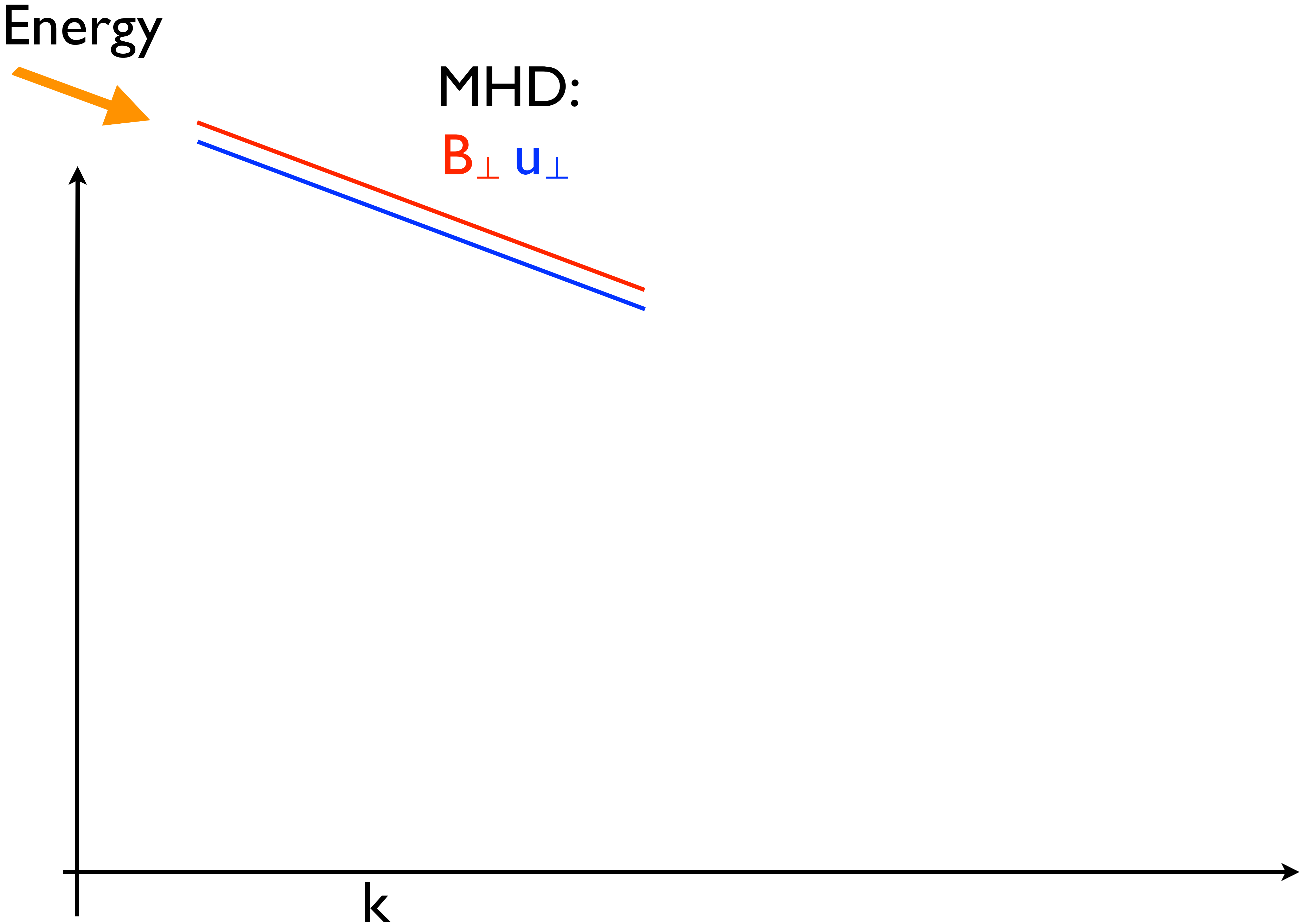
Kiyani et al. ApJ 2013

electric field spectrum

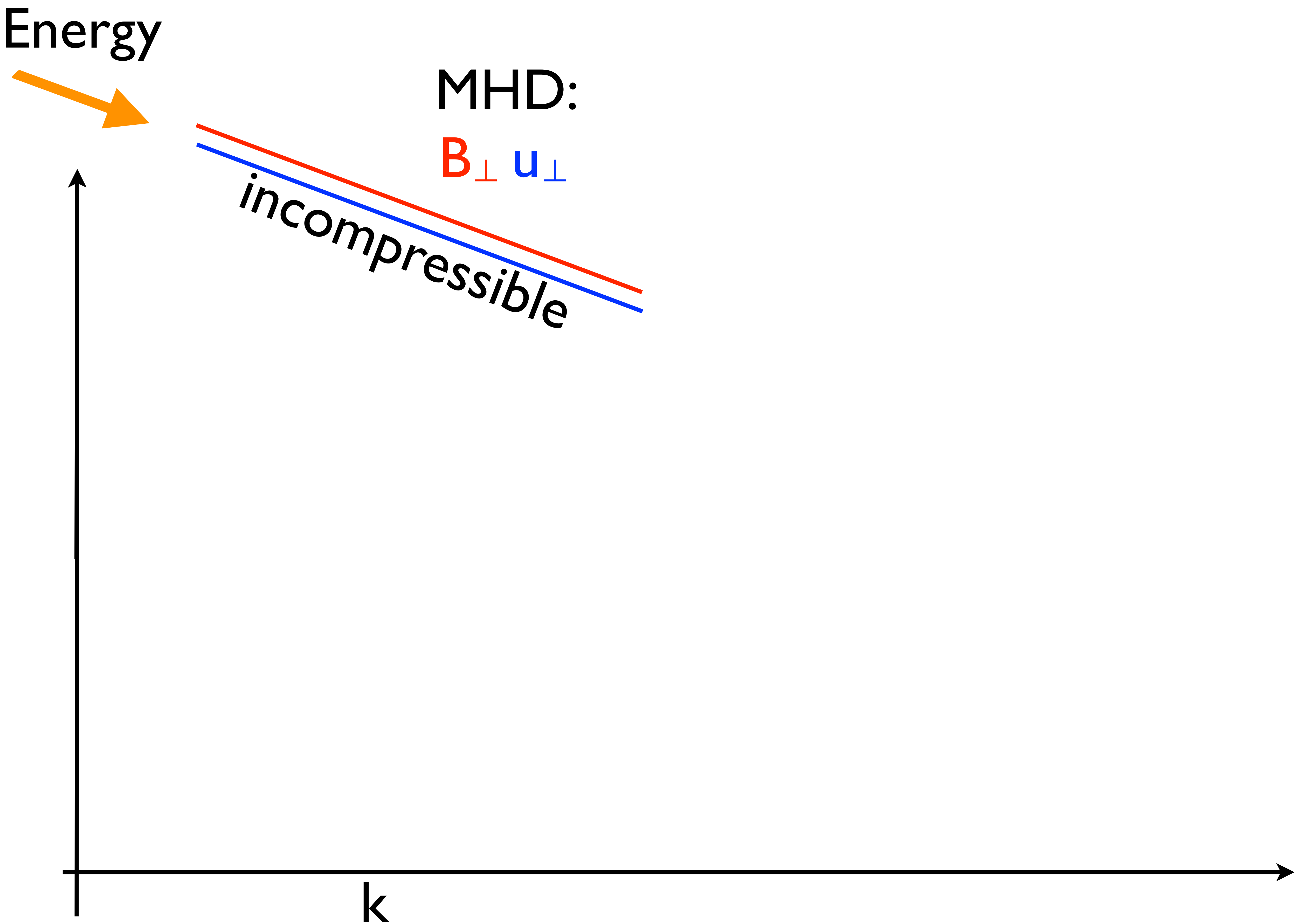


Bale et al PRL 2005

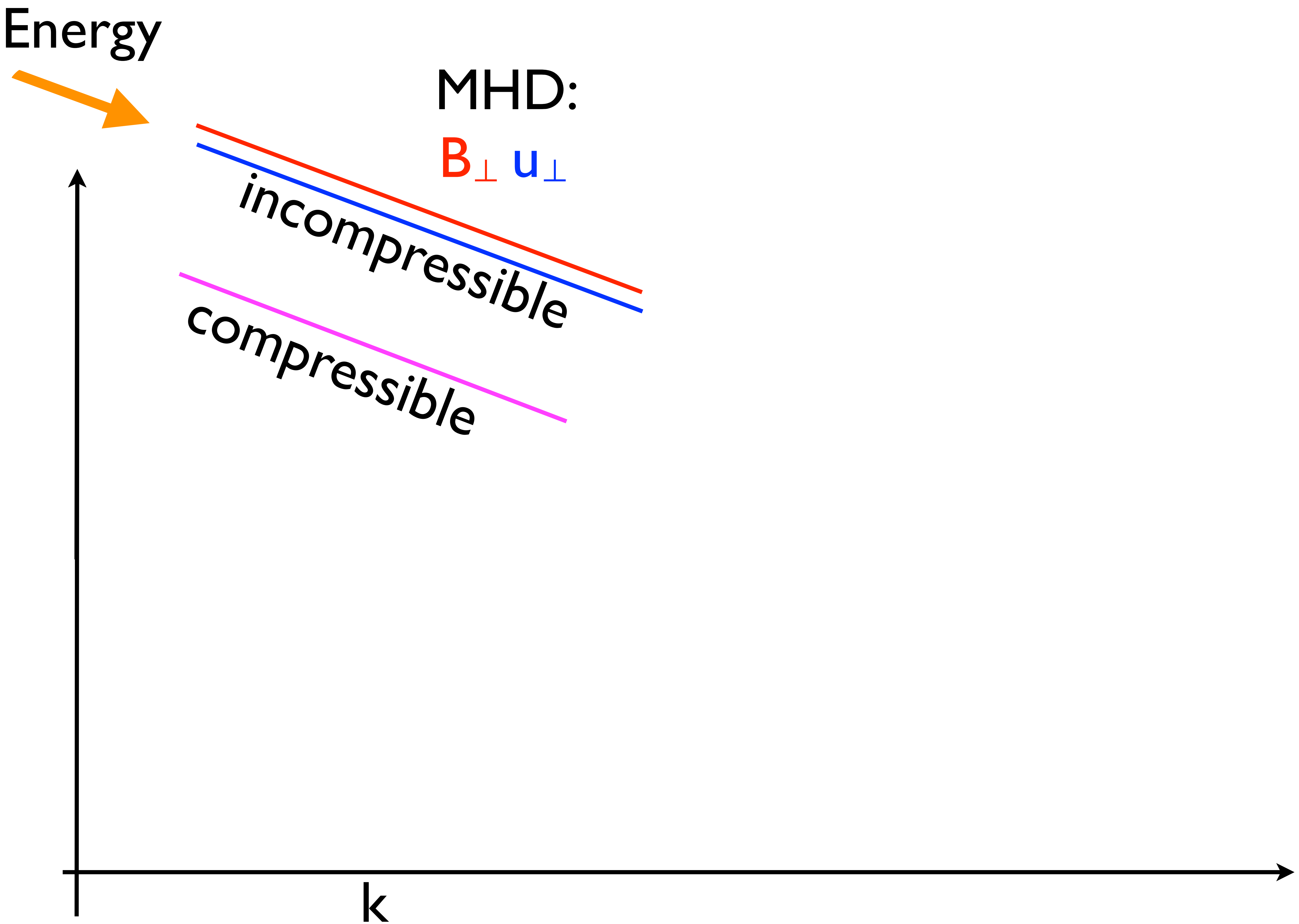
MHD-kinetic turbulent cascade



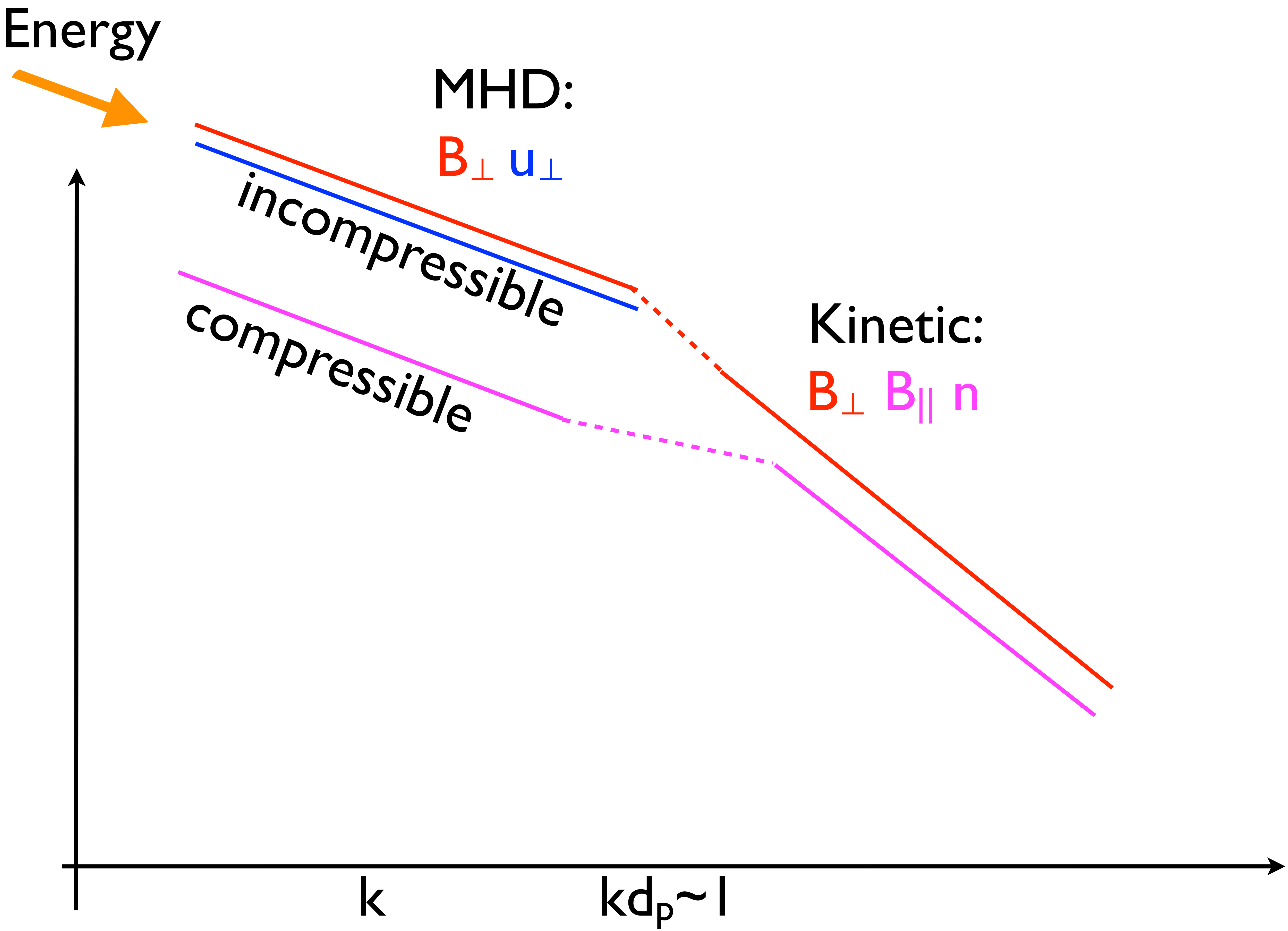
MHD-kinetic turbulent cascade



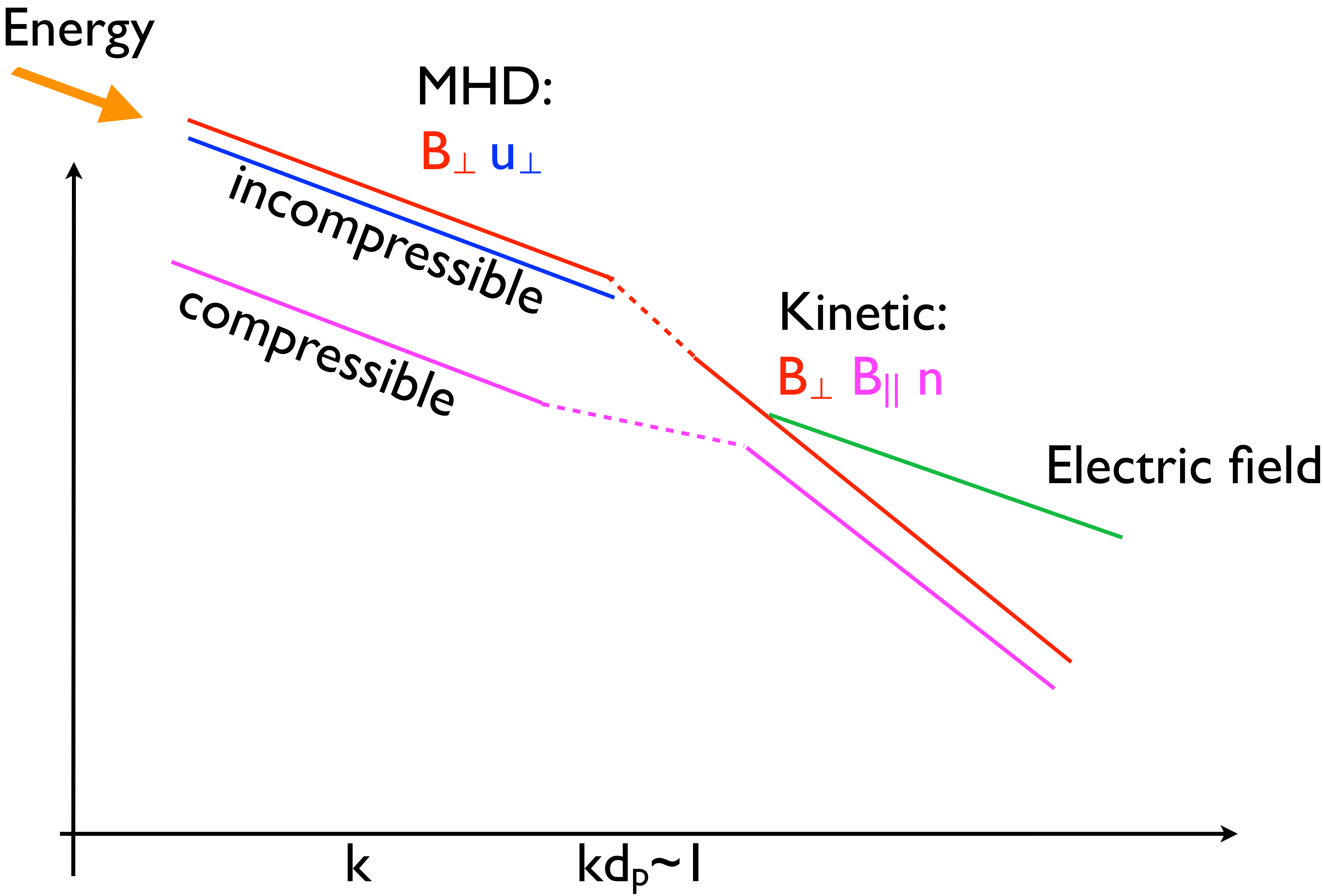
MHD-kinetic turbulent cascade



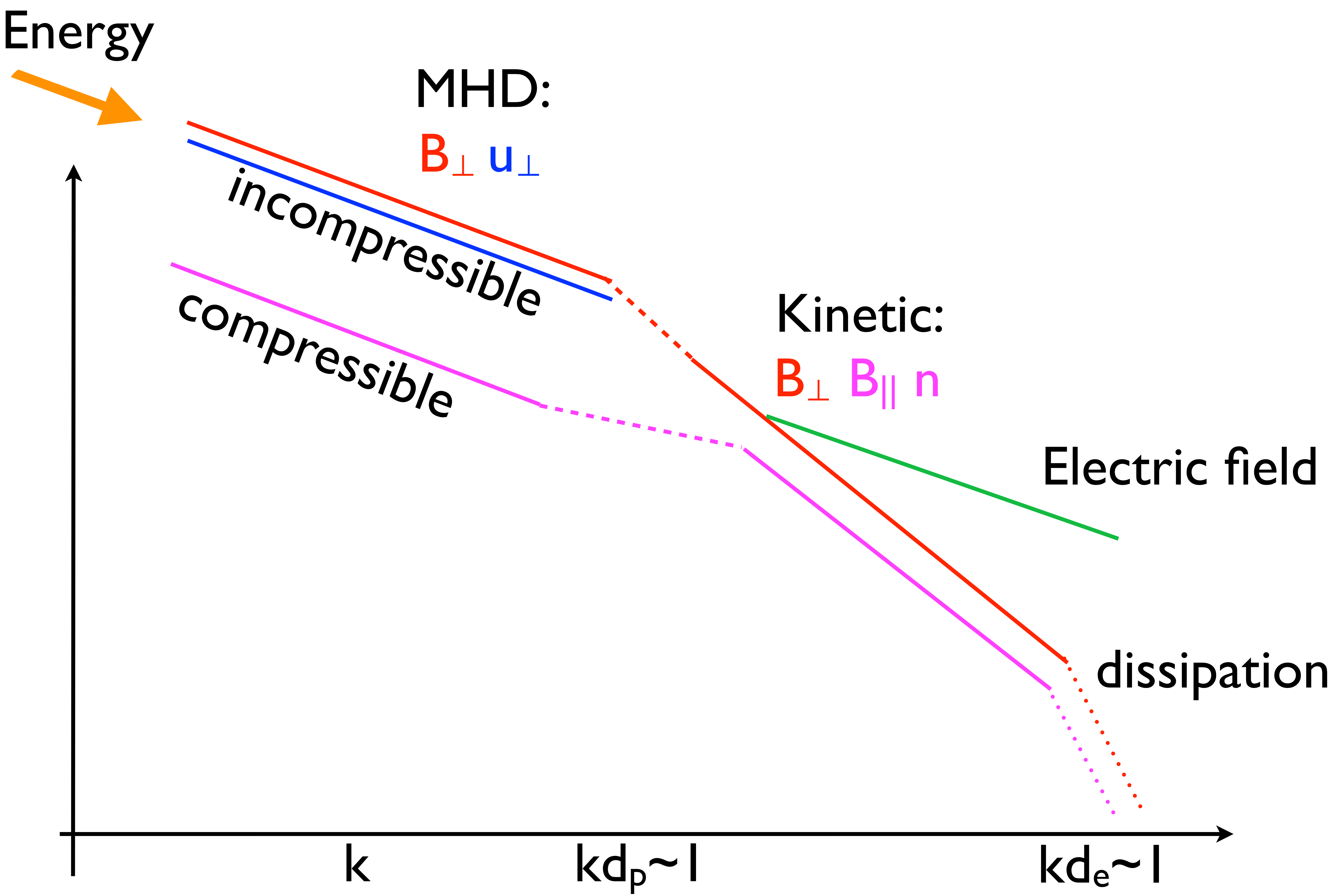
MHD-kinetic turbulent cascade



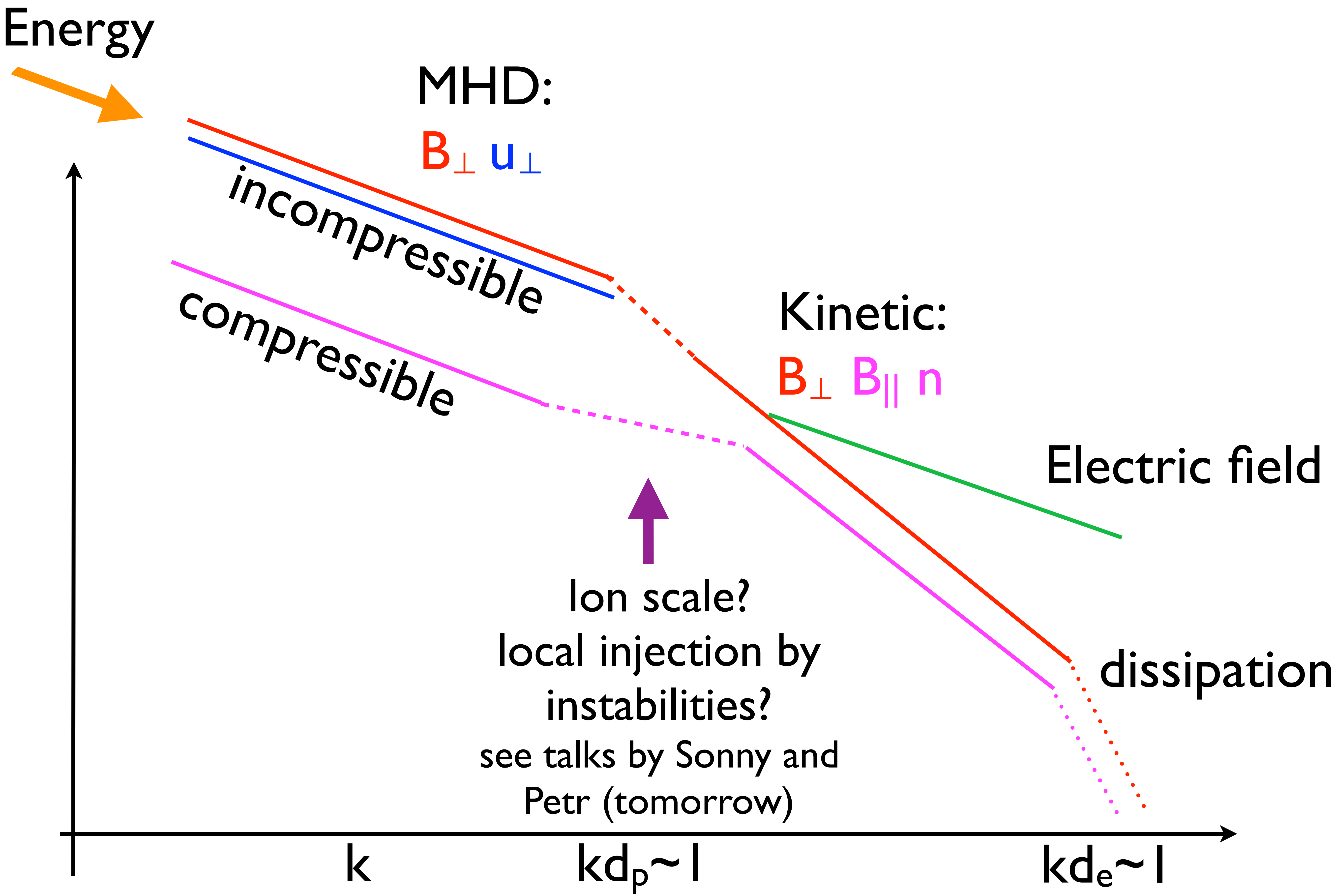
MHD-kinetic turbulent cascade



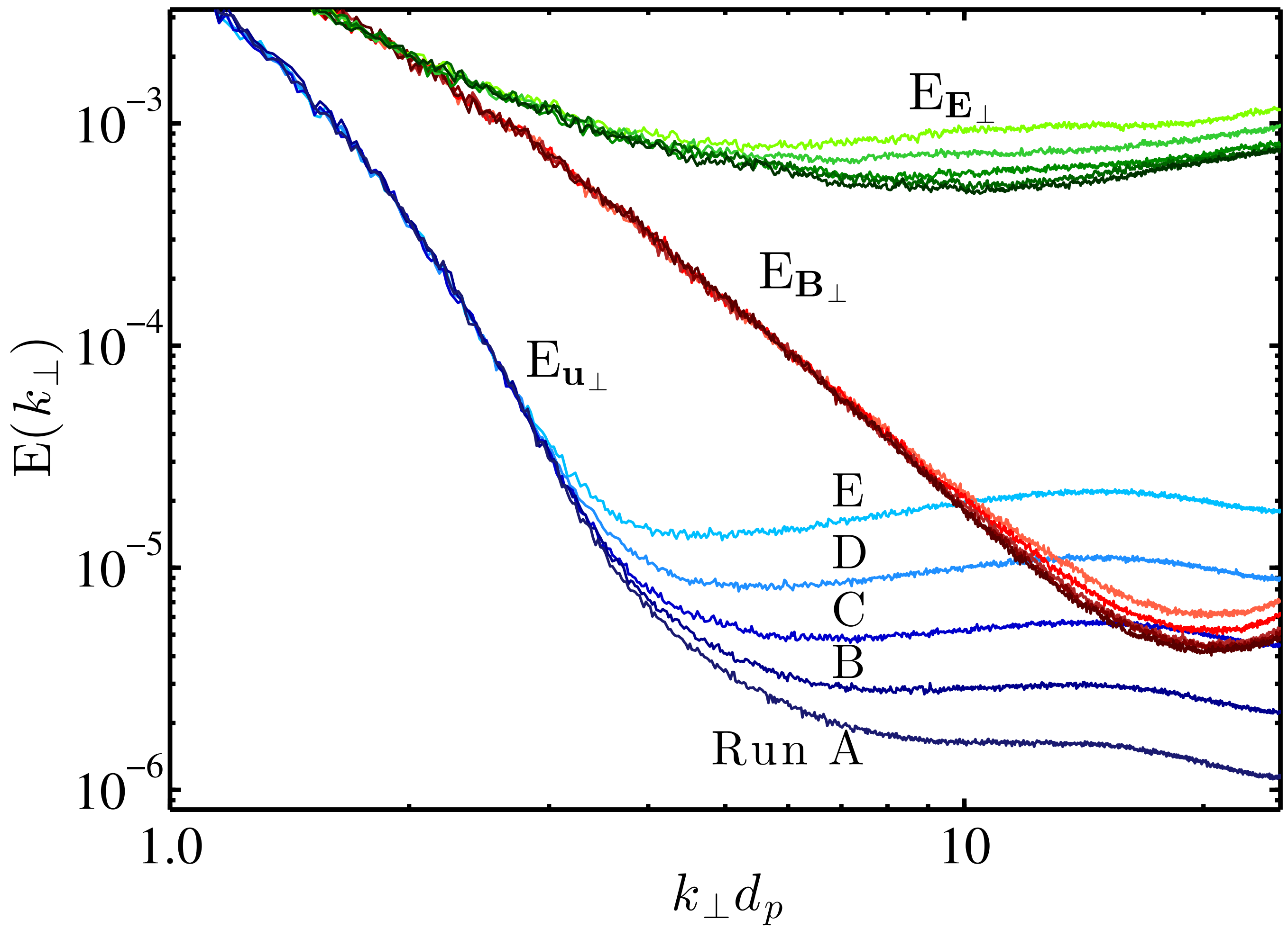
MHD-kinetic turbulent cascade



MHD-kinetic turbulent cascade



Stability of results: resistivity and ppc

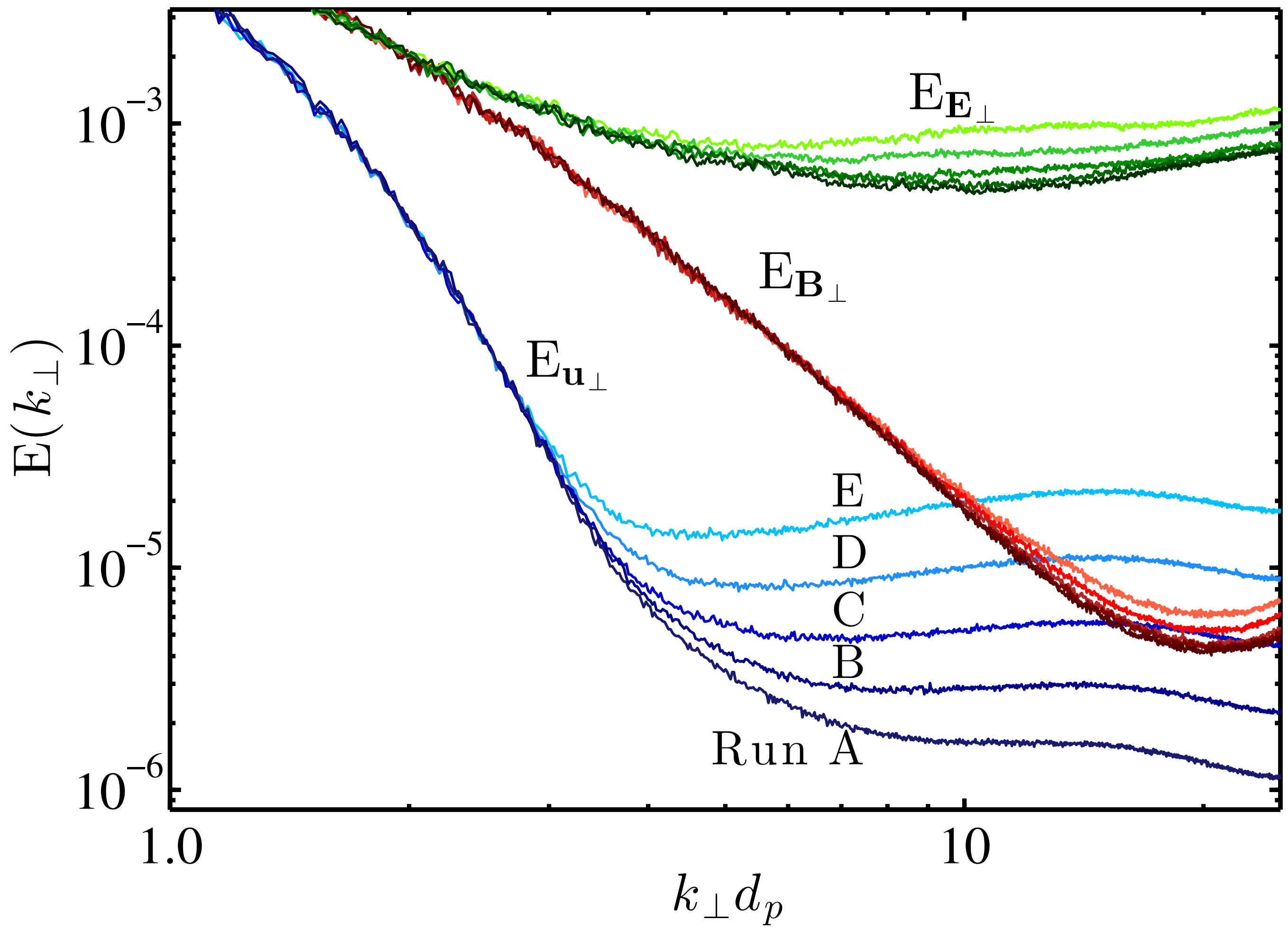


Run	$\Delta x/d_p$	$L_{\rm box}/d_p$	η	ppc
A	0.125	256	5×10^{-4}	8000
B	0.125	256	5×10^{-4}	4000
C	0.125	256	5×10^{-4}	2000
D	0.125	256	5×10^{-4}	1000
E	0.125	256	5×10^{-4}	500
F	0.125	256	1×10^{-4}	8000
G	0.125	256	1×10^{-3}	8000
H	0.250	512	1×10^{-3}	8000
I	0.500	1024	2×10^{-3}	8000

Number of particles per cell influences the spectral properties of different fields

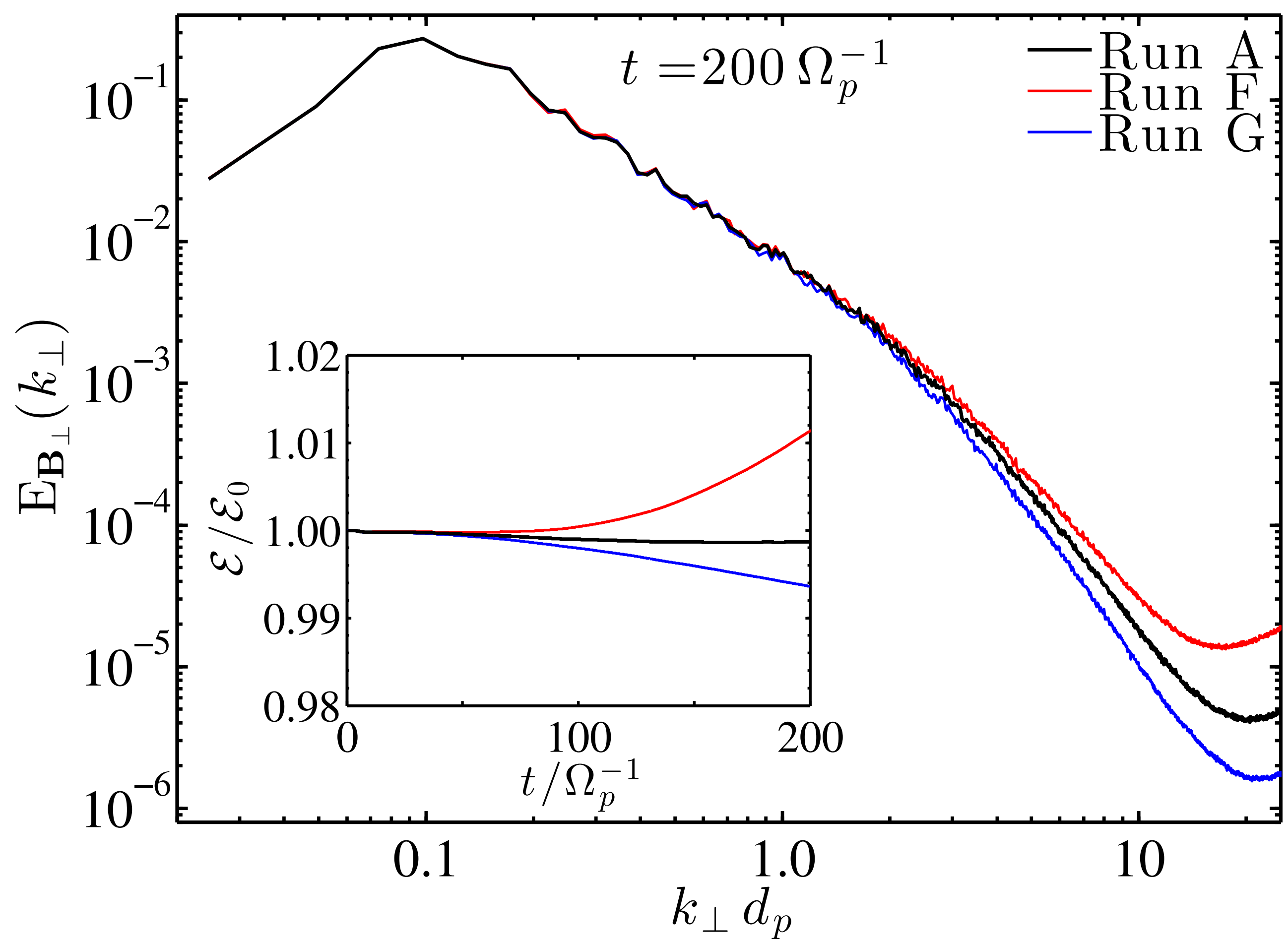
Stability of results: resistivity and ppc

Run	$\Delta x/d_p$	$L_{\rm box}/d_p$	η	ppc
A	0.125	256	5×10^{-4}	8000
B	0.125	256	5×10^{-4}	4000
C	0.125	256	5×10^{-4}	2000
D	0.125	256	5×10^{-4}	1000
E	0.125	256	5×10^{-4}	500
F	0.125	256	1×10^{-4}	8000
G	0.125	256	1×10^{-3}	8000
H	0.250	512	1×10^{-3}	8000
I	0.500	1024	2×10^{-3}	8000



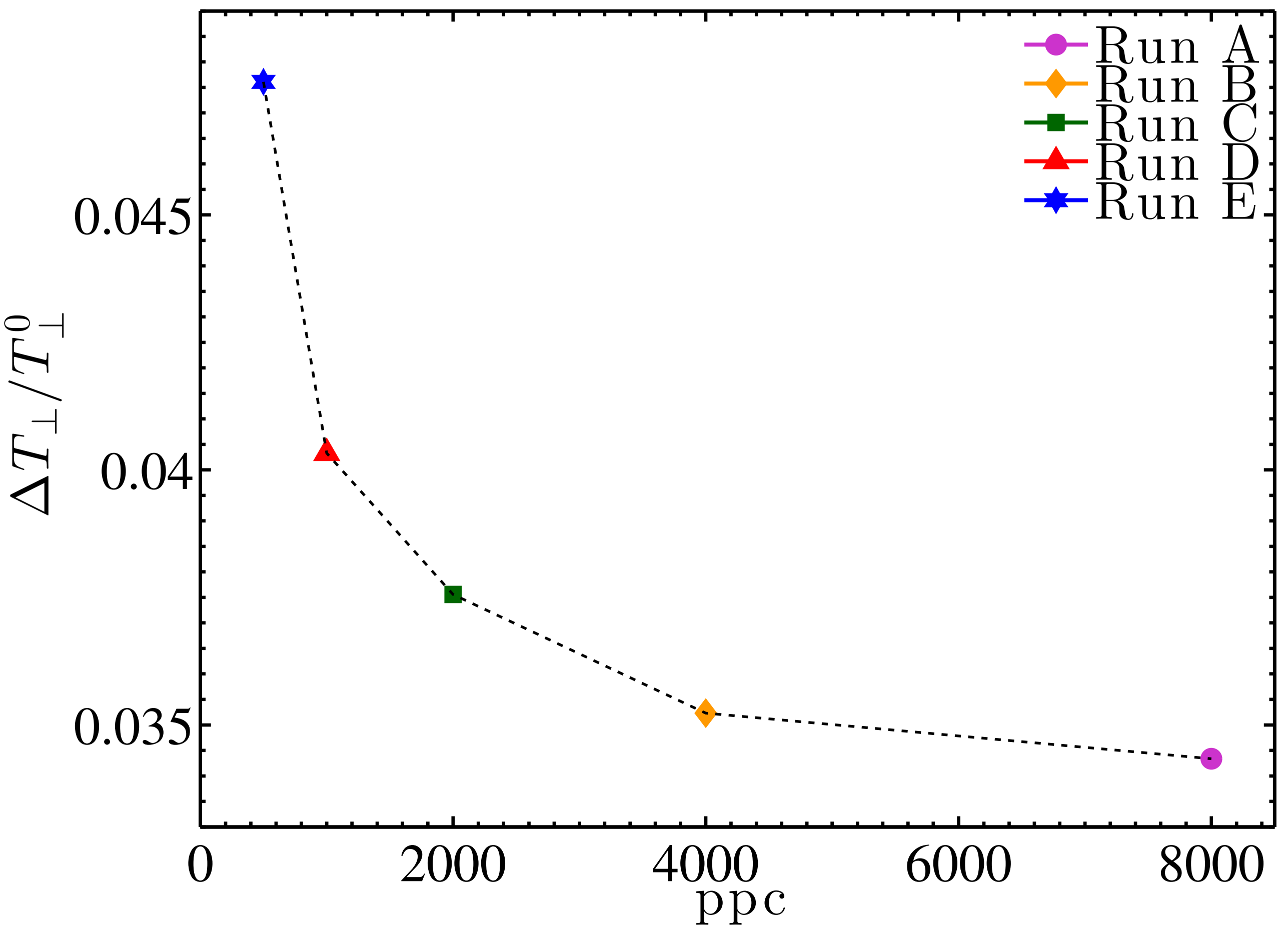
Number of particles per cell influences the spectral properties of different fields

Fine tuning of resistivity is needed to satisfy energy conservation



Stability of results: particle heating

Number of particles per cell influences the amount of perpendicular heating.
We see convergence for ppc>4000

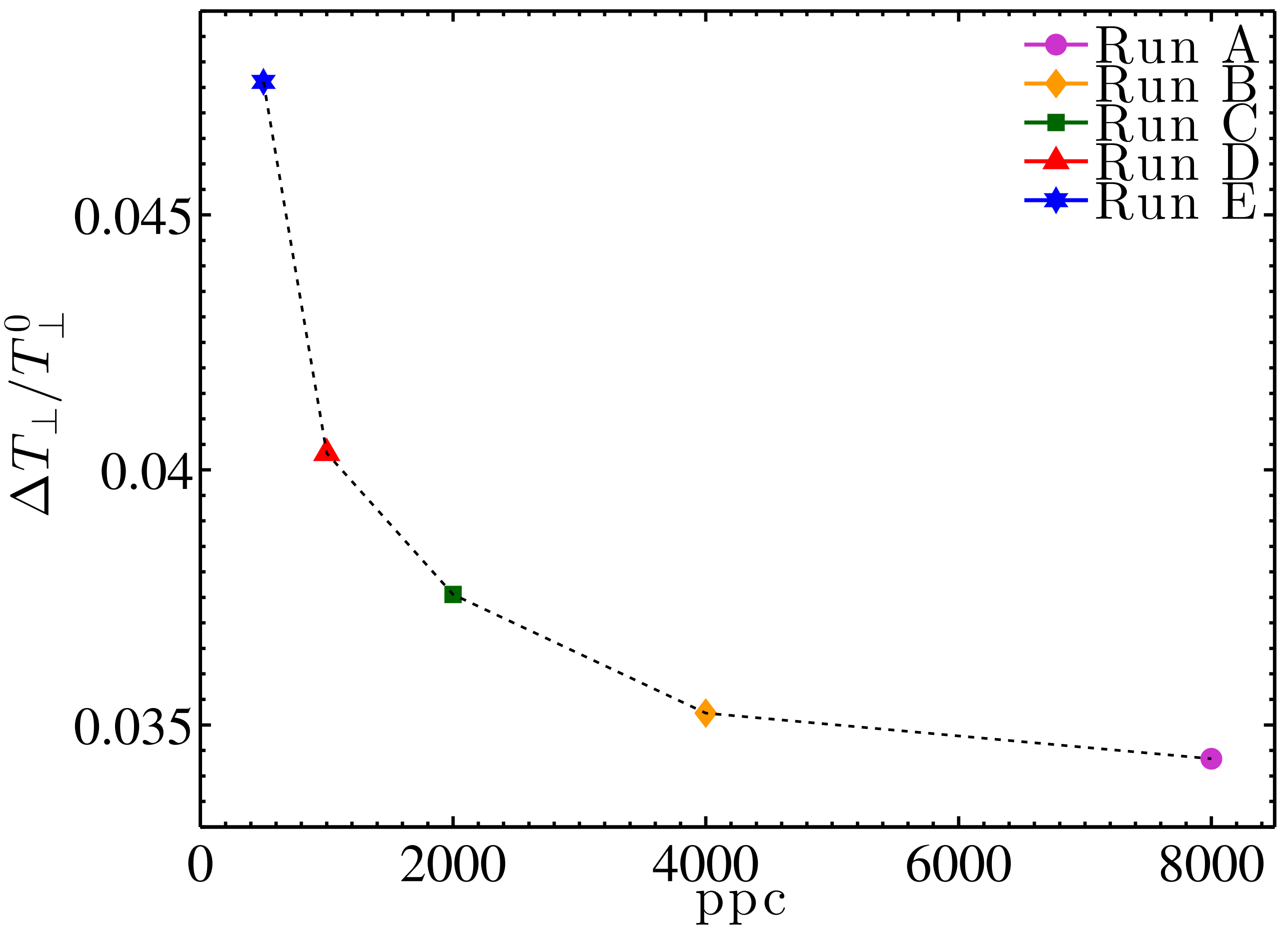


Run	$\Delta x/d_p$	L_{box}/d_p	η	ppc
A	0.125	256	5×10^{-4}	8000
B	0.125	256	5×10^{-4}	4000
C	0.125	256	5×10^{-4}	2000
D	0.125	256	5×10^{-4}	1000
E	0.125	256	5×10^{-4}	500
F	0.125	256	1×10^{-4}	8000
G	0.125	256	1×10^{-3}	8000
H	0.250	512	1×10^{-3}	8000
I	0.500	1024	2×10^{-3}	8000

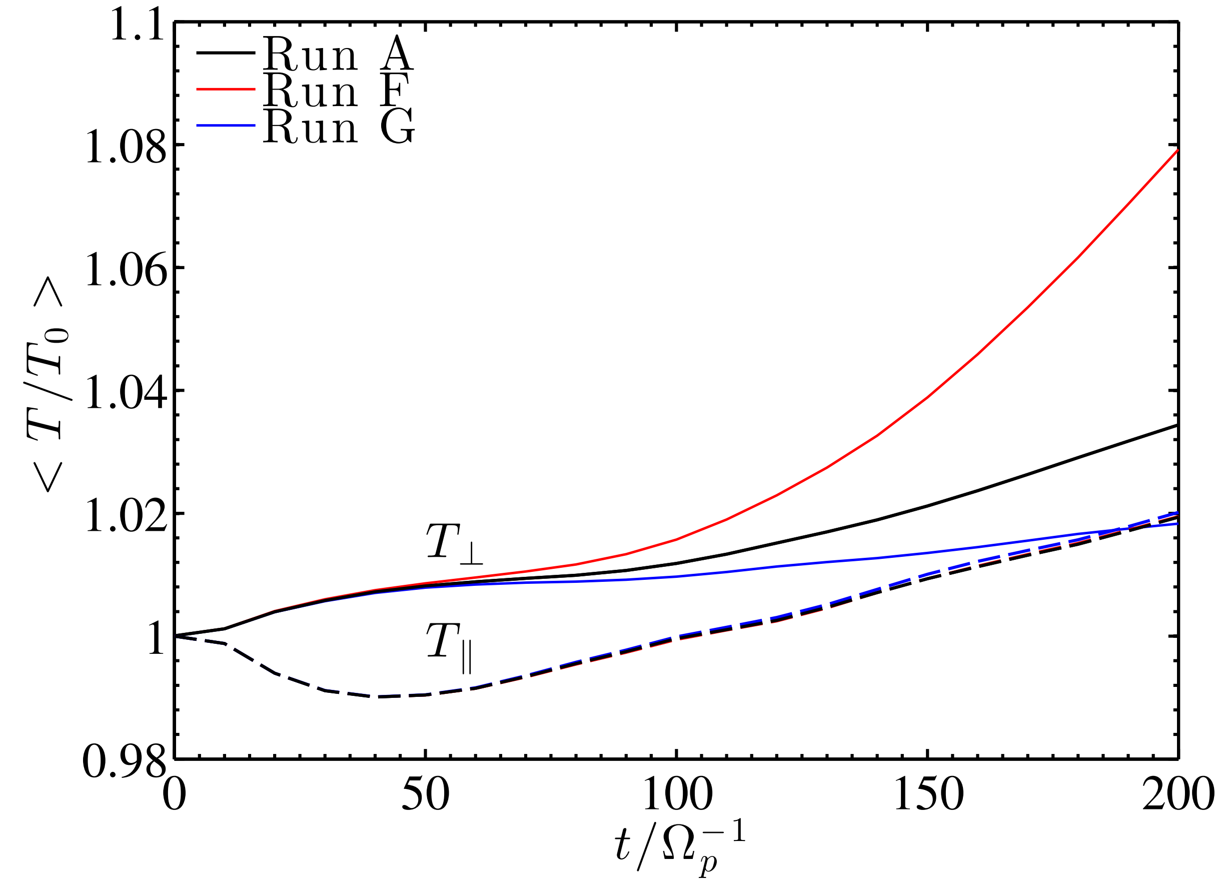
Stability of results: particle heating

Number of particles per cell influences the amount of perpendicular heating.
We see convergence for ppc>4000

Run	$\Delta x/d_p$	L_{box}/d_p	η	ppc
A	0.125	256	5×10^{-4}	8000
B	0.125	256	5×10^{-4}	4000
C	0.125	256	5×10^{-4}	2000
D	0.125	256	5×10^{-4}	1000
E	0.125	256	5×10^{-4}	500
F	0.125	256	1×10^{-4}	8000
G	0.125	256	1×10^{-3}	8000
H	0.250	512	1×10^{-3}	8000
I	0.500	1024	2×10^{-3}	8000



Resistivity affects perp. heating

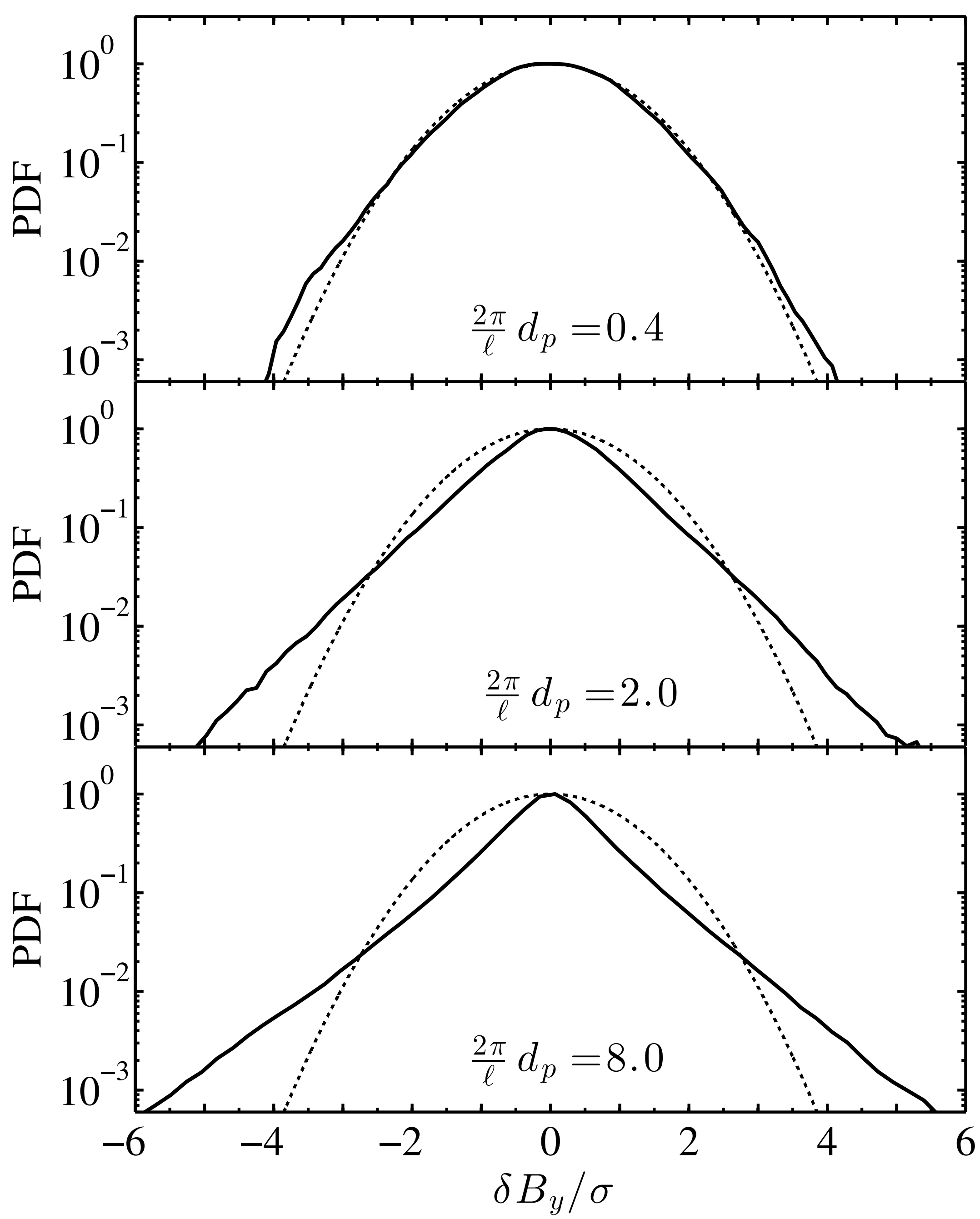


Conclusion

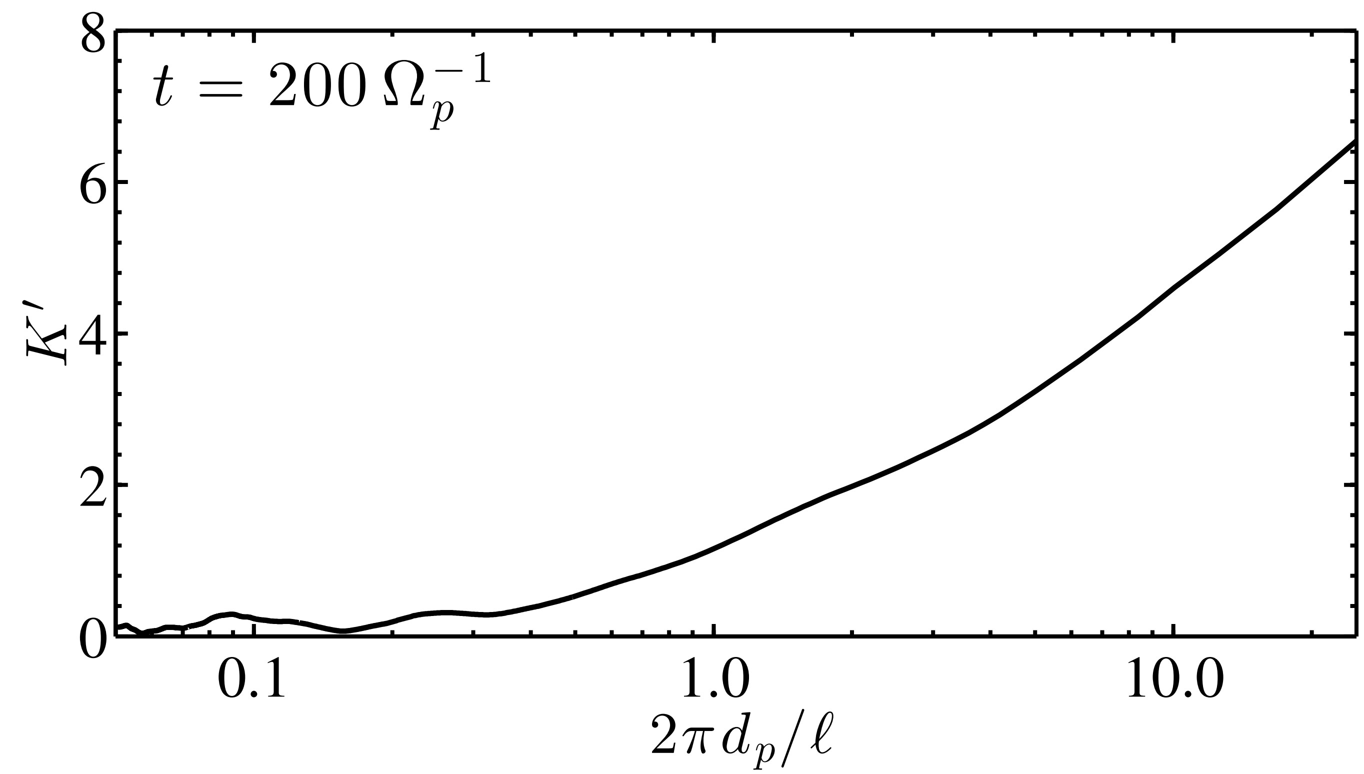
- Self-consistent description of transition from MHD to kinetic turbulence. One full decade for each regime separated by a smooth break.
- 2D geometry can capture most of SW properties. Why??
  3D is needed to validate results.
- MHD with residual energy ~k^{-2} (-5/3, -3/2, -2)
- A cascade at kinetic scales consistent with KAW turbulence (fluctuations not waves!)
- Some proton heating in both parallel and perp directions is observed. Parallel one is very solid, perp. heating is strongly affected by resolution. Some care has to be used in general to interpret simulation results!
  More work is needed to understand the origin of the heating

Intermittency

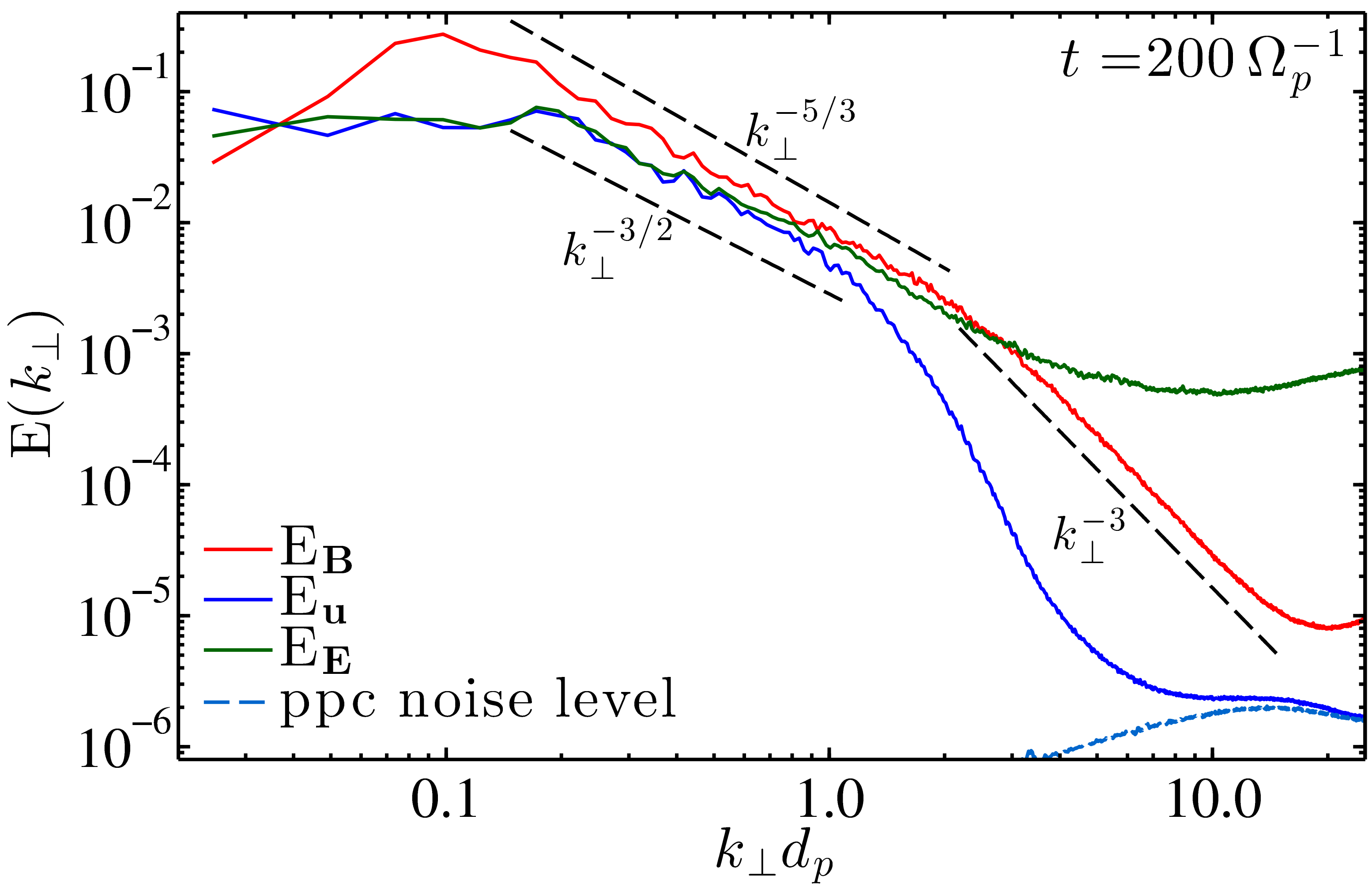
Non-Gaussian distribution of δB increasing with scale



Excess of Kurtosis



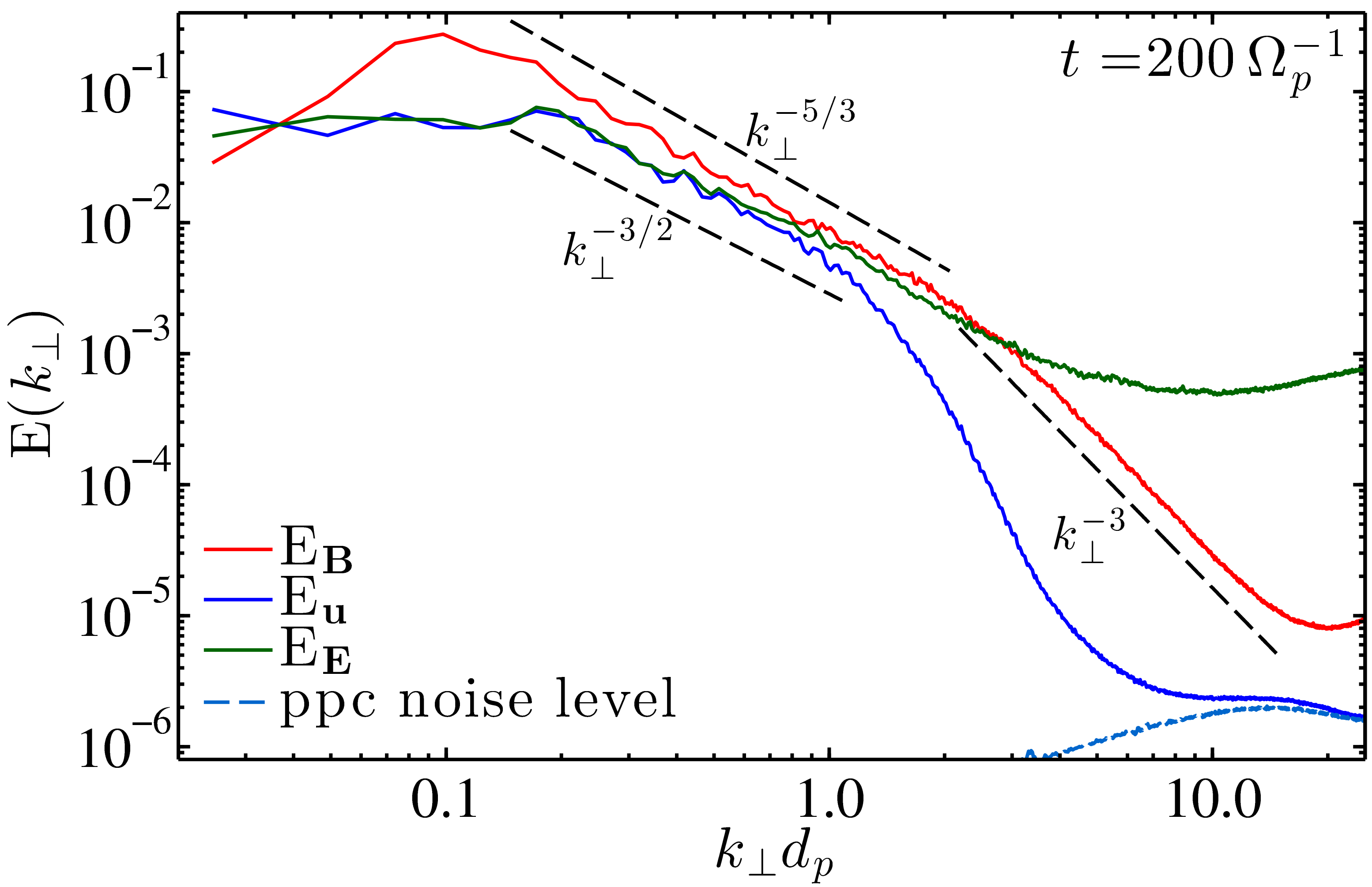
Electric field (inertial range)



MHD range: E=-VxB ~ $\delta v B_0 + \delta b U_0$

Plasma frame: $E_{SW} = \delta v B_0 \propto k^{-3/2}$

Electric field (inertial range)



E is frame dependent

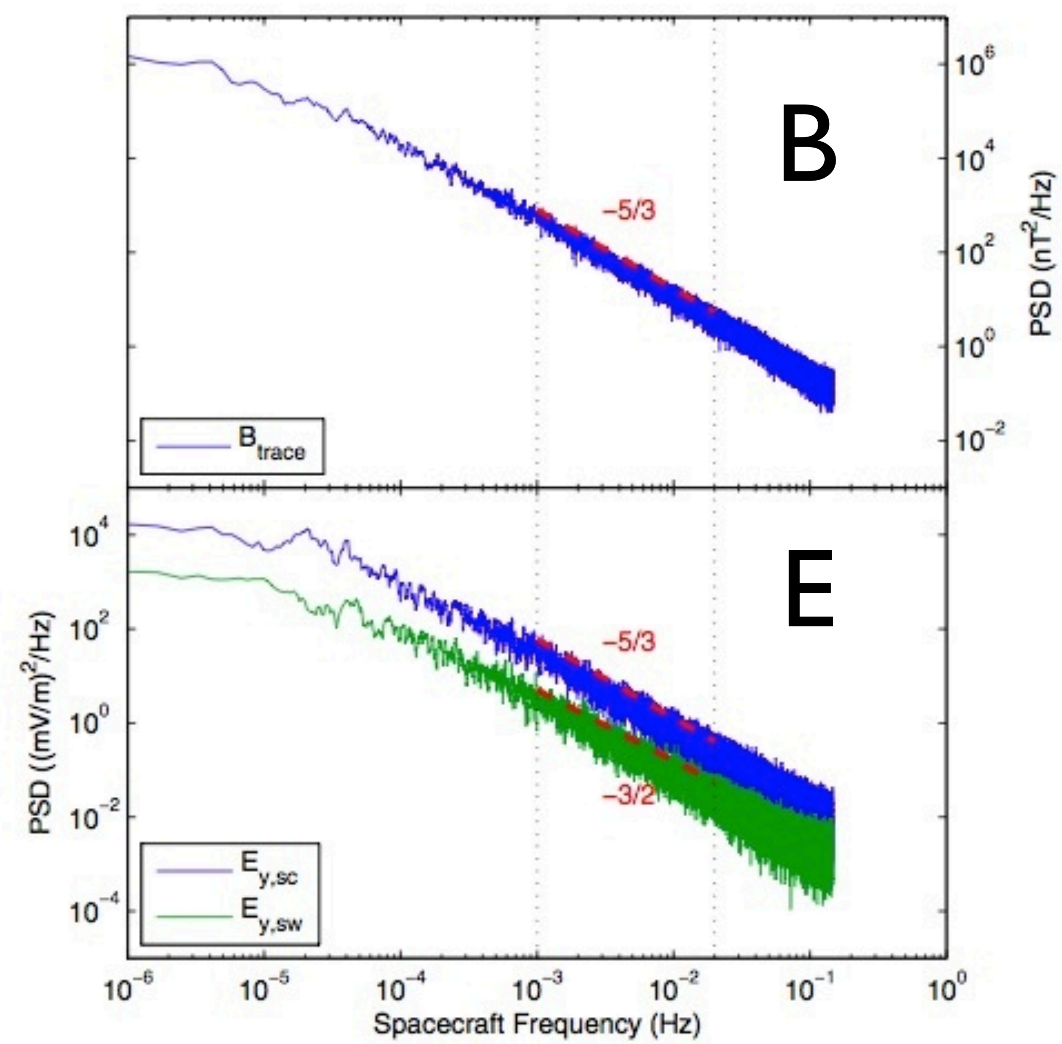


Figure 1. Sample power spectra from *P2* 2010 days 217–230. The dotted lines show the range of scales that the spectral index is fitted to. Gradients of −5/3 and −3/2 are marked for reference.

(A color version of this figure is available in the online journal.)

Chen et al 2011

MHD range: $E=-V\times B \sim \delta v B_0+\delta b U_0$

Plasma frame: $E_{SW}=\delta v B_0 \propto k^{-3/2}$

s/c frame:

$E_{s/c}=\delta v B_0+\delta b U_0 \sim \delta b U_0 \propto k^{-5/3}$

Electric field - all terms

$$\underbrace{\boldsymbol{E}}_{} = \underbrace{-\boldsymbol{u} \times \boldsymbol{B}}_{\boldsymbol{E}_{\mathrm{MHD}}} + \underbrace{\boldsymbol{J} \times \boldsymbol{B}/n}_{\boldsymbol{E}_{\mathrm{Hall}}} \underbrace{-\boldsymbol{\nabla} \mathrm{p}_e/n}_{\boldsymbol{E}_{\mathrm{pe}}} + \underbrace{\eta \boldsymbol{J}}_{\boldsymbol{E}_\eta}$$