Rapidly rotating turbulence and its role in planetary dynamos

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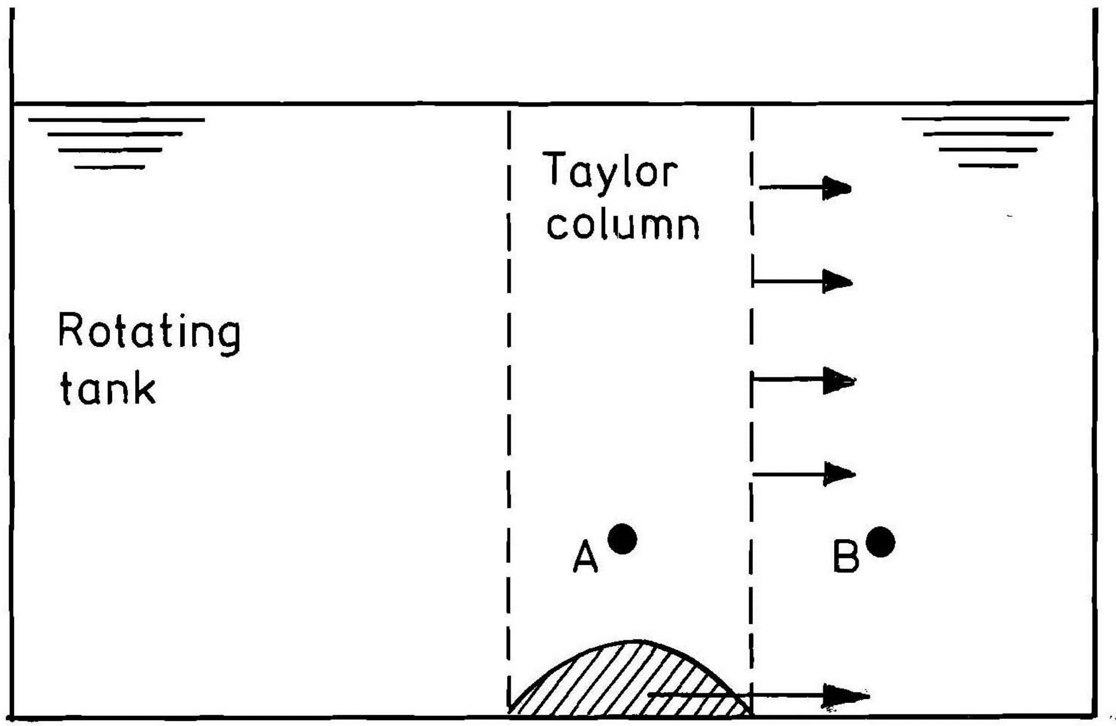
An old idea …(G I Taylor, 1921)

Rotation-dominated flow: pressure gradient balances Coriolis force

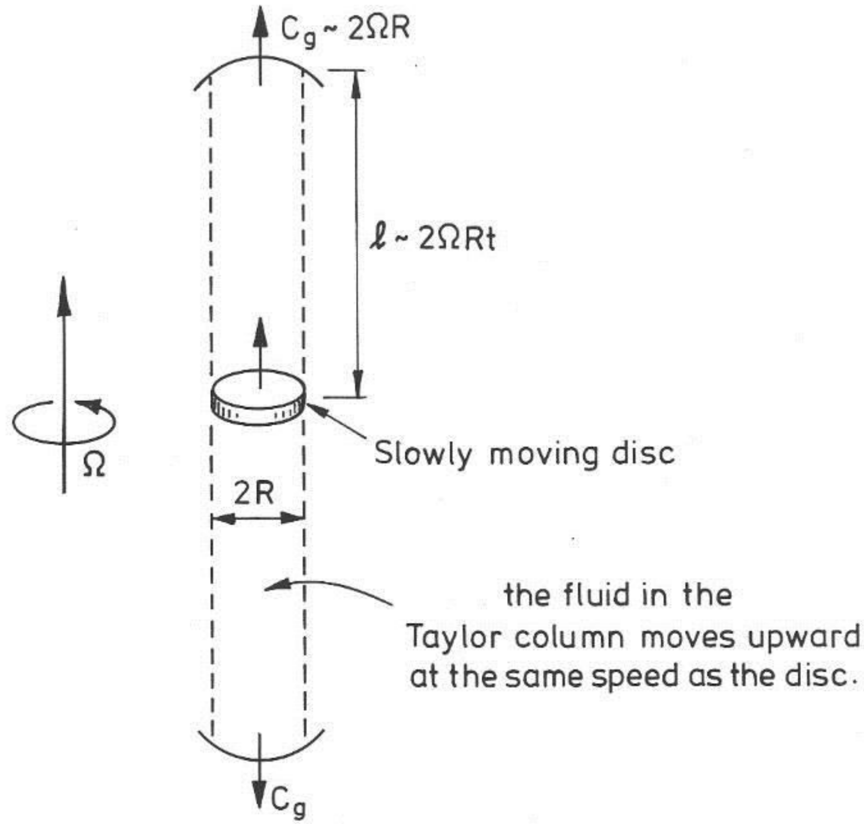
$\nabla p = 2$[redacted]$\times$[redacted] Geostrophic balance

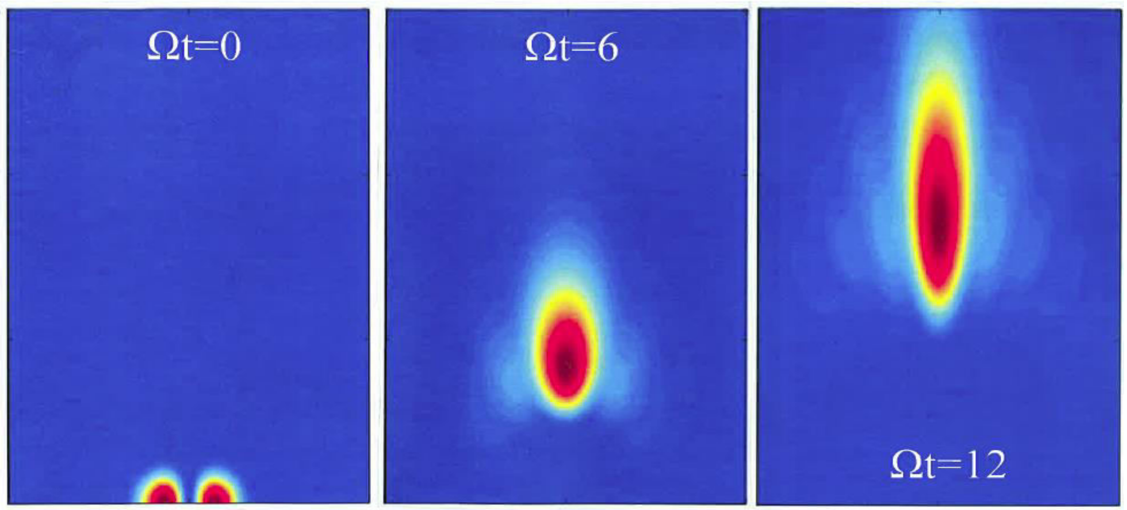
requires $\frac{\partial}{\partial z}$[redacted]$= 0$ 2D flow



Zero (low) frequency inertial wave-packets create columnar structures

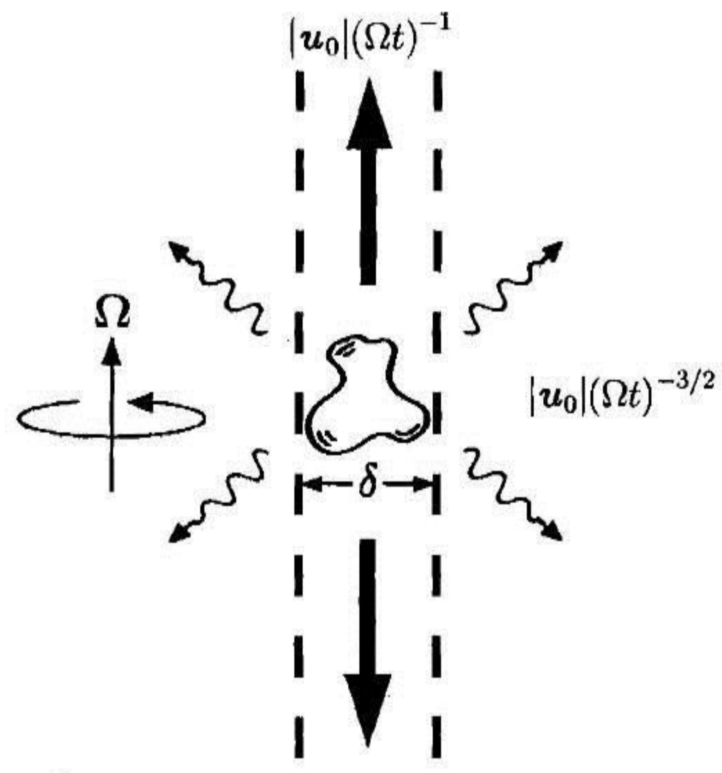
e.g. transient Taylor column





Spontaneous formation of columnar eddy from a localised disturbance.

Caused by spontaneous self-focussing of radiated energy onto rotation axis



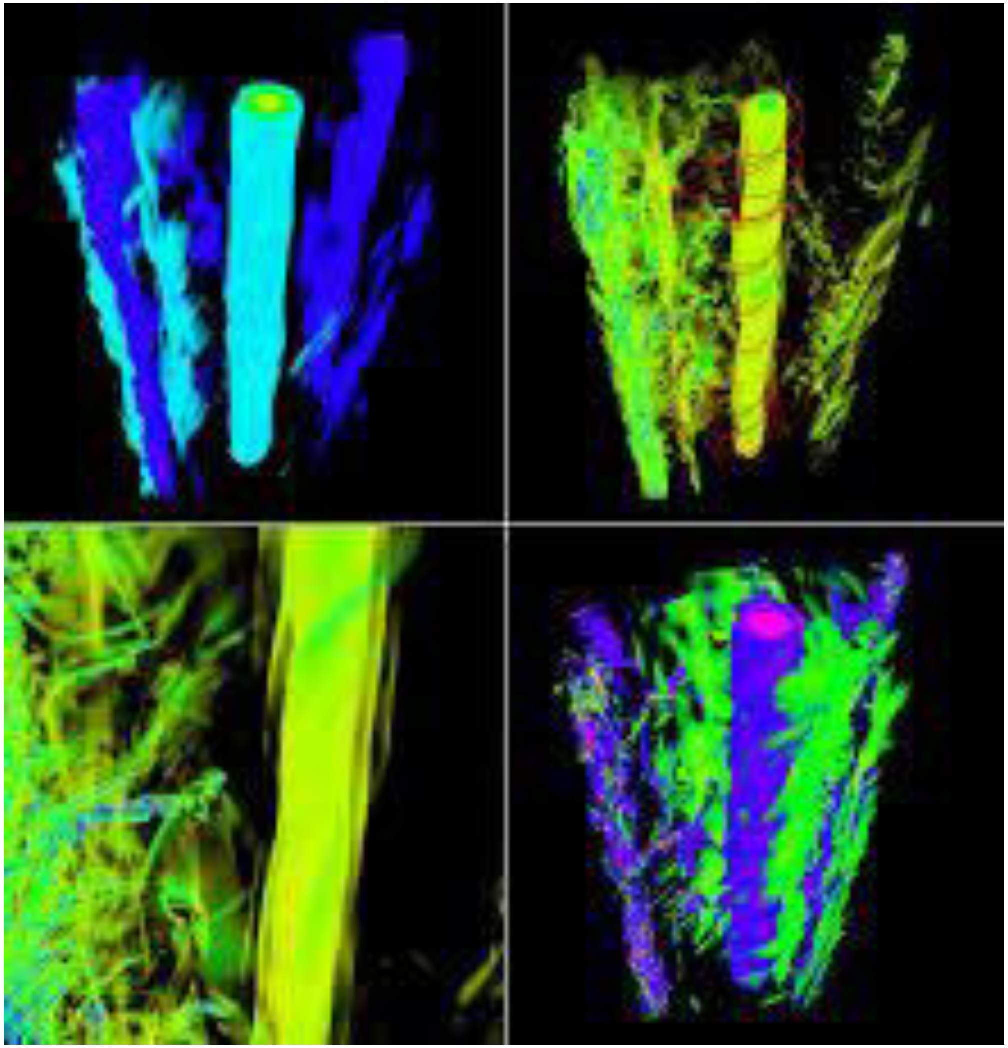
Reason: angular momentum conservation.

Eddy grows and propagates at the group velocity of zero-frequency inertial wave packets

Davidson et. al (JFM, 2006)

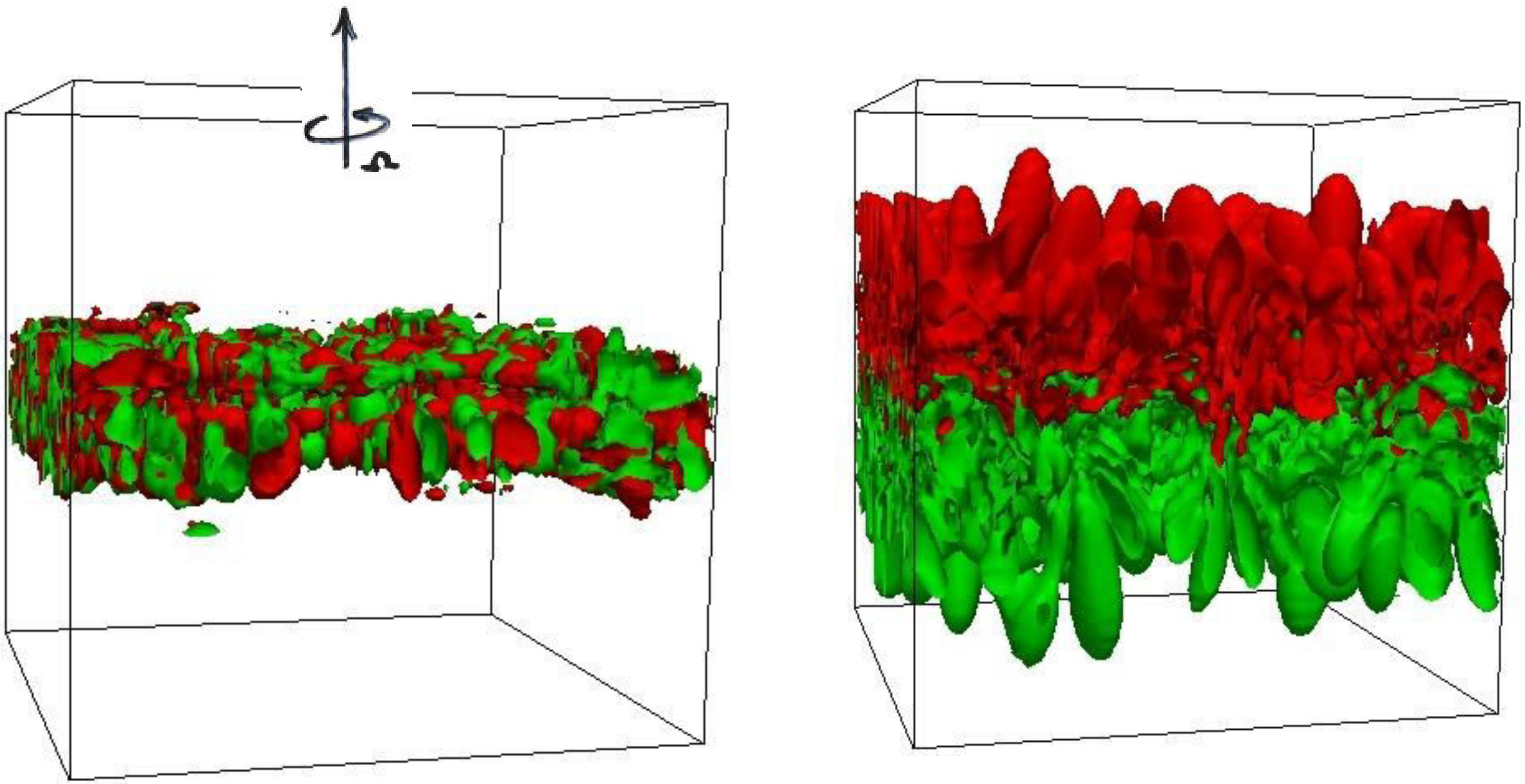
Can this theory explain the ubiquitous occurrence of columnar vortices in rotating turbulence?

The turbulence community do not think so ! … But…



DNS of rotating turbulence from NCAR

Initial condition consisting of a slab of turbulence at Ro ~ 0.1



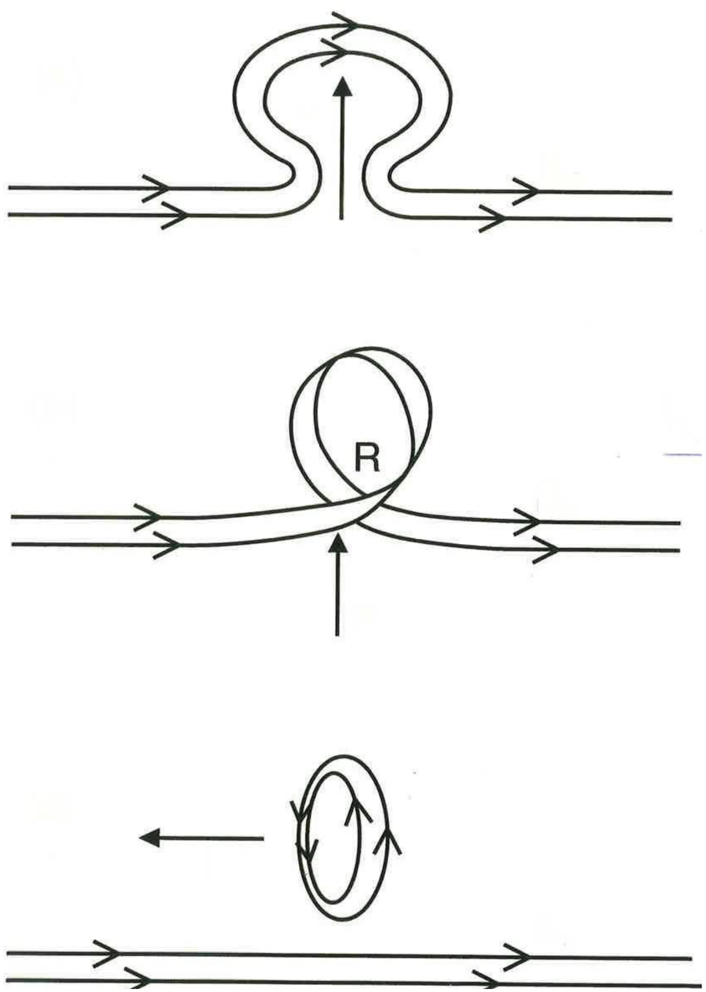
Spontaneous emergence of columnar vortices in form of wave packets.

Iso-surfaces of helicity. Red is negative, green positive.

Wave packets spatially segregate helicity.

(h > 0 means right-handed spirals, h < 0 means left-handed)

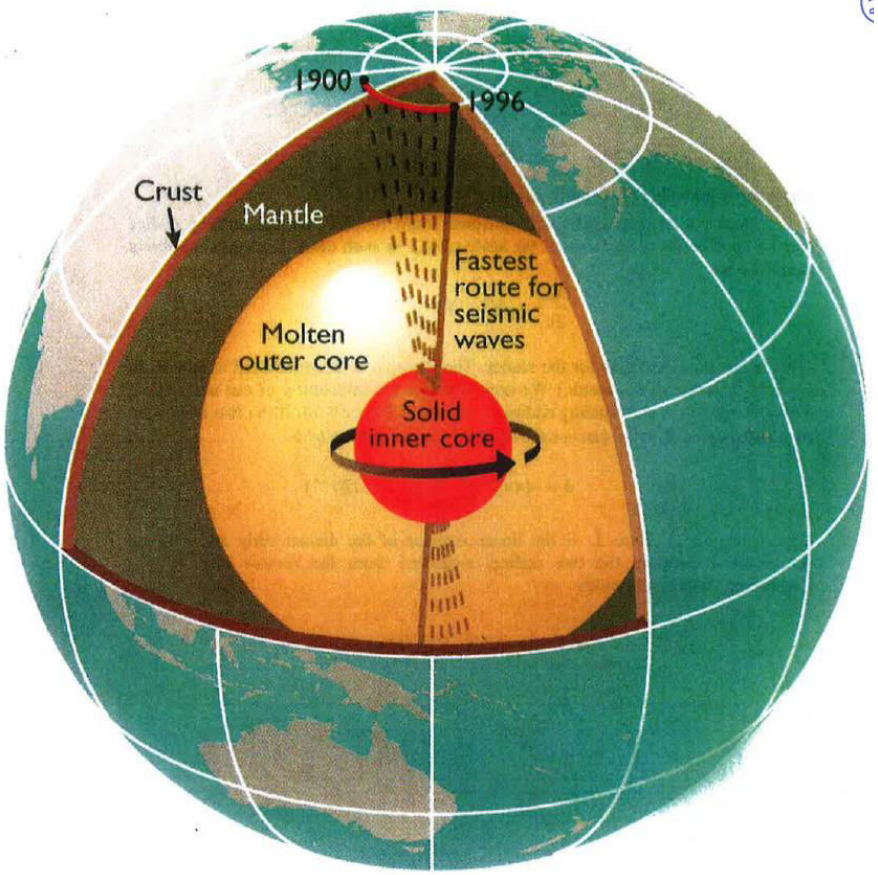
Helicity crucial for field generation in planetary dynamos

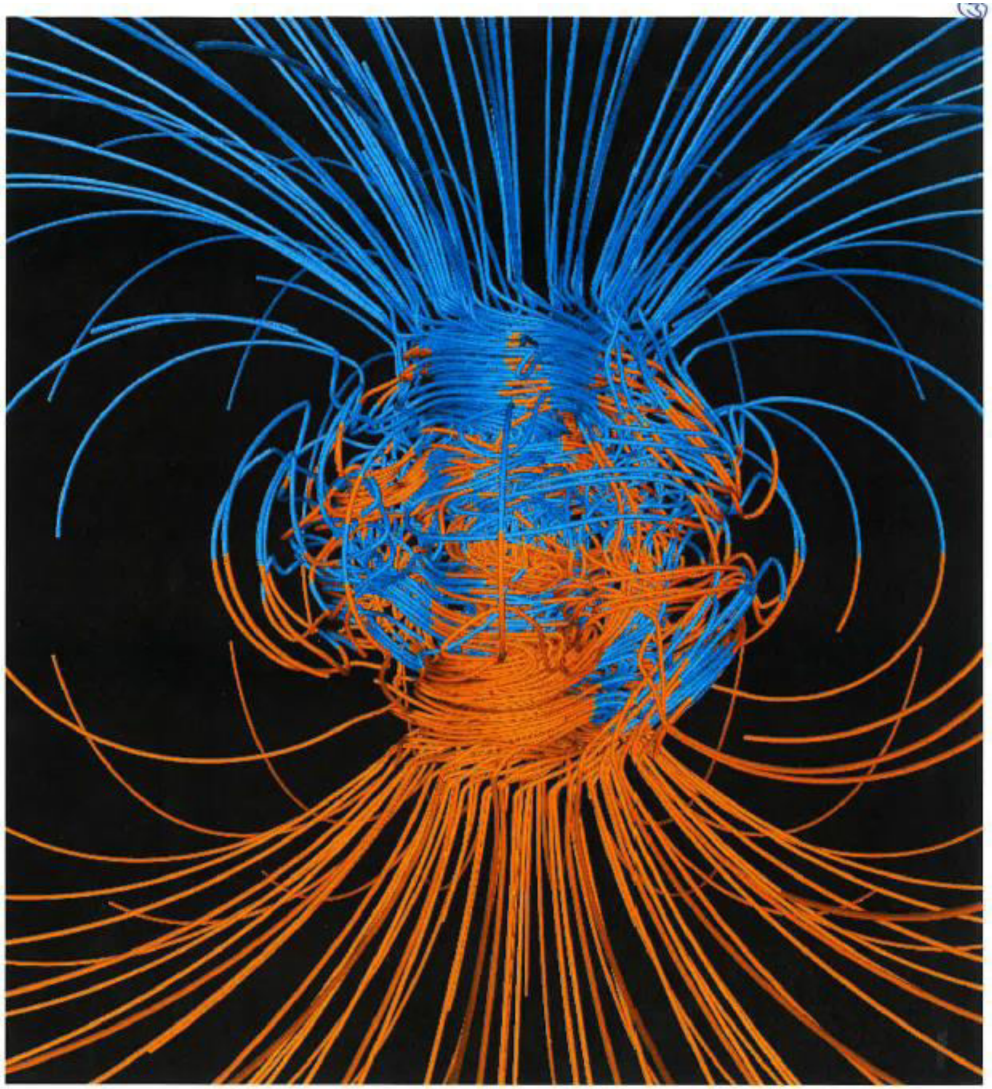


For dynamo action in a planet need a **robust** mechanism of:

- Helicity generation
- Helicity segregation (north, south)
- Dynamo simulations show negative helicity in the north, positive in the south
- Source of helicity hotly disputed

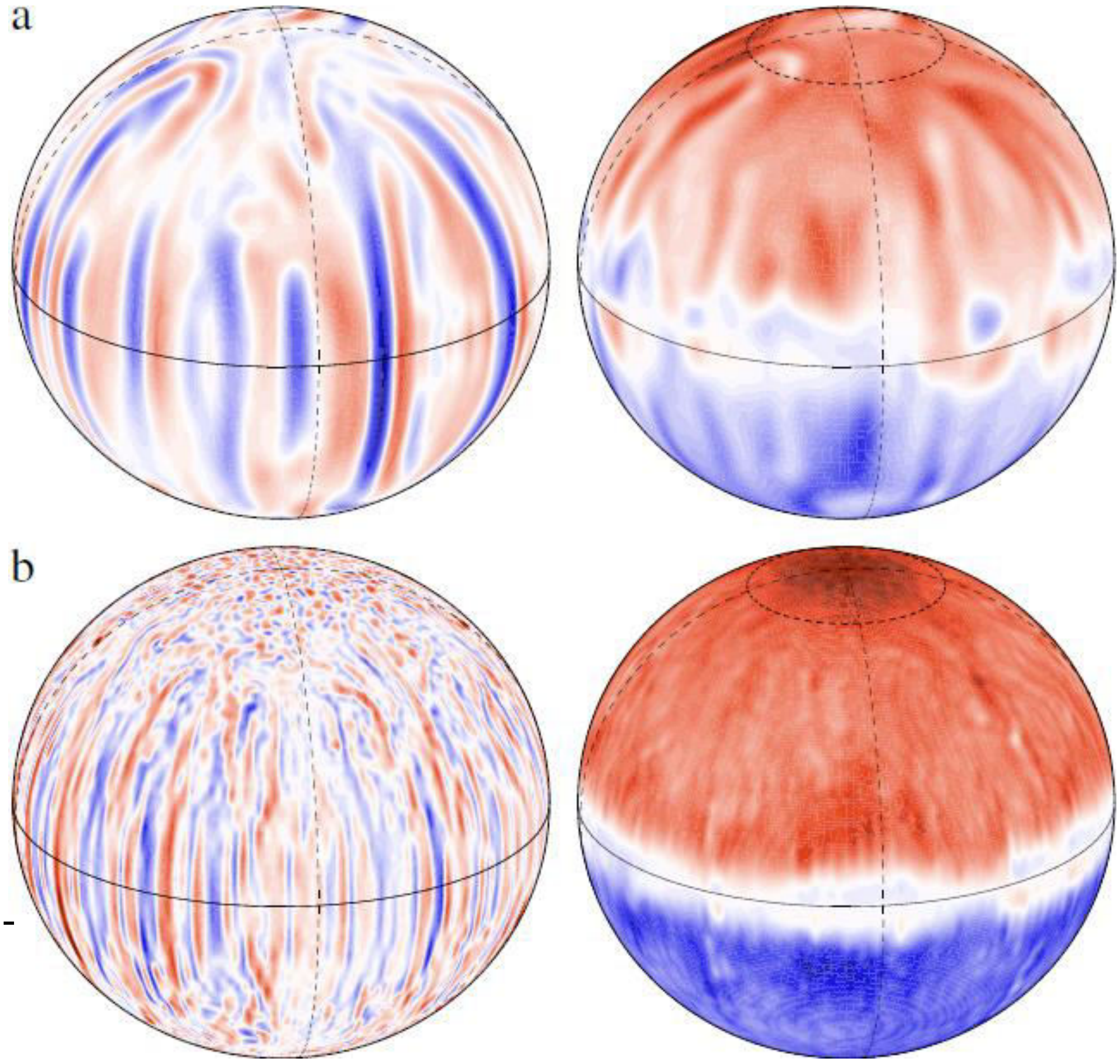
Can we construct a planetary dynamo from zero-frequency inertial wave packets?





Typical results of dynamo simulations (Sreenivasan, 2010)

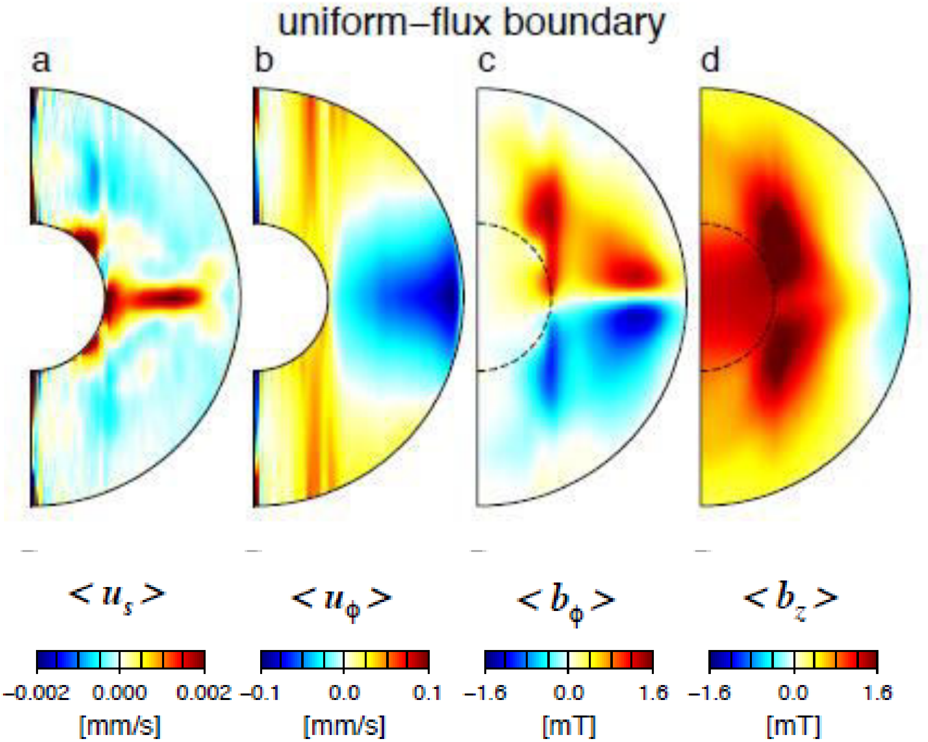
Weakly forced
10 times critical



Moderately forced
50 times critical

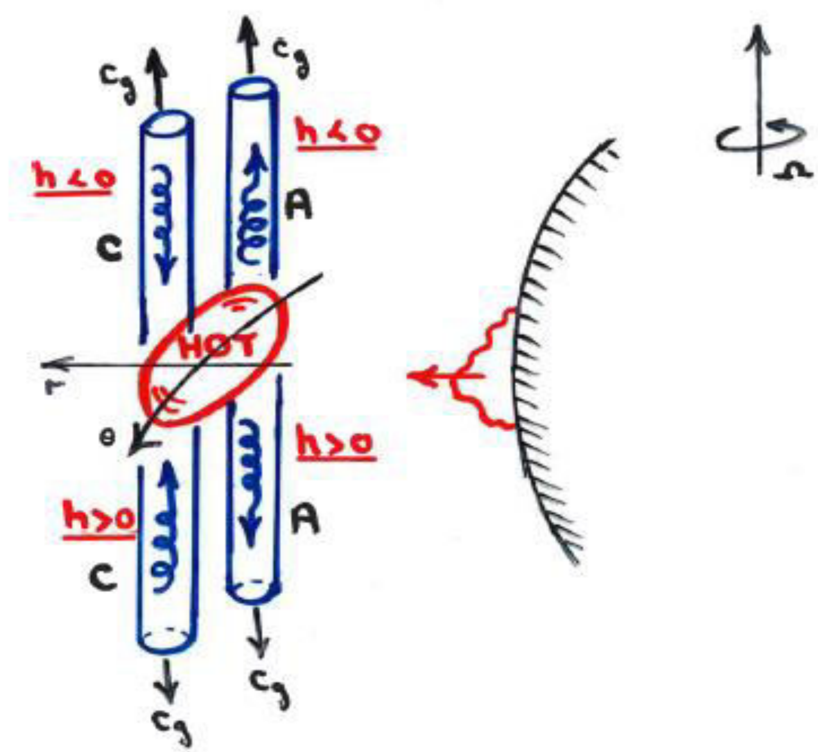
Note alternating cyclones-anticyclones

Note the Earth is a million times critical !



Results from numerical simulations with uniform-flux boundary
(Sakuruba & Roberts, 2009)

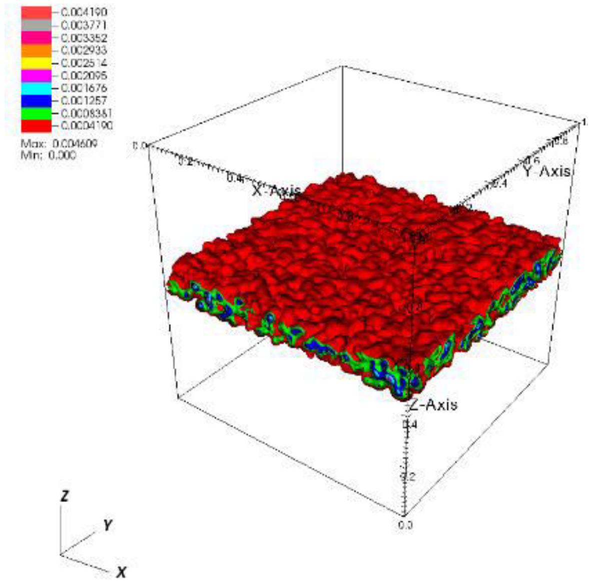
Note the strong equatorial jet



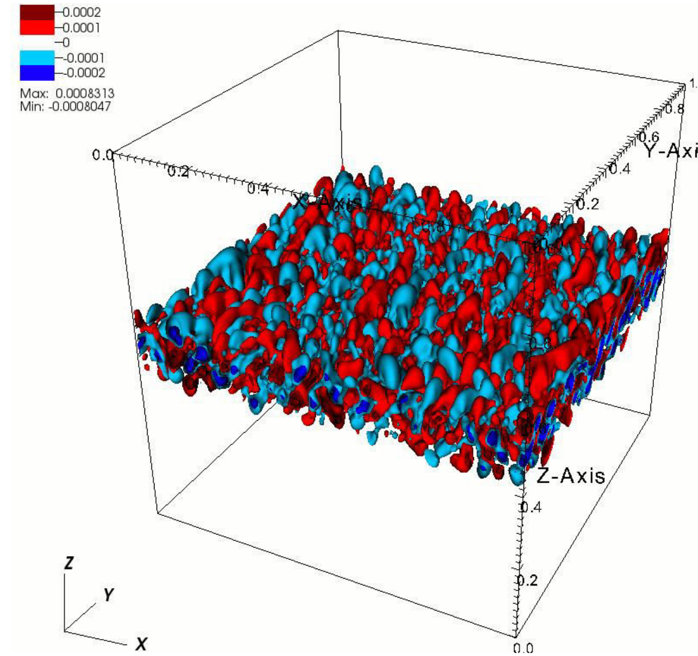
Dispersion pattern of low-frequency inertial wave packets from a buoyant blob

Note pairing of cyclone and anti-cyclone above and below

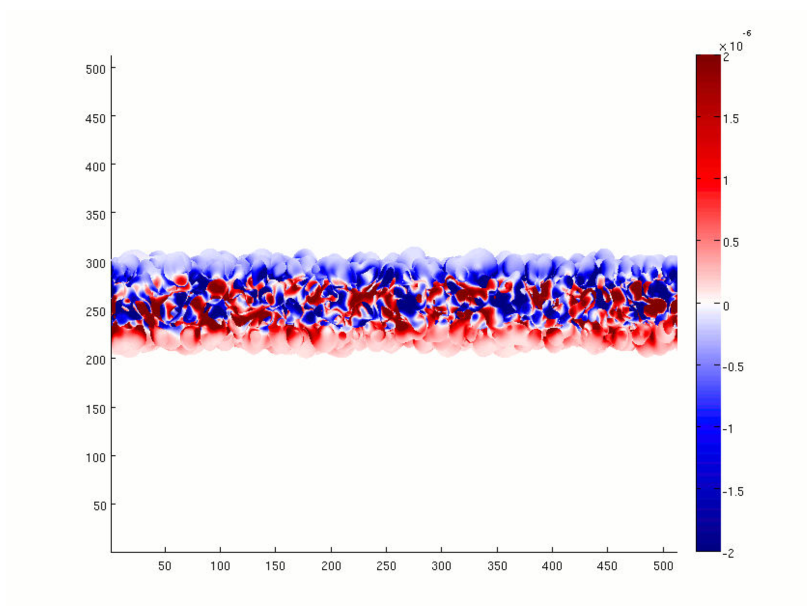
(Davidson, Geo. J. Int. 2014)



Buoyancy field at Ωt=0



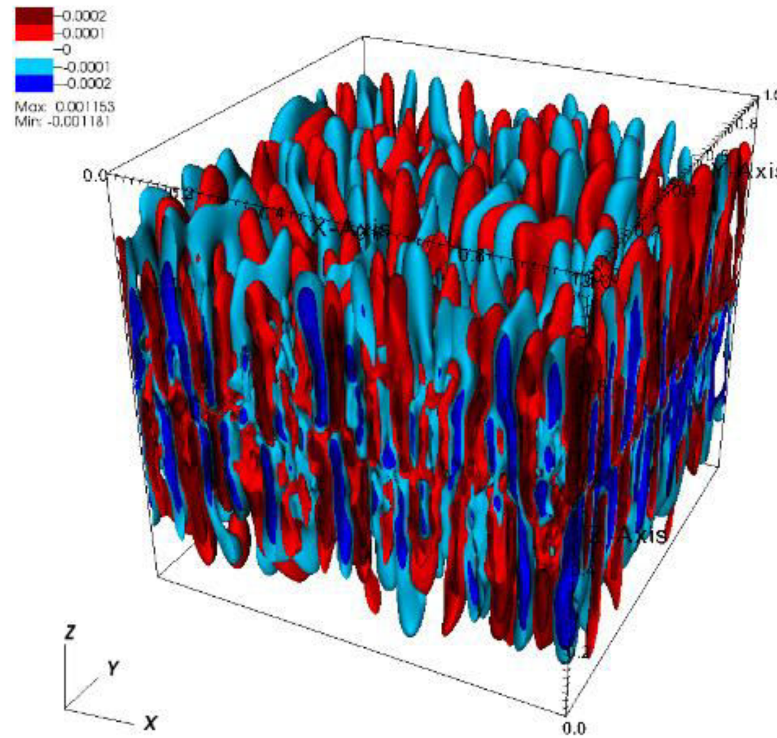
u_z iso-surfaces



Normalized u_z^2 coloured by helicity

Iso-surfaces of u_z (positive is red, negative is blue) for $\Omega t=12$

Davidson & Ranjan (2014) *Submitted*



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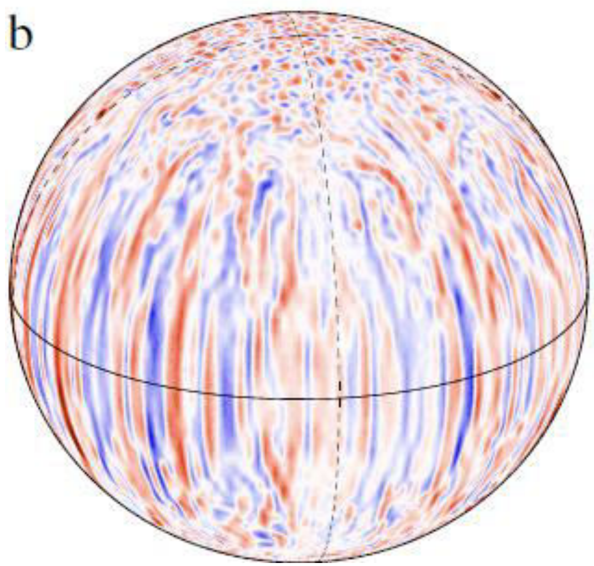
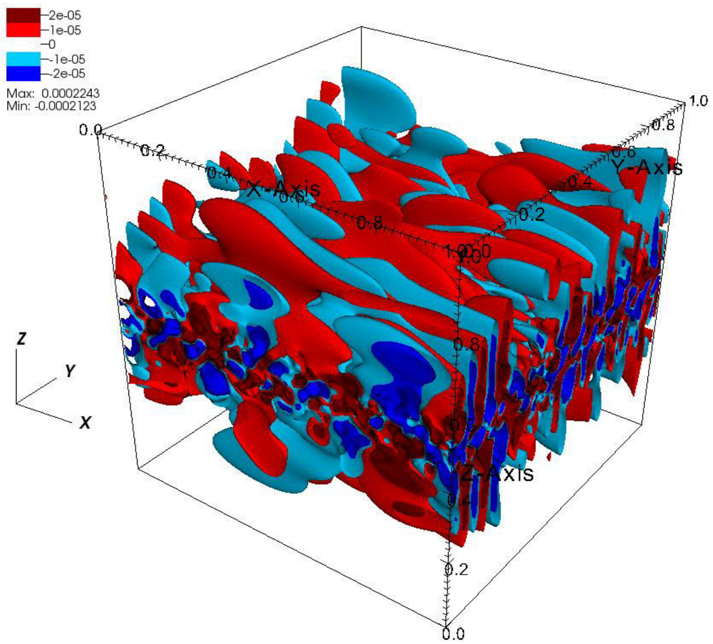
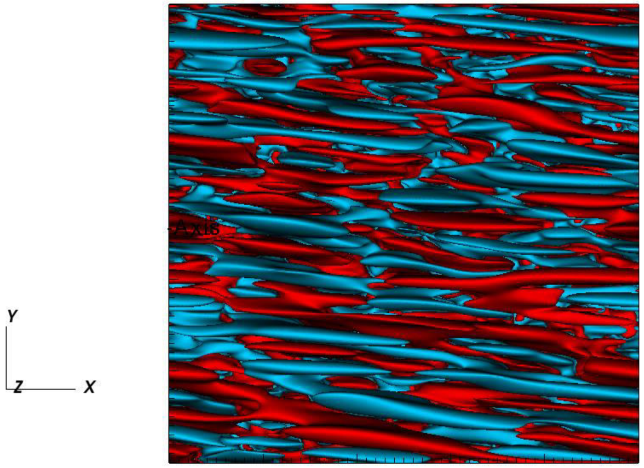


Compare!

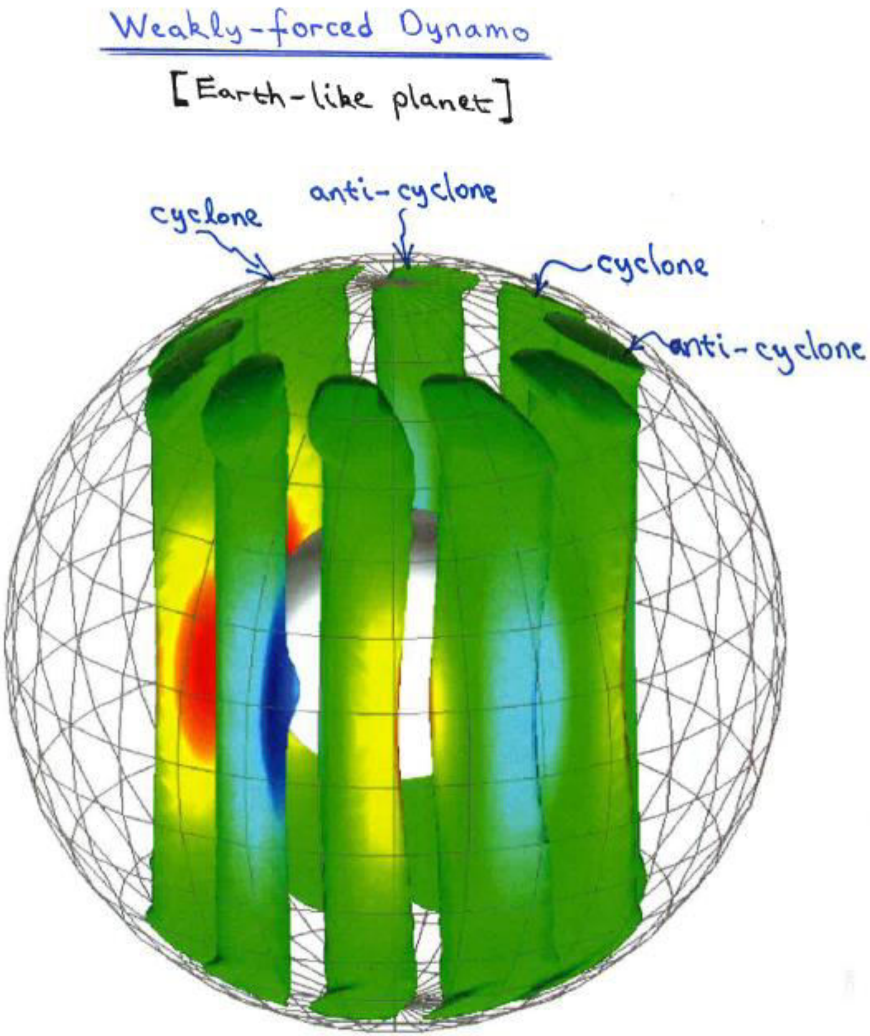
If we include the dynamic influence of the mean magnetic field, the wave packets become anisotropic.

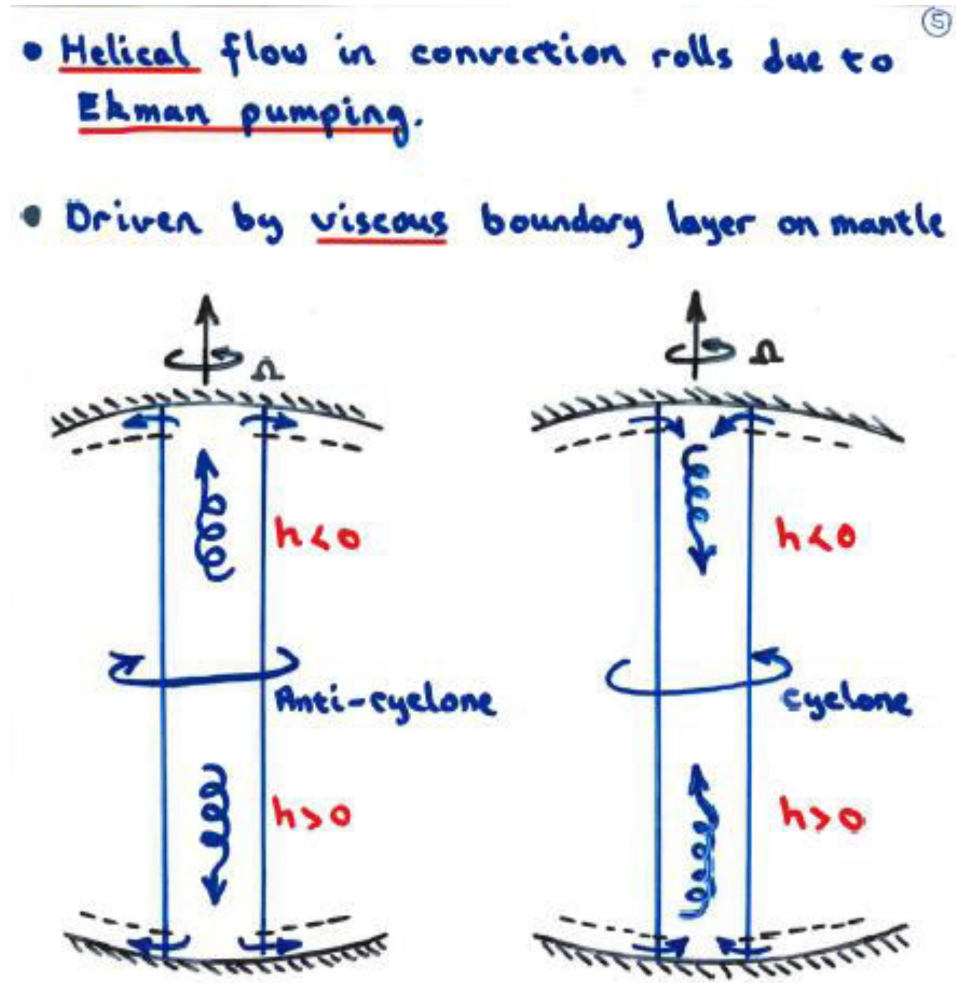




Mean magnetic field along *x*-axis

Compare with the popular cartoon for geo-dynamo based on weakly-forced, highly-viscous simulations





What is the source of helicity in real planets ?

3 problems for the viscous Ekman pumping mechanism

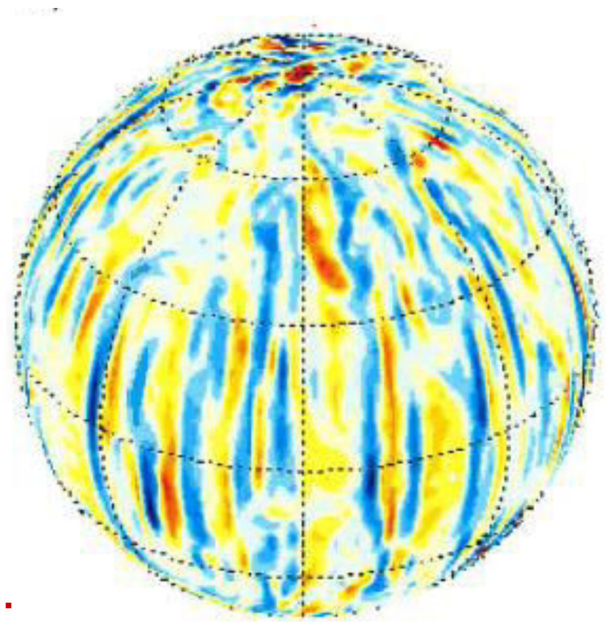
- Viscous stress is tiny, $Ek \sim 10^{-15}$

- Mercury, Earth, Jupiter, Saturn have similar **B**-fields, both in structure (dipolar, aligned with Ω) and magnitude

Planet	Mercury	Earth	Jupiter	Saturn
$\frac{\bar{B}_z/\sqrt{\rho\mu}}{\Omega R_C}$	5.5 x 10^{-6}	13 x 10^{-6}	5.2 x 10^{-6}	2.2 x 10^{-6}

This suggests similar dynamo mechanisms despite different B.C.s

- As forcing gets stronger, lose the ‘Swiss-watch’ assembly of convection rolls



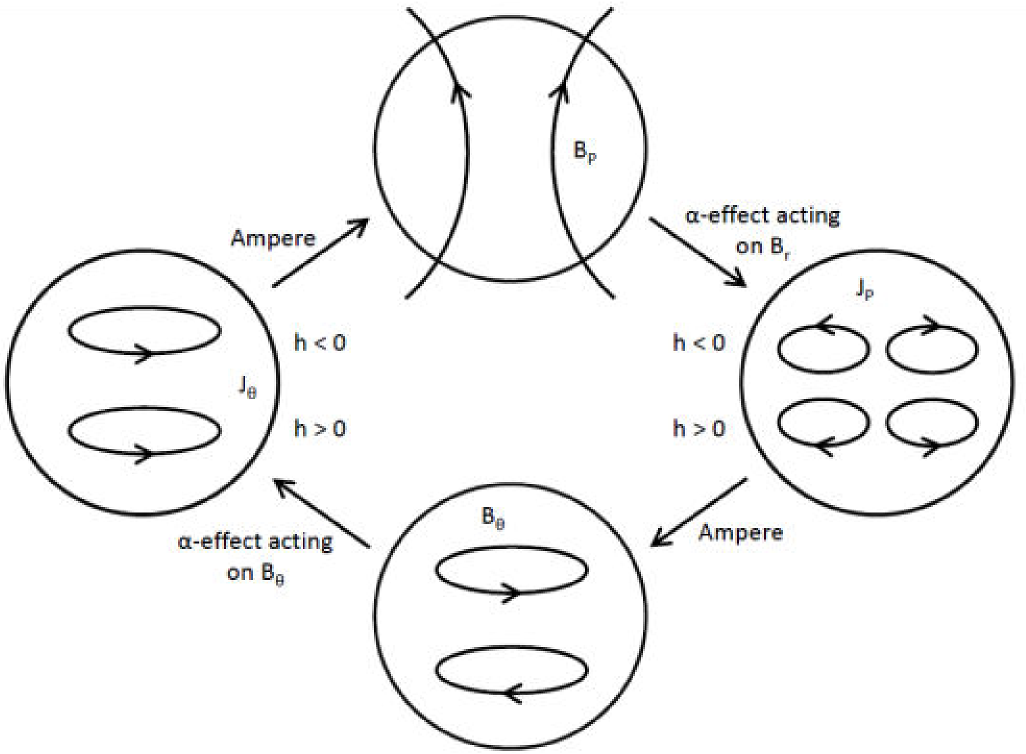
u_r

More realistic model of helicity generation should be:

- Independent of viscosity
- Internally driven (independent of B.C.)
- Robust but random

How about zero-frequency inertial wave packets ?

An old idea revisited… an inertial-wave dynamo



- Can form a self-consistent α^2-dynamo operating outside tangent cylinder based on inertial wave packets initiated near/on the Equatorial plane
- Can deduce scaling laws for Elsasser number, Rossby number and magnetic Reynolds number as a function of planetary size, rotation and core heat flux
- Results consistent with the more strongly forced numerical dynamos

Scaling Laws for Inertial-wave α^2 Dynamo

Input:

- α effect (modelled as on previous slide)
- Curl (**J** X **B**) ~ Curl (**u** x **B**) ~ Curl (buoyancy)
- Inertial wave packets dissipate before reaching mantle

Dimensionless parameters:

$$\Lambda = \frac{\sigma B^2}{\rho\Omega} \qquad R_\lambda = \frac{\Omega R_C^2}{\lambda}$$

R_C = radius of core, λ = magnetic diffusivity

Rayleigh-type number based on average rate of working of the buoyancy force: $Ra_Q = \frac{\bar{P}}{\Omega^3 R_C^2}$ $\qquad \bar{P} \sim \frac{g\beta}{\rho c_p}\frac{Q_T}{4\pi R_C^2}$

Predictions :

$$\Lambda \sim Ra_Q R_\lambda^2 \frac{\delta}{R_C} \qquad Rm = \frac{u\delta}{\lambda} \sim \Lambda^{1/2}$$

δ = mean width of columnar vortices

$$\mathrm{Ro} = \frac{u}{\Omega R_C} \sim Ra_Q^{1/2}\sqrt{\frac{R_C}{\delta}} \qquad \frac{B^2/\mu}{\rho u^2} \sim \frac{\Omega\delta^2}{\lambda}$$

But, what determines δ ?

Scaling laws cont.

- The scaling laws are consistent with results of the more rapidly rotating numerical simulations (Davidson, 2014)

- Predictions for Earth assuming B ~ 30 Gauss & Q ~2 T Watts:
  Ro ~ 10^{-5},   Magnetic energy / KE ~ 100

Summary

We have a simple, predictive cartoon of the geodynamo based on helicity generation by low-frequency inertial wave packets

Thank You

References

P.A. Davidson & A.Ranjan (2014). Planertary dynamos driven by helical waves: Part 2. *Geophysical Journal International* (Submitted)

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A.Ranjan and P.A. Davidson (2014). *Evolution of a turbulent cloud under rotation. Journal of Fluid Mechanics, 756, 488-509*

P.A. Davidson (2013). Turbulence in rotating, stratified and electrically conducting fluids, Cambridge University Press

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α^2-Dynamo

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Exact integral relationships from Induction Equation

$$\int_{V_N} \left(\overline{B}_\theta / r\right) dV \sim \frac{1}{\lambda} \int_{V_N} \frac{z}{r} \left\langle \mathbf{v} \times \mathbf{b} \right\rangle_r dV \qquad \int_{V_C} \overline{B}_z dV = \frac{1}{3\lambda} \int_{V_C} r \left\langle \mathbf{v} \times \mathbf{b} \right\rangle_\theta dV$$

(*r, θ, z*) coordinates, λ = magnetic diffusivity, <~> = azimuthal average,
b, v are non-axisymmetric components of **B, u**

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Induced emf (alpha effect)

$$\left\langle \mathbf{v} \times \mathbf{b} \right\rangle + \frac{\delta^2}{\lambda} \left\langle \mathbf{v} \times \frac{\partial \mathbf{b}}{\partial t} \right\rangle = \pm \frac{2|\delta|}{\lambda} \left\langle \mathbf{v}_\perp^2 \overline{\mathbf{B}}_\perp - (\mathbf{v}_\perp \cdot \overline{\mathbf{B}}_\perp) \mathbf{v}_\perp \right\rangle$$

Assuming:

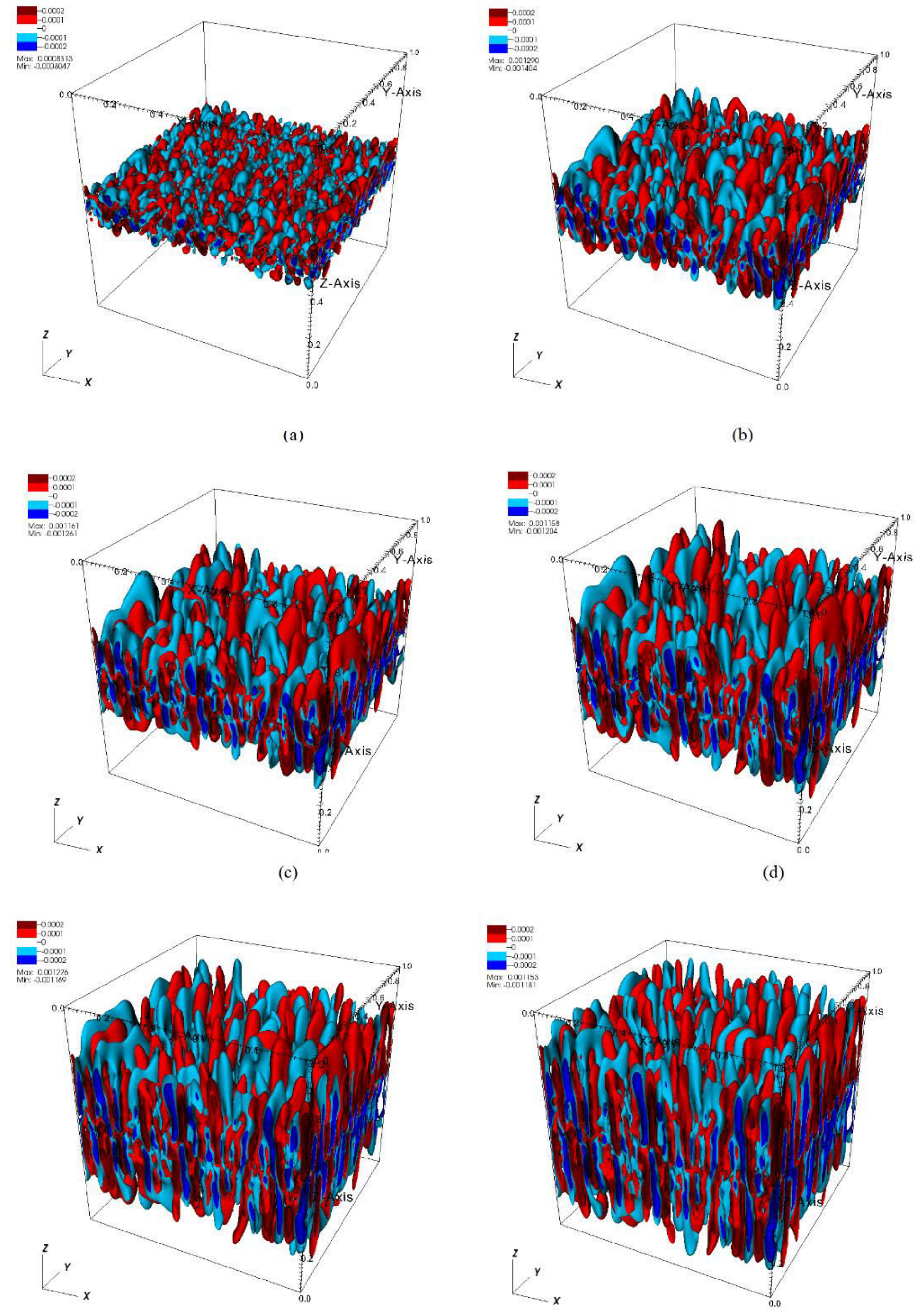
- **B** varies slowly on the scale of δ, blob size
- axial gradients in **v** are very small
- the fluctuations in velocity have maximal helicity,
- the fluctuations are statistically homogeneous, at least locally;
- **b** is much smaller than the local mean field (first order smoothing)

Resulting scaling relationships

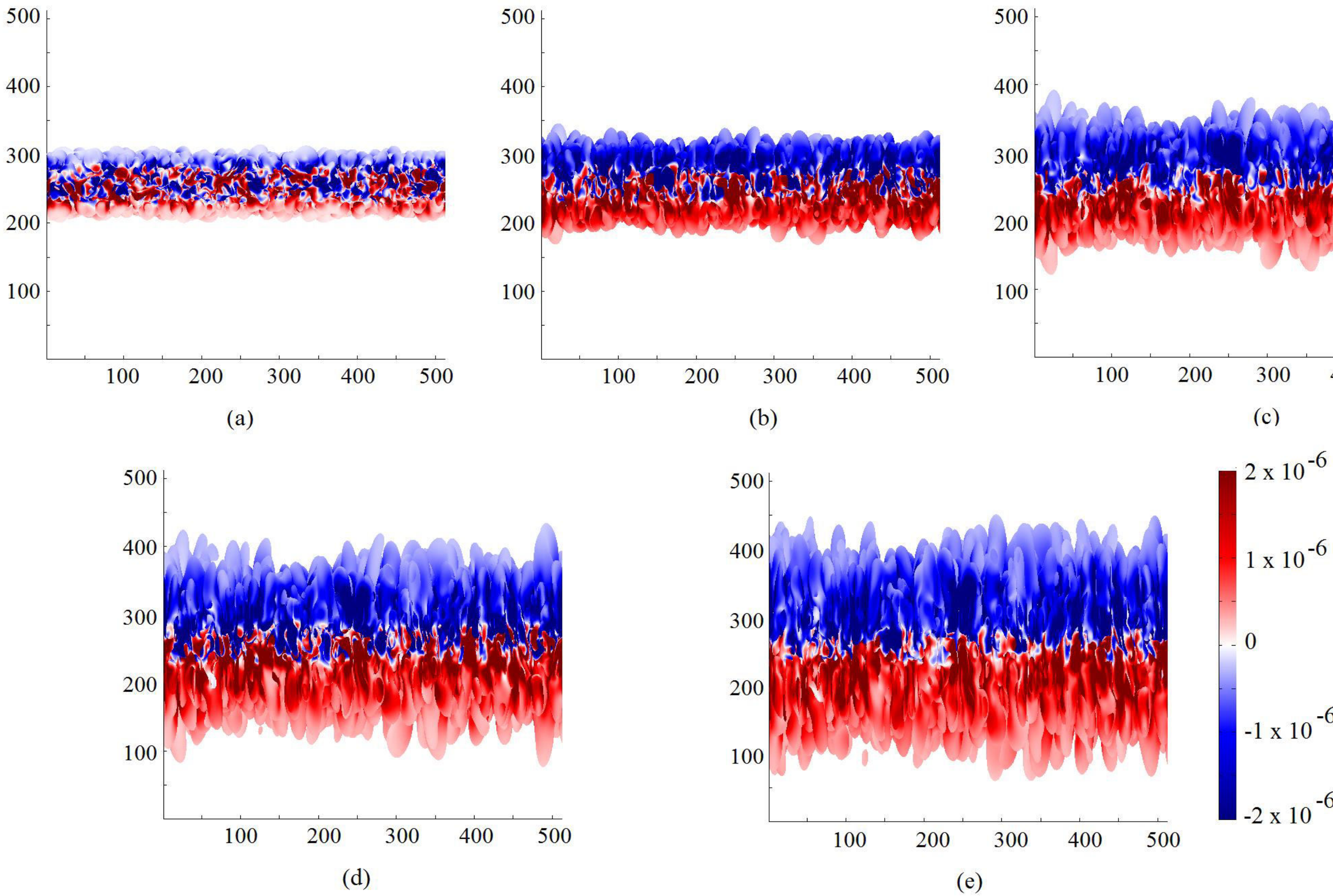
$$\left\langle \mathbf{v} \times \mathbf{b} \right\rangle_r \sim \pm \frac{\lambda^{-1} |\delta| \left\langle \upsilon_\theta^2 \right\rangle \overline{B}_r}{1 + \left(\delta^2 \varpi / \lambda\right)^2} \qquad \left\langle \mathbf{v} \times \mathbf{b} \right\rangle_\theta \sim \pm \frac{\lambda^{-1} |\delta| \left\langle \upsilon_r^2 \right\rangle \overline{B}_\theta}{1 + \left(\delta^2 \varpi / \lambda\right)^2}$$

ϖ = wave frequency

Iso-surfaces of u_z (positive is red, negative is blue) for Ωt=2-12



(a) (b) (c) (d) (e) (f)



Iso-surfaces of energy coloured by helicity (positive is red, negative is blue) for Ωt=2-10