Self-consistent Castaing distribution of inertial range fluctuations in solar wind turbulence

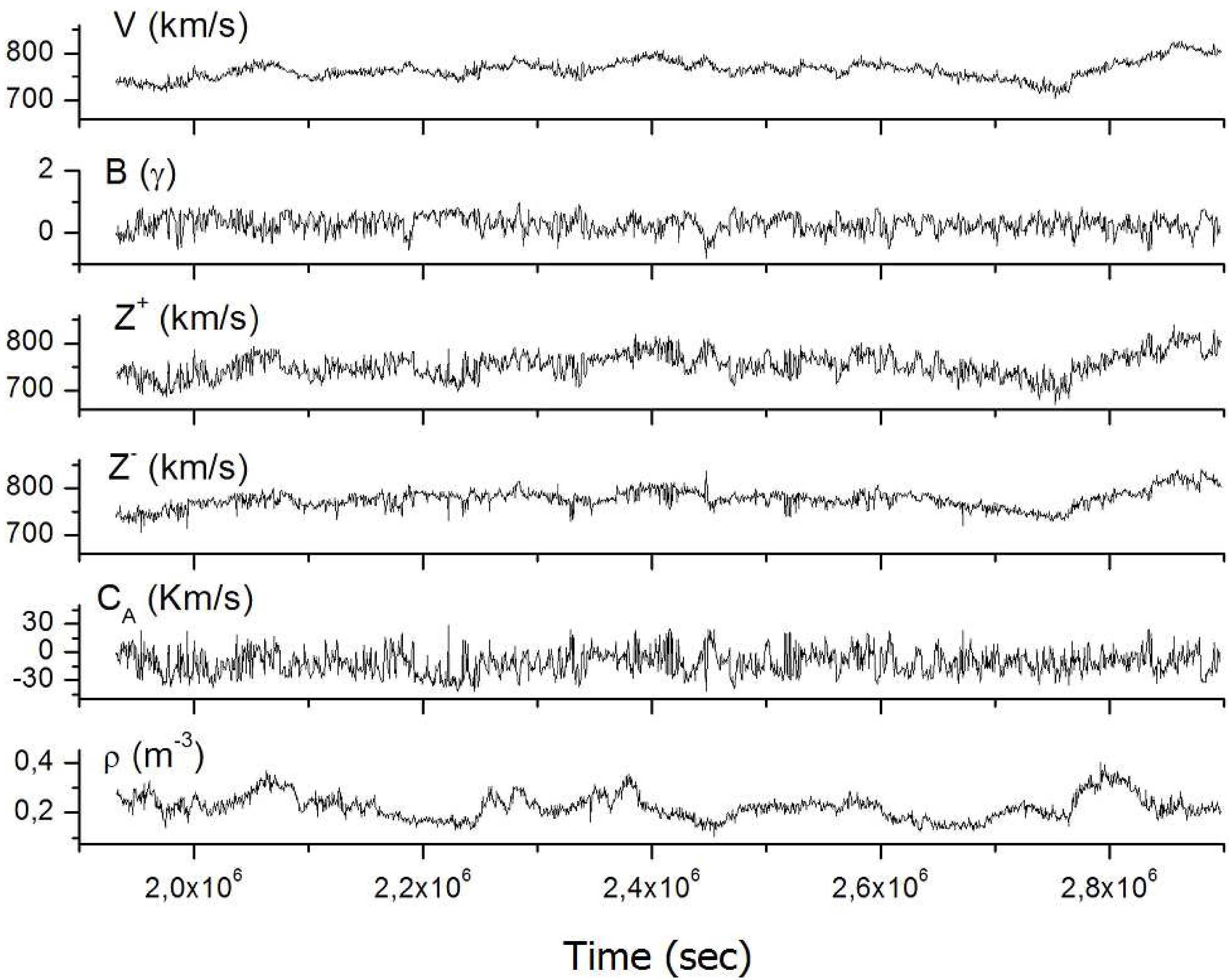
Luca Sorriso-Valvo, Raffaele Marino

Nanotec/CNR, Rende, Italy

Silvia Perri, Luisa Lijoi and Vincenzo Carbone

Dipartimento di Fisica, Università della Calabria, Rende, Italy

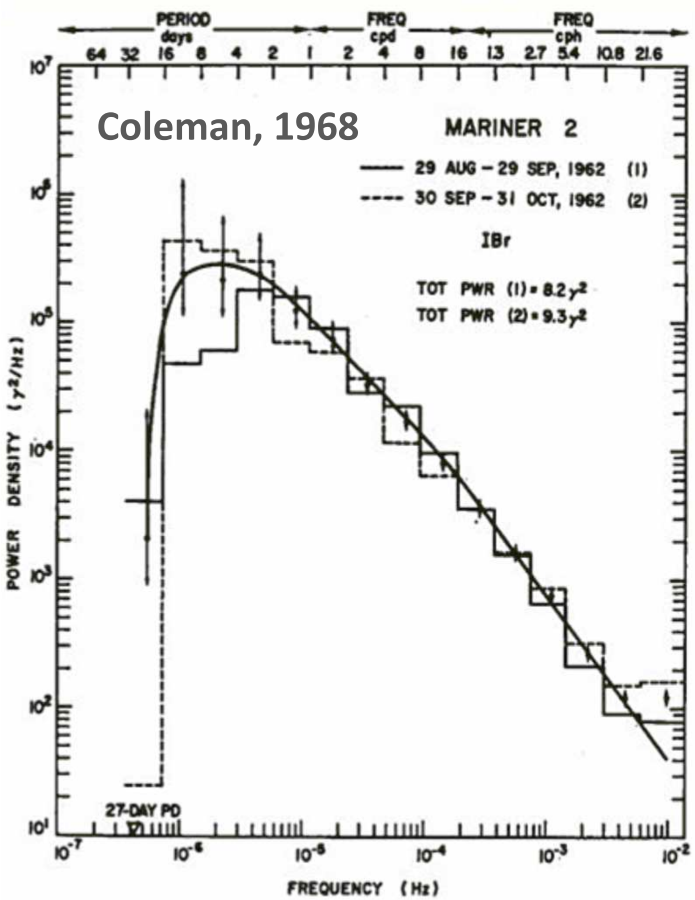
The solar wind as a wind tunnel

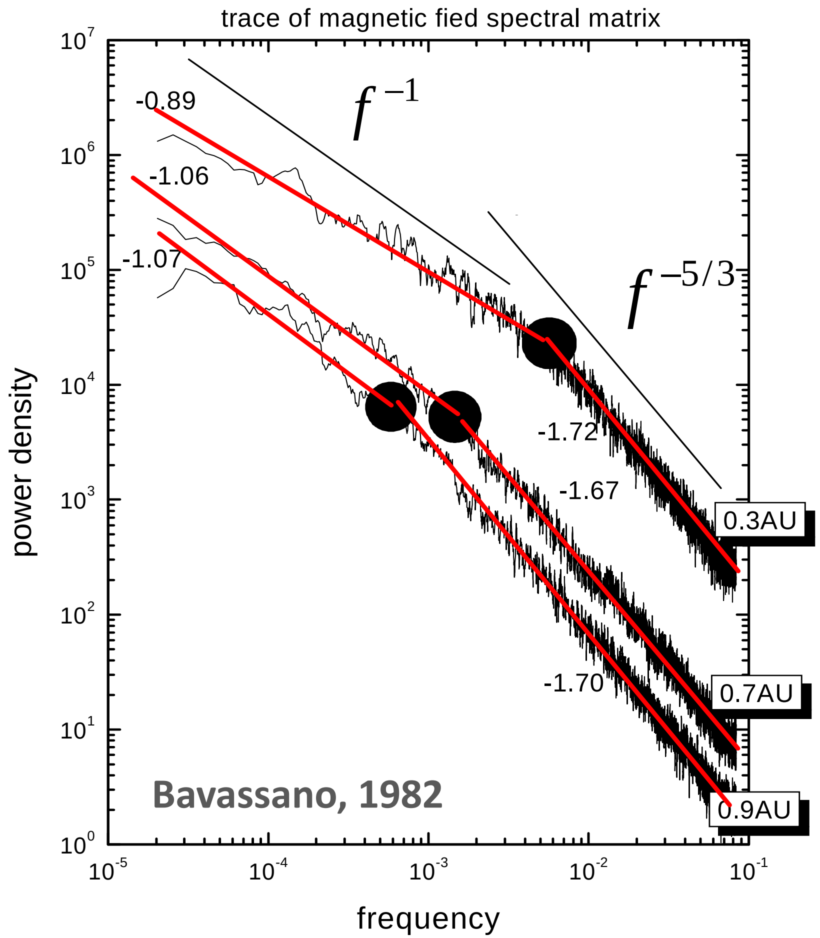


Bruno & Carbone, Liv. Rev. Solar Phys., 2005/2013; Alexandrova et al., Space Sci. Rev., 2013; Matthaeus & Velli, Space Sci. Rev., 2011

Turbulent fluctuations in the solar wind

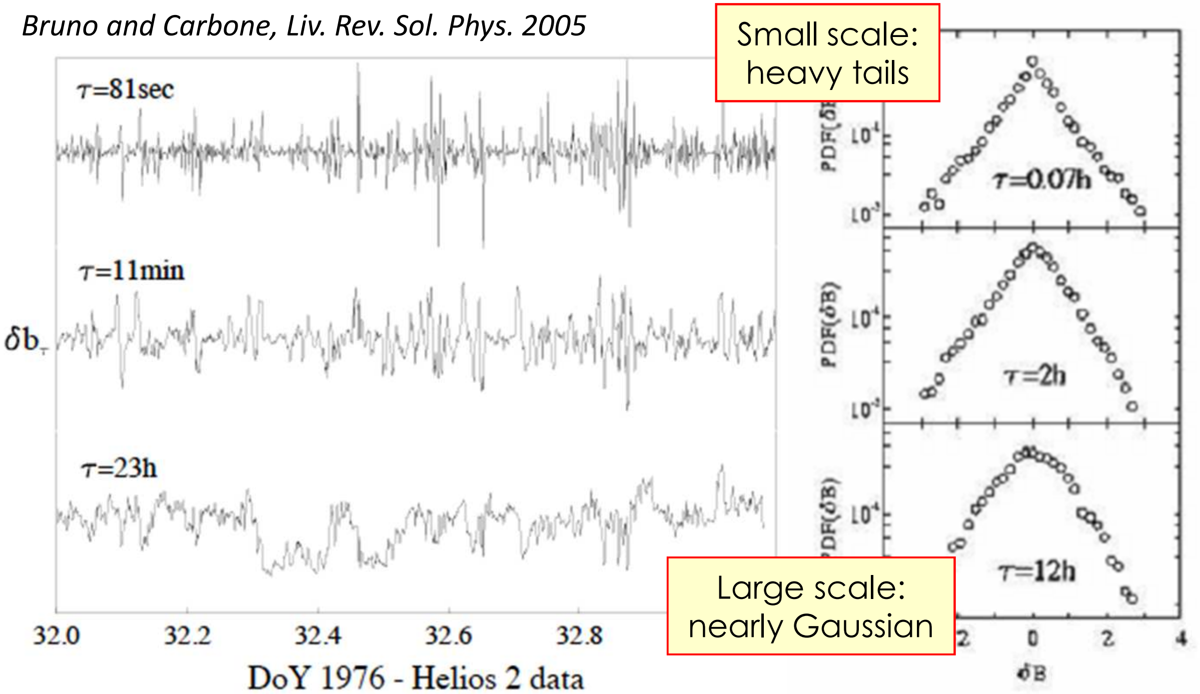
Early observations of magnetic spectra: turbulent solar wind.
Evidence of radial evolution.





Intermittency in solar wind turbulence

Bruno and Carbone, Liv. Rev. Sol. Phys. 2005



Tu & Marsch, Space Sci. Rev. 1995; Sorriso-Valvo et al., GRL 1999

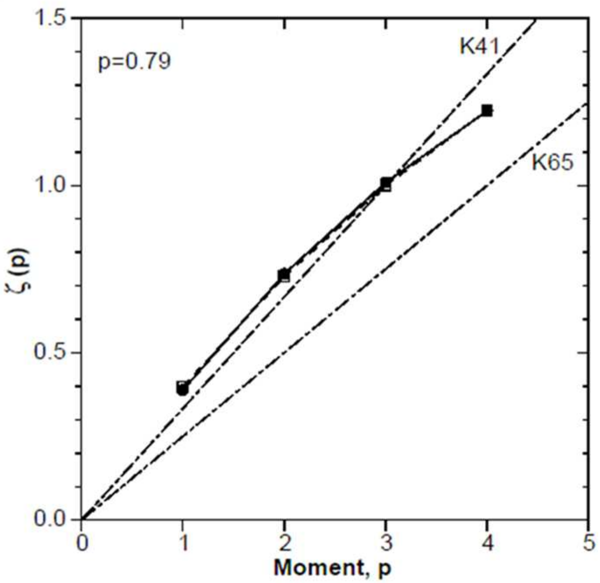
Solar wind turbulence is characterized by strong intermittency.

Intermittency:
Rare, bursty events dominate the smaller scales statistics, resulting in non-Gaussian, scale dependent PDFs fat tails.

Higher moments of PDFs (structure function) have scaling exponents that depend non-linearly on the order (K41).

Intermittency in turbulence is due to the multifractal, inhomogeneous nature of the energy cascade.

Carbone, Ann. Geophys. 1994; Carbone et al., JGR 1996



Early works and ~established results: cascade

MHD turbulence satisfies the linear prediction for the scaling of the mixed third-order moment – analogous of the 4/5th law for Navier-Stokes turbulence

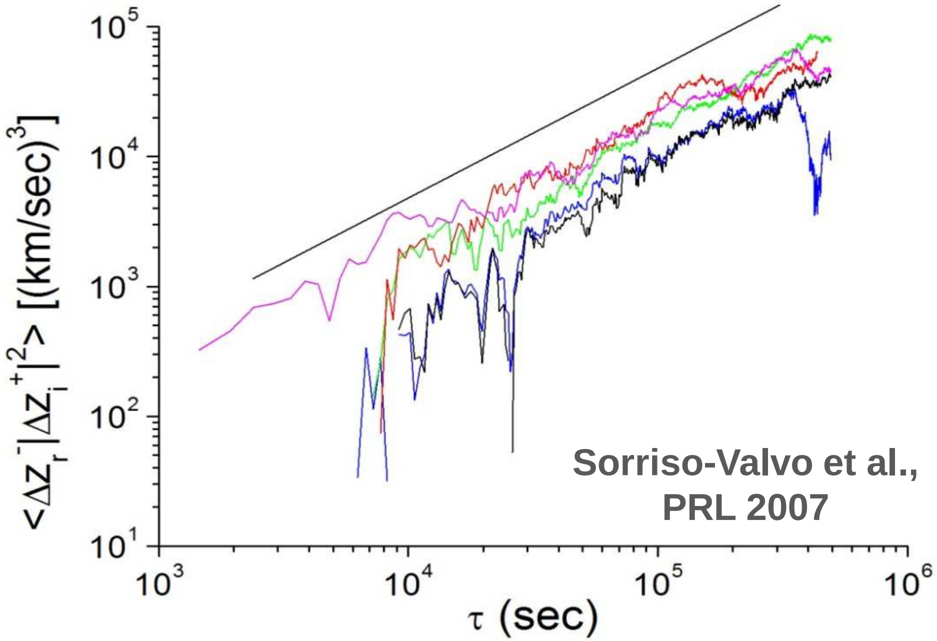
$$\frac{\partial \mathbf{z}^{\pm}}{dt} \mp (\mathbf{B}_0 \cdot \nabla)\, \mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla)\, \mathbf{z}^{\pm} = -\nabla P + D^{\pm}$$

$$z^{\pm} = \boldsymbol{v} \pm (4\pi\rho)^{-1/2} \boldsymbol{b}$$

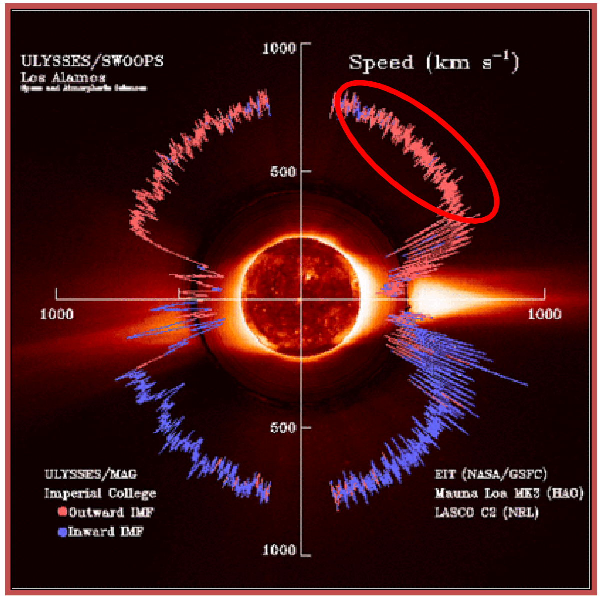
H: homogeneity, isotropy, stationarity and negligible dissipation (inertial range)

$$Y^{\pm}(l) = \langle |\Delta z^{\pm}(l;\ r)|^2 \Delta z_r^{\mp}(l;\ r) \rangle = -\frac{4}{3}\epsilon^{\pm} l$$

Politano & Pouquet, GRL 1998



Sorriso-Valvo et al., PRL 2007



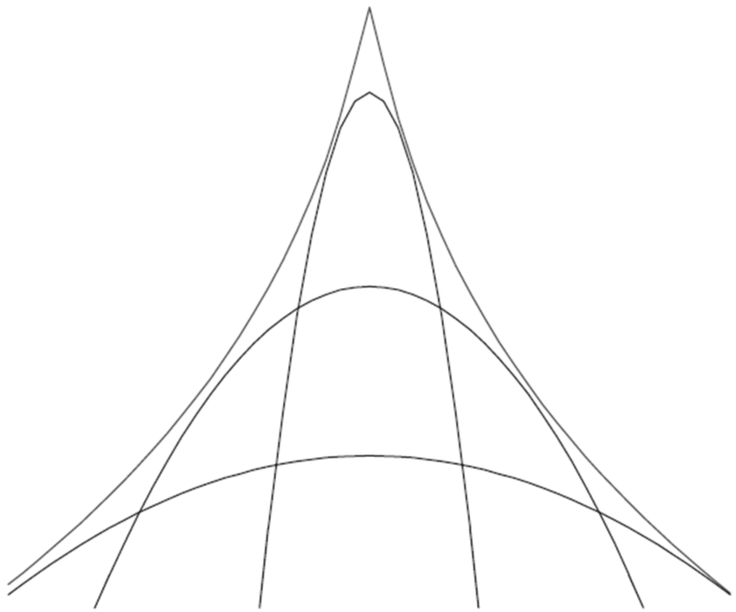
Castaing multifractal model for intermittent PDFs

Multifractal picture: each portion of space with given properties (fractal dimension, energy transfer…) has Gaussian fluctuations (e.g. Δb) at all scales.

The PDF is then the superposition of Gaussians, each with given width (variance) σ, and with weight L determined by the size of each subset.

This can be formulated by introducing the distribution of weights $L(\sigma)$, and then computing the convolution with generic Gaussian of width σ, $G(\sigma,\Delta b)$:

$$P(\Delta b) = \int L(\sigma)\ G(\Delta b,\sigma)\ d\sigma$$



Castaing et al., Phys. D 1990; Carbone et al., Riv. Nuovo Cimento 2004

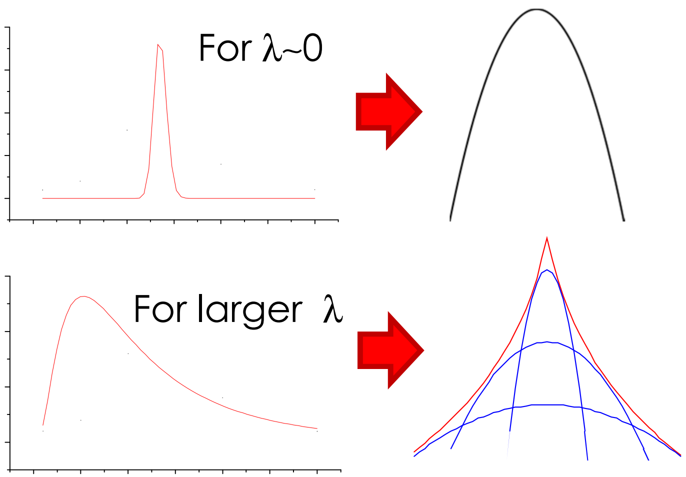
Phenomenology: lognormal variance distribution

Castaing's main hypotheses:

1) From dimensional considerations, the scale dependent distribution of the fluctuations variances σ can be associated to the energy transfer rate ε's.
2) In the framework of a inhmogeneous multiplicative energy cascade, the latter can be supposed to be lognormal (using CLT, Kolmogorov 1962).

$$P(\Delta b) = \int L(\sigma)\ G(\Delta b,\sigma)\ d\sigma$$

$$L(\sigma) = \frac{1}{\sqrt{2\pi}\lambda} \exp\left[-\frac{\ln^2\left(\sigma/\sigma_0\right)}{2\lambda^2}\right]$$

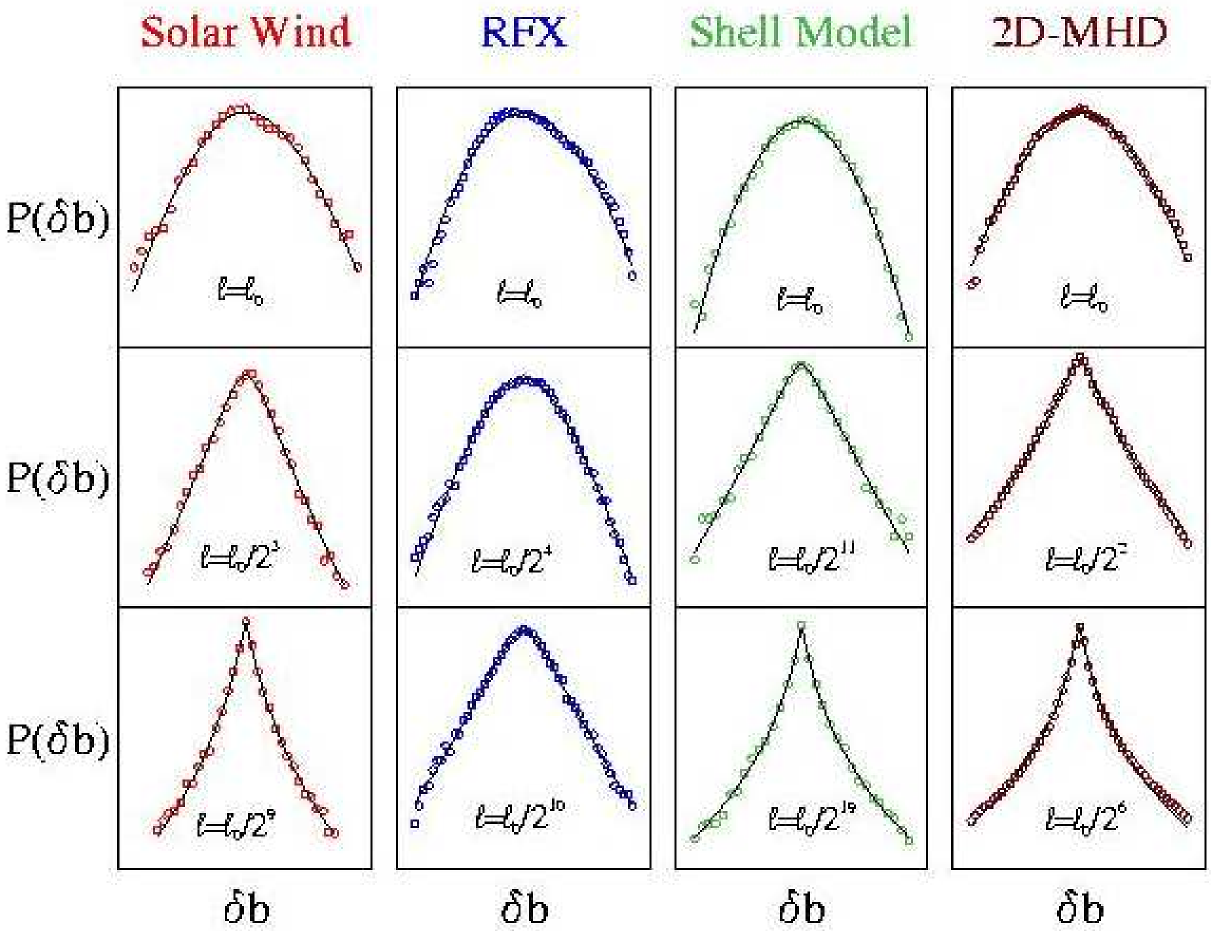


Castiang distribution is a three parameter function, which are related to the moments of 2° (σ_0), 3° (skewness factor) and 4° ($\lambda = f(K)$) order.

Castaing et al., Phys. D 1990; Carbone et al., Riv. Nuovo Cimento 2004

Fit of PDFs with Castaing model

Castaing PDF has been successfully used to describe intermittency in plasmas



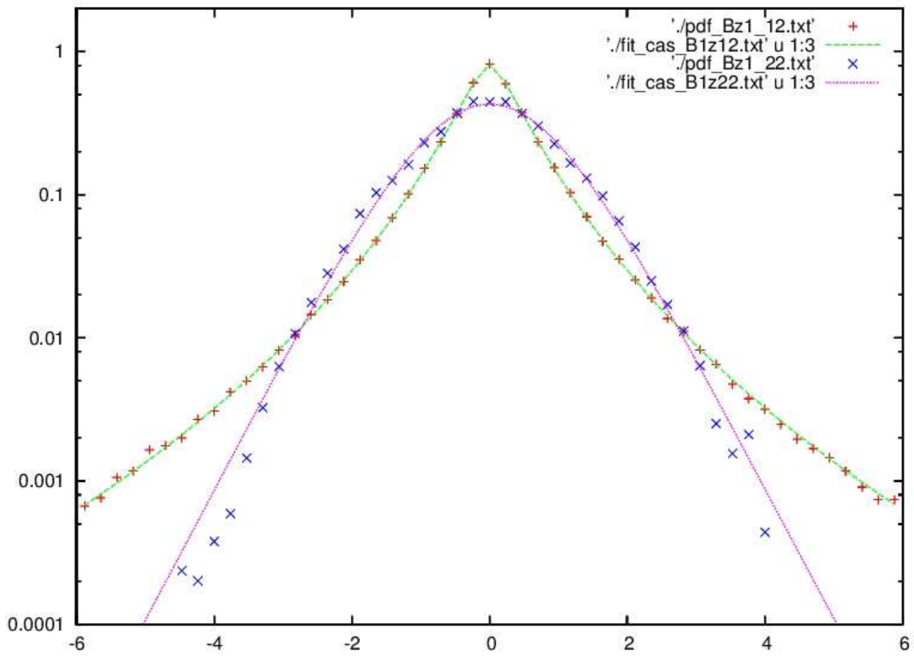
Is it possible to reproduce the intermittent PDFs self-consistently, without the two Castaing assumptions?

In other words: we want to check Castaing's hypotheses.

Parameter scaling

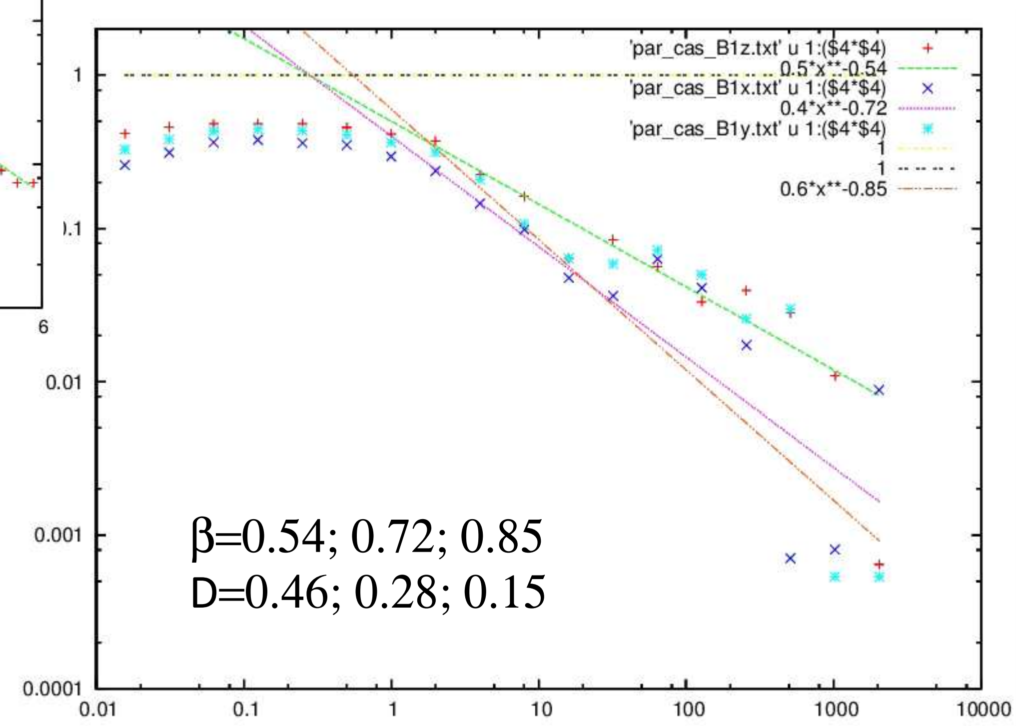
Castaing PDF has been successfully used to describe intermittency in plasmas

[Sorriso-Valvo et al., 1999,2000,2001; Padhye et al. 2001; Pagel & Balogh 2003; Stepanova et al. 2003; Hnat et al. 2003; Carbone et al. 2004; Leubner & Voros 2005°,b…]



An example from Cluster data: highly turbulent magnetosheath, downstream the quasi-parallel shock

[Retinò et al. 2007; Sundqvist et al.2007]

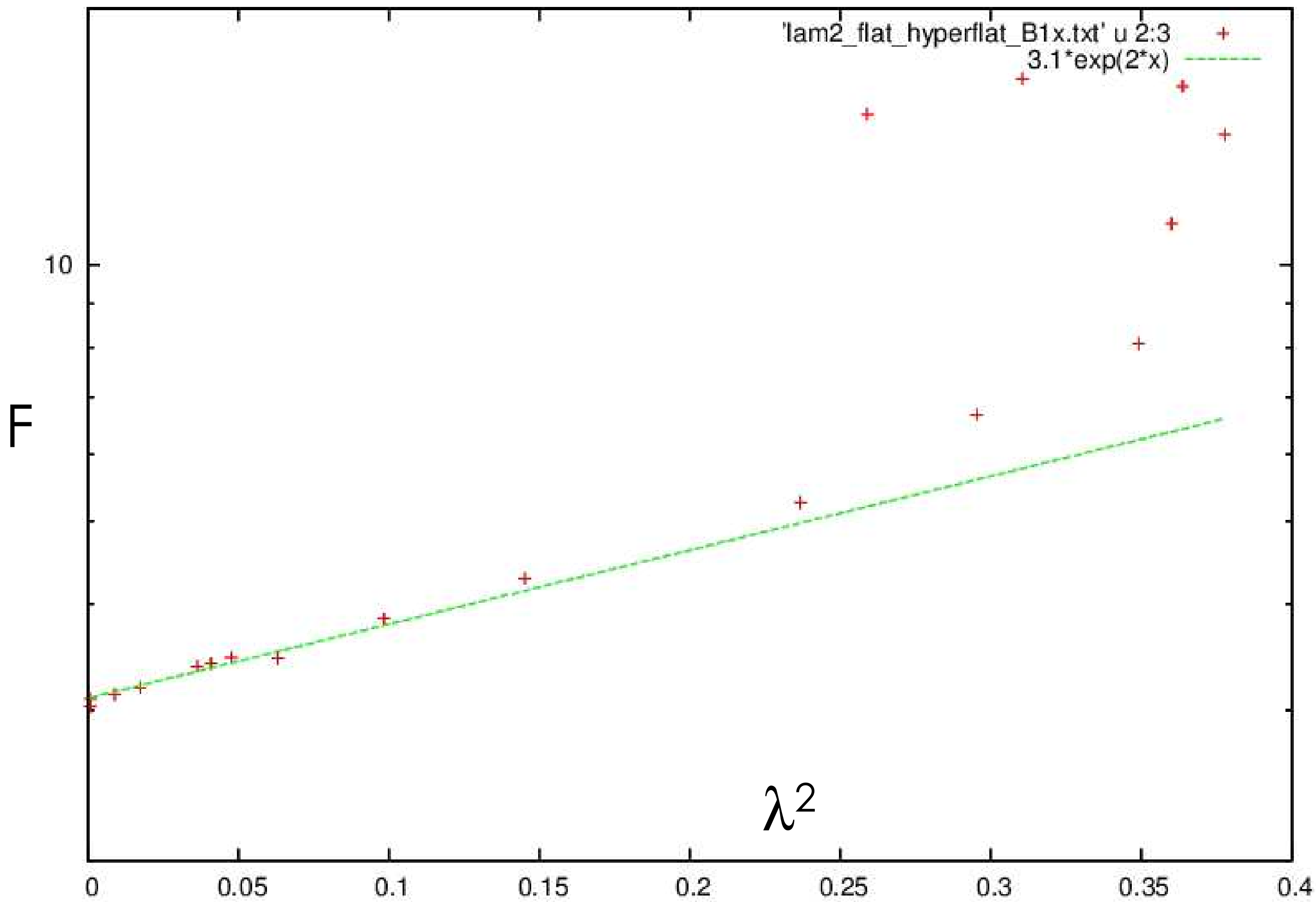


Power-law scaling of the parameter $\lambda^2 \sim \tau^{-\beta}$: the scaling exponent $\beta = 1 - D$ gives the co-dimension of the most intermittent structures

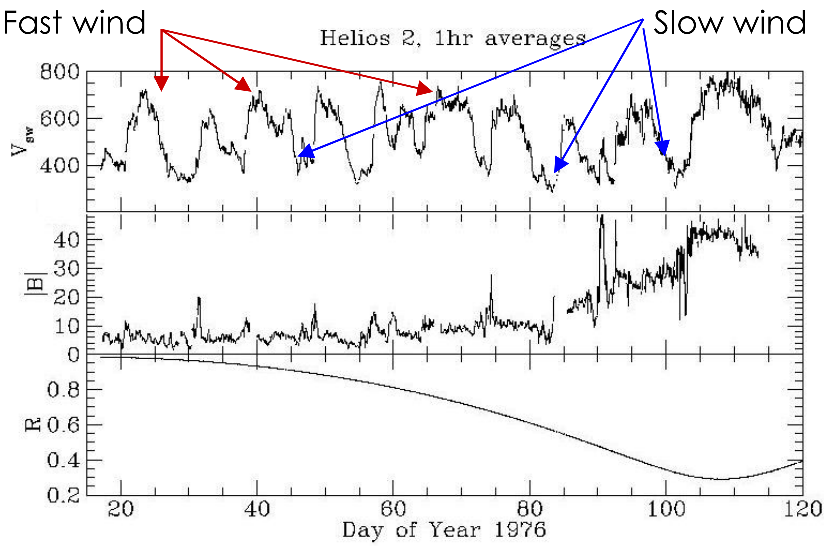
Relation with flatness

It is possible to predict the scaling of the Flatness in terms of λ^2 by simply integrating the moments of the Castaing distribution, $F = \frac{\int x^4 P(x)\,dx}{\left[\int x^2 P(x)\,dx\right]^2}$, so that

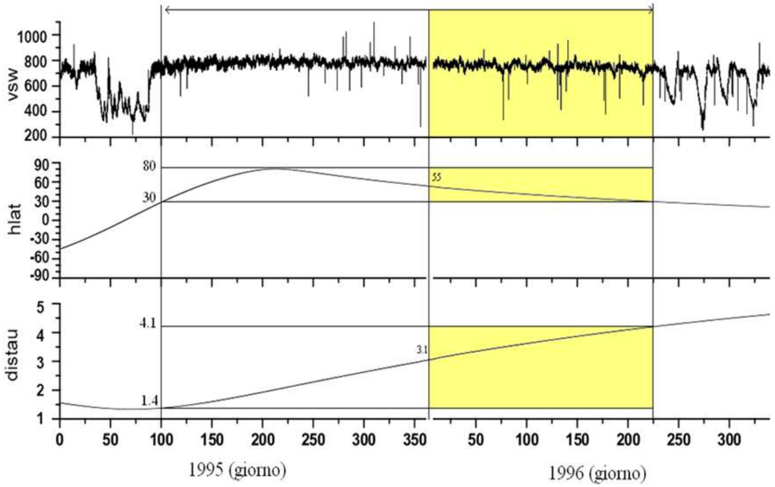
$$F \sim 3\sigma^4_0 \exp(2\sigma_0 \lambda^2)$$



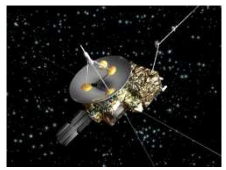
Solar wind data: Helios 2, Ulysses



- **Helios 2**: 10^4 pts
- 81 s resolution velocity, magnetic field and density
- Separated stationary 6 fast and 5 slow streams @1-0.3AU



- **Ulysses**: 4×10^4 pts
- 8 min resolution velocity, magnetic field and density
- Stationary fast wind @3.5AU



Self-consistent distribution
Step 1: the local energy transfer rate

Proxy of the local energy transfer rate from the Yaglom relation for MHD, evaluated from fluctuations at the smallest (resolution) scale:

$$Y^{\pm} = \left\langle \Delta z_{\ell}^{\mp} \left| \Delta z_{i}^{\pm} \right|^{2} \right\rangle = -\frac{4}{3} \varepsilon^{\pm} \ell$$

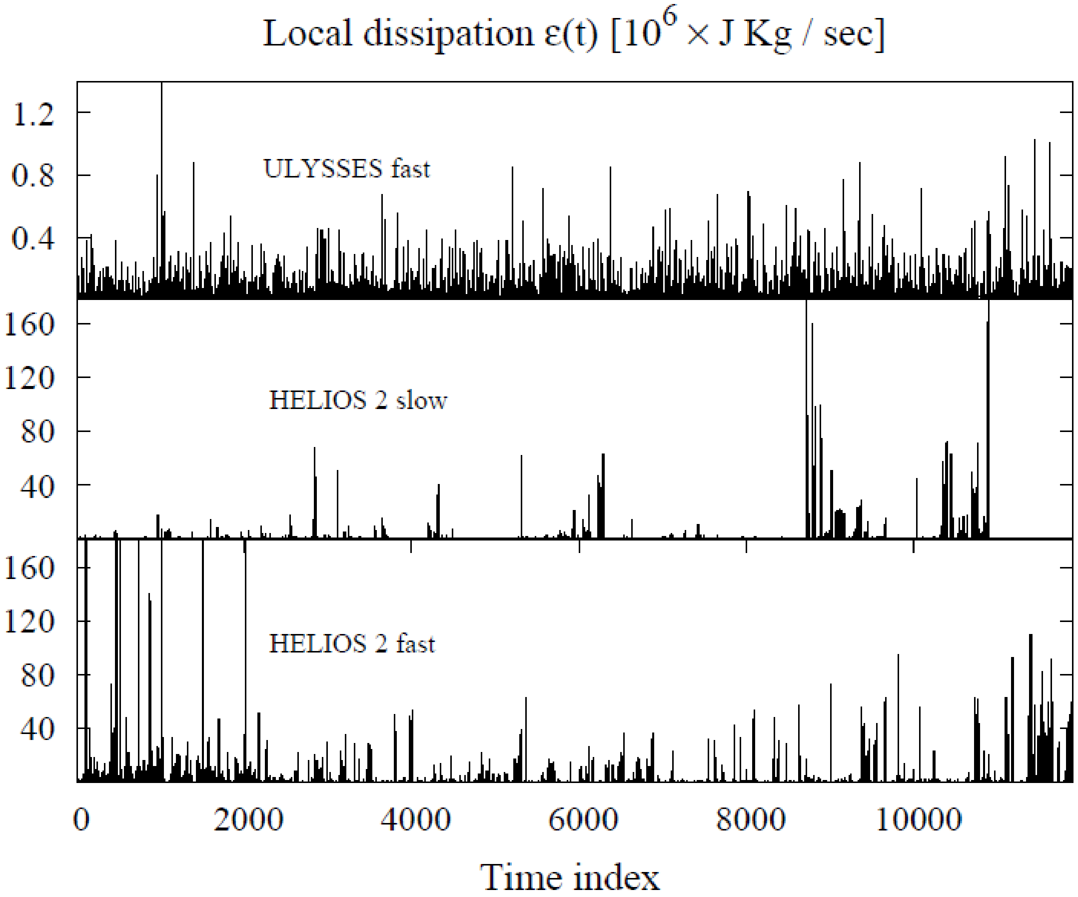
⬇

$$\varepsilon^{\pm}(x) = \frac{(\delta z^{\pm}(x,\ell))^{2} \delta z_{r}^{\mp}(x,\ell)}{\tau \langle v \rangle}$$

⬇

$$\varepsilon_{tot} = \frac{\varepsilon^{+} + \varepsilon^{-}}{2}$$

Politano & Pouquet, GRL 1995



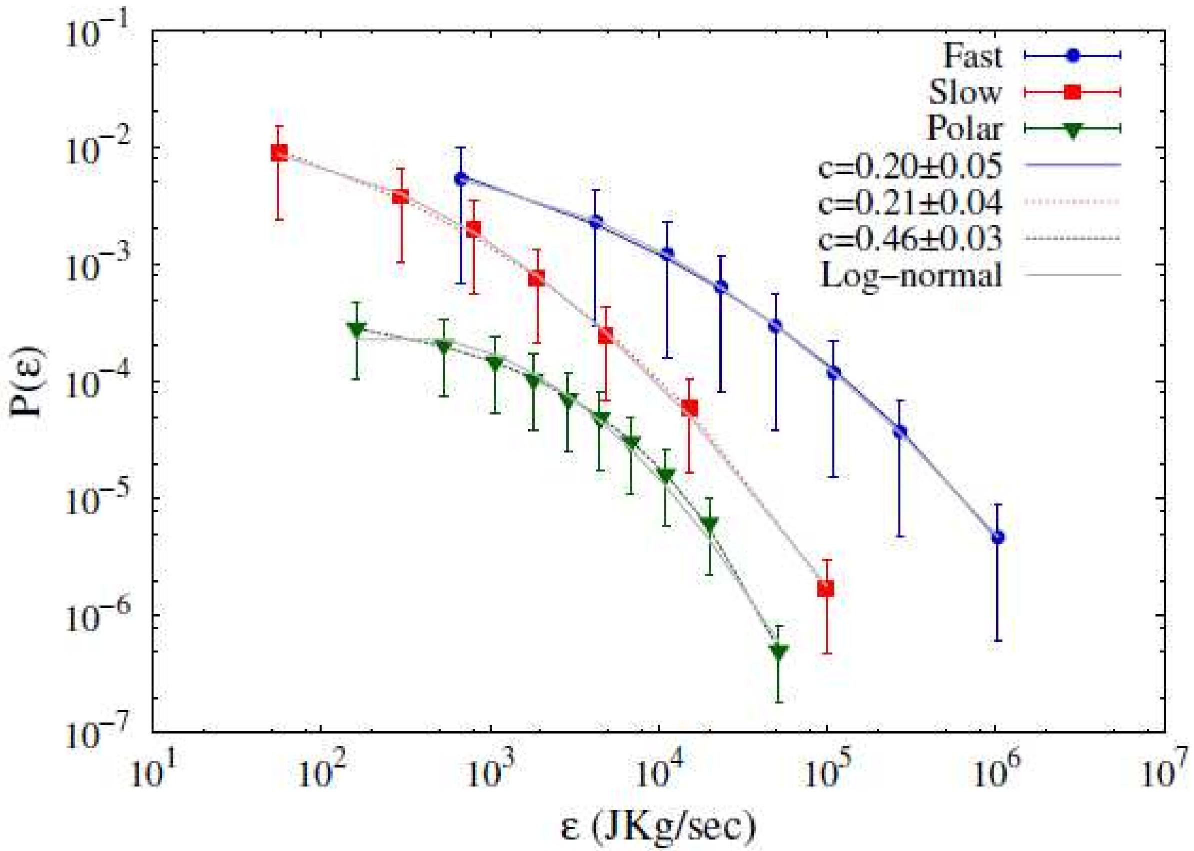
Inhomogeneous distribution of the energy transfer rate: intermittency.

Step 2: the distribution of ε

P(ε) is well reproduced by a Log-Normal distribution, but also (and better) by a stretched exponential: $P(\varepsilon) \propto \exp(-a\varepsilon^{c})$

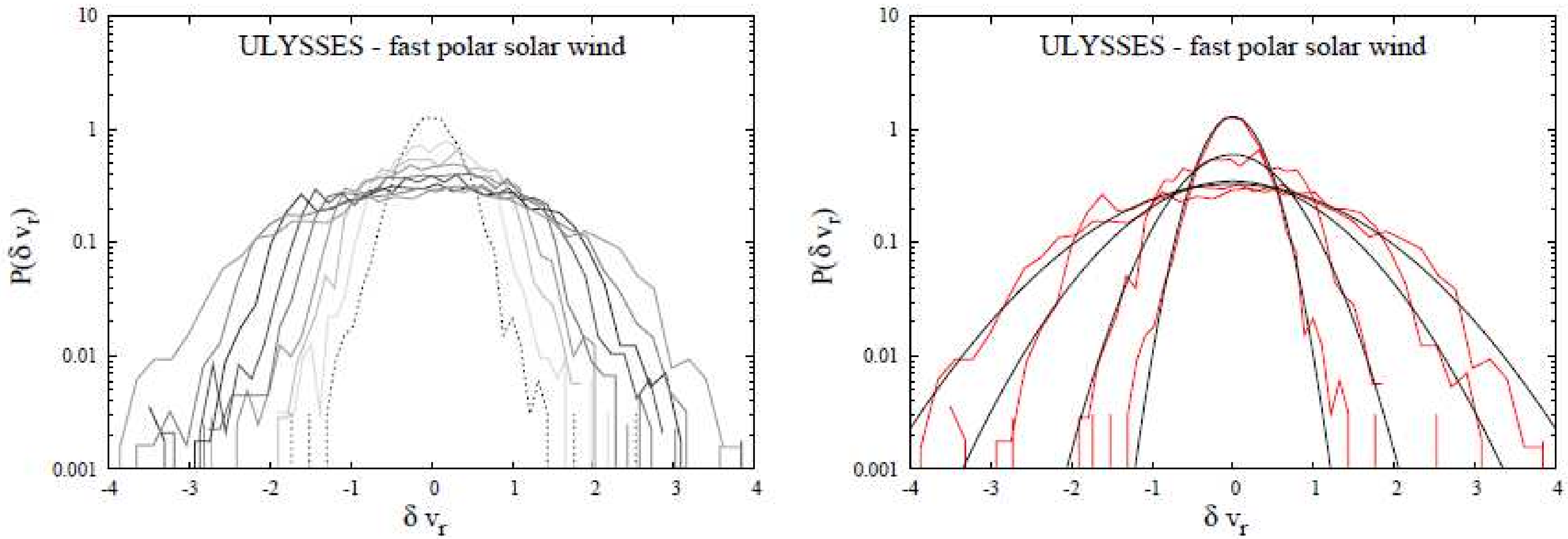
Compatible with the prediction of the Extreme Deviations Theory for a multifractal fragmentation process (multiplicative cascade).

(Frisch & Sornette, J. Phys. I France, 1997)



Step 3: conditioned PDFs

PDFs of fields fluctuations conditioned to given value of ε are (roughly) Gaussian at all scales. This supports Castaing's model.

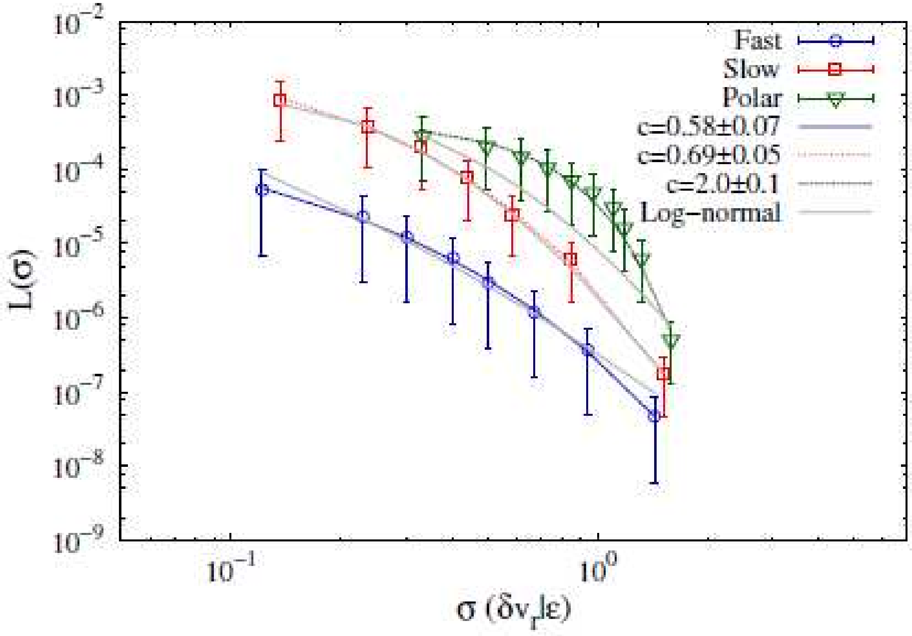


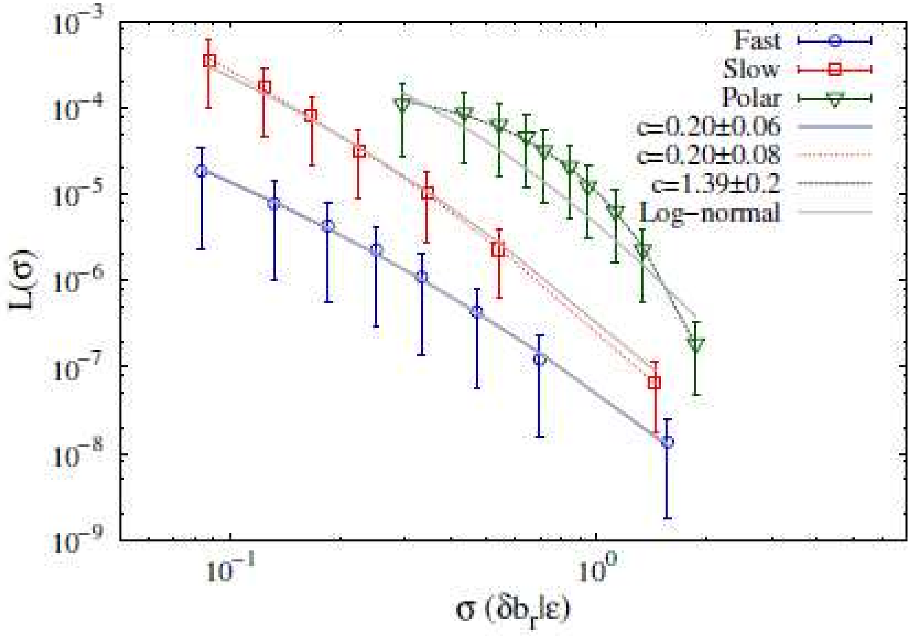
Conditioned PDFs can be fitted with Gaussian curves and their conditioned standard deviations σ can be obtained.

$$P(\Delta v) = \int L(\sigma)\ \ G(\Delta v, \sigma)\ d\sigma$$

Step 4: self-consistent L(σ)

The distribution of variances L(σ) have been estimated self-consistently from the dataset. The distributions are well represented by stretched exponential. Log-normal fits are not satisfactory.





$$P(\Delta v) = \int L(\sigma)\ \ G(\Delta v,\sigma)\ d\sigma$$

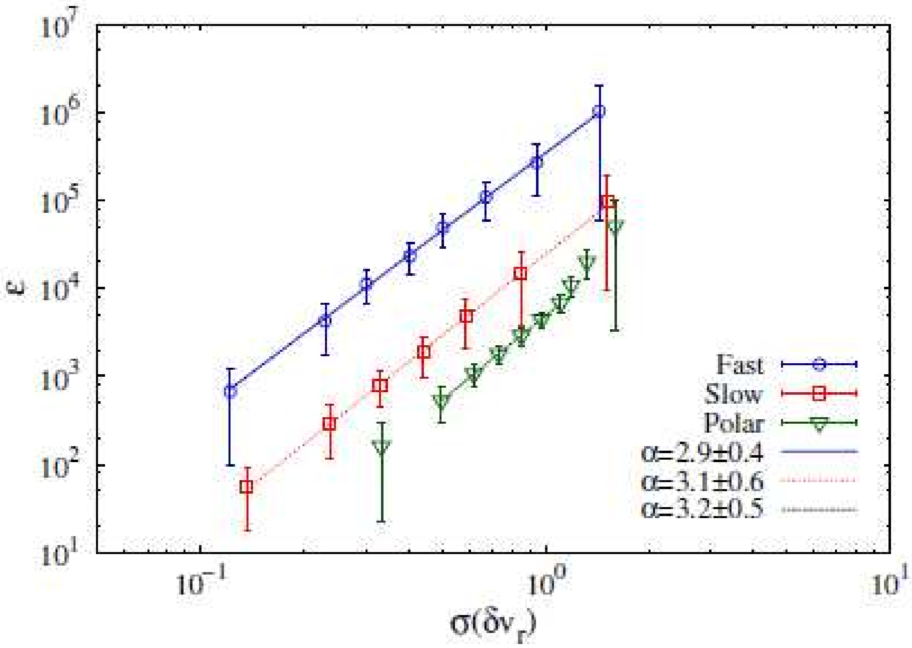
Check for dimensional consistency: P(ε) vs L(σ)

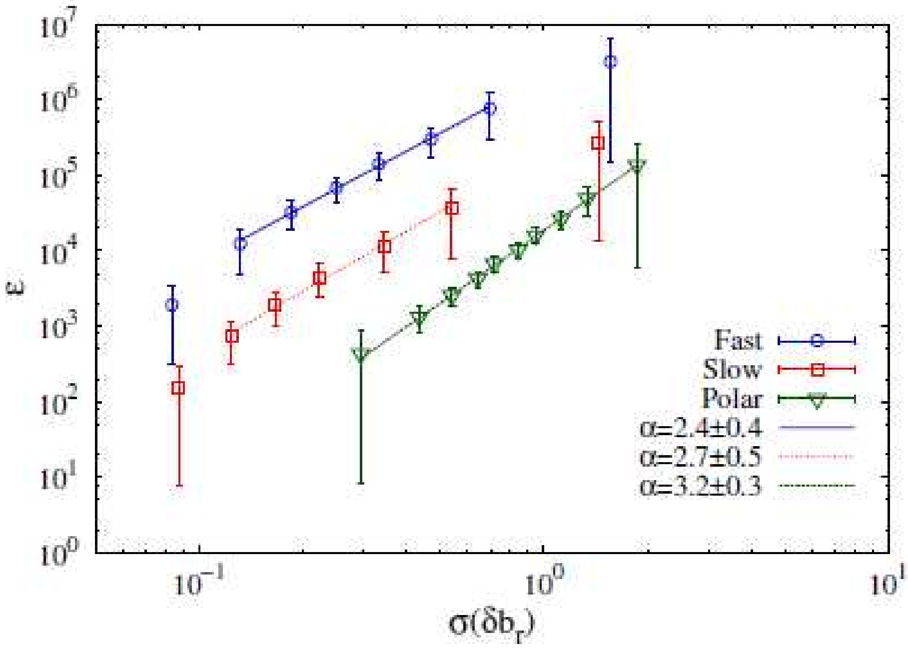
It is possible to check that a very clear power-law relation exist, $\varepsilon \sim \sigma^{\alpha}$

We find $\alpha \approx 3$, consistent with the cubic dependence expected from the definition of the local energy transfer rate used here.

This was not observed in laboratory fluids, where $\alpha = 5$ was instead found.

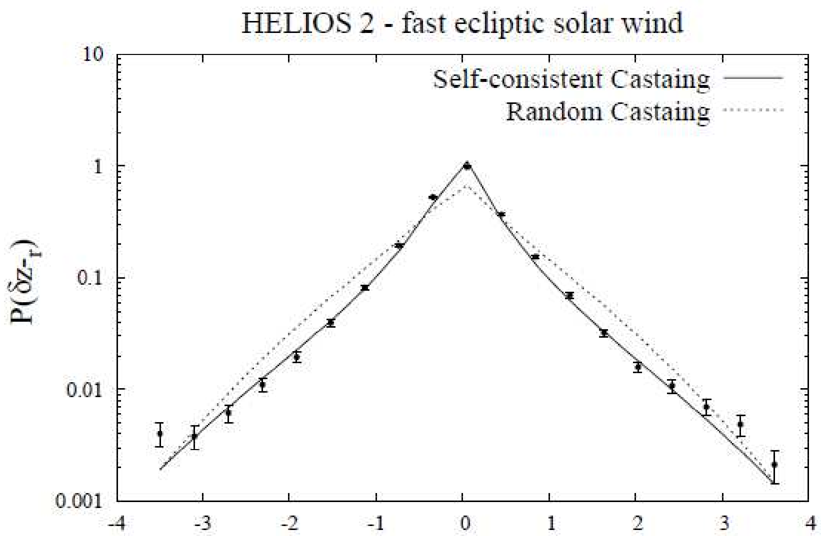
Castaing et al. 1990; Naert et al. 1998

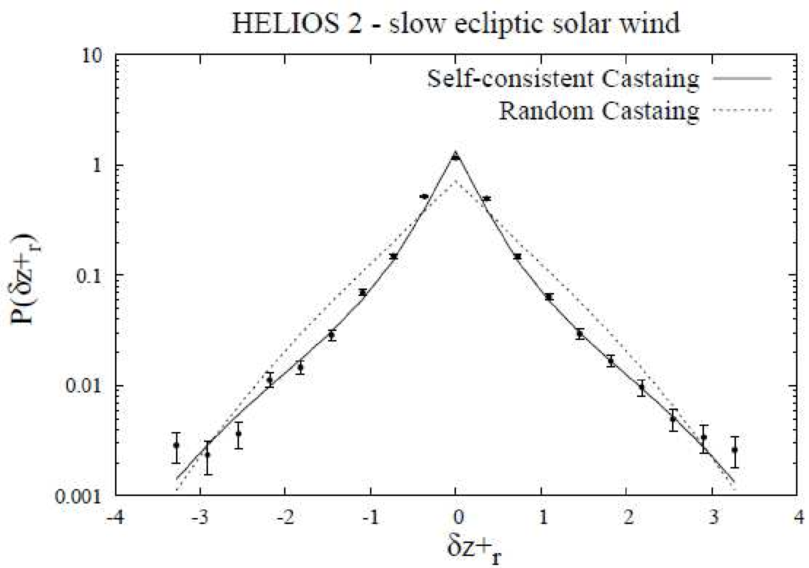


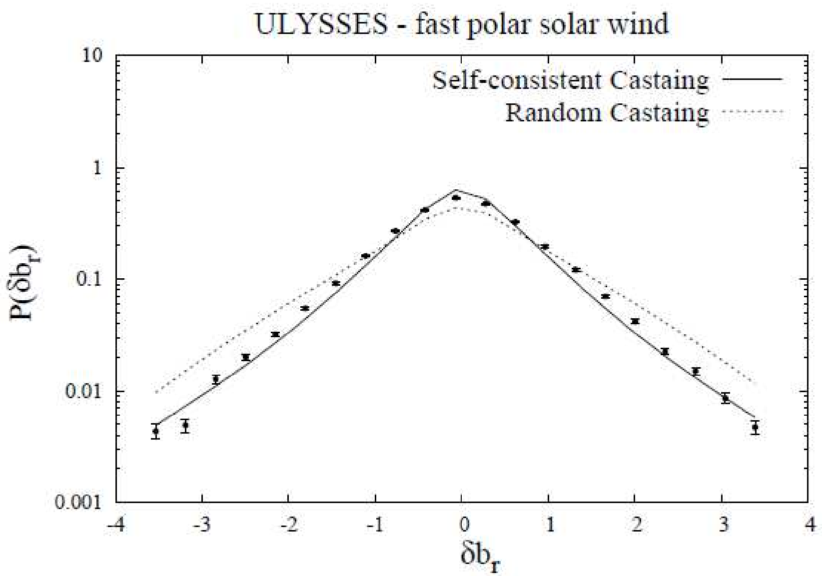


Step 5: self-consistent Castaing PDF

The parameter-free Castaing convolution can finally be computed using L(σ), and fitted (amplitude) to the experimental PDFs of field fluctuations.







$$P(\Delta\psi_\tau) = A_{norm}\sum_{i=1}^{N_{bin}} L(\sigma)P_0(\Delta\psi_\tau, \sigma, a_s)$$

The shape of the PDF is well reproduced.

A test to check that PDF shape cannot be reproduced by a random superposition of gaussians. 10^4 random realizations of L(σ) in the same range of the data fail to fit the PDFs (χ^2 is one order of magnitude larger)

Conclusions

- A proxy of the local energy dissipation rate has been evaluated for three samples of solar wind, and its statistical properties described through log-normal and stretched exponential functions.

- PDFs of the solar wind fluctuations conditioned to the energy dissipation rate have been estimated: they are Gaussian at all scales, so that intermittency is removed.

- Variances of those Gaussian PDFs are distributed as stretched exponentials (not log-normal), and a cubic relationship exist between ε and σ.

- The empirical distributions of σ have been used to build up the PDFs of field fluctuations, consistently with a multifractal cascade in solar wind turbulence.

Sorriso-Valvo et al., APJ, in press