On the origin of the magnetic excess in MHD turbulence

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MHD turbulent flows with equipartition spontaneously develop a magnetic excess.

The *amount* of magnetic excess *and its scaling* with wavenumber naturally results from a balance between a source (magnetic stretching) and a loss (Alfvén propagation)

The relation so obtained makes the ratio of magnetic excess to total energy to vary as the ratio of Alfvén time to nonlinear time.

Solar Wind scalings (slopes $m_{tot} = 5/3$, $m_{res}=2$, *Chen et al 2013*) is a solution of this scenario.
It is recovered here also by direct simulations of expanding MHD equations

Meudon Turbulence Workshop 2015

Scenario

1. Alfvén effect → equipartition $u \approx b$

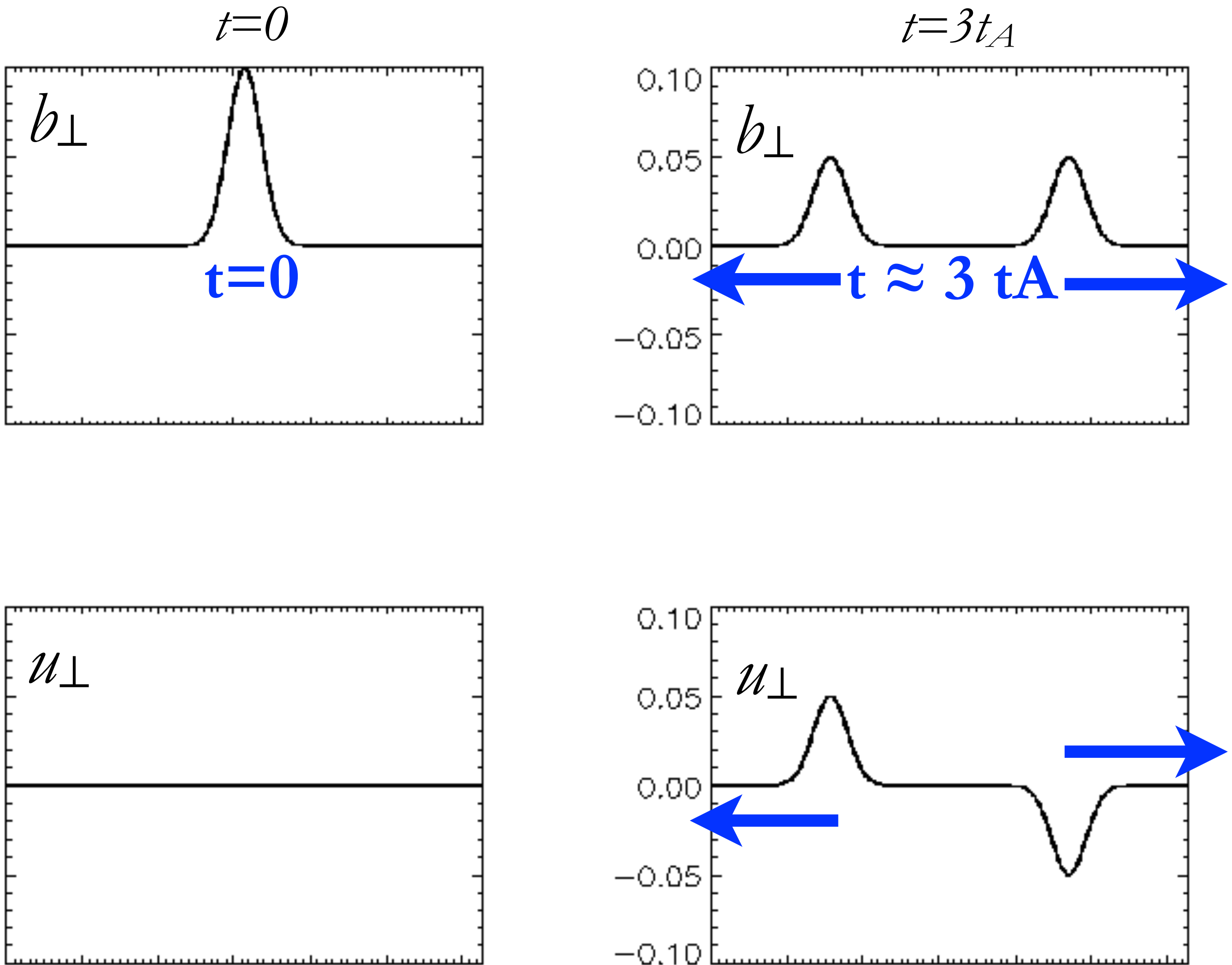
equipartition time = Alfvén time

$$\partial b/\partial t = (B^\circ.\nabla)u$$
$$\partial u/\partial t = (B^\circ.\nabla)b$$

$$\Downarrow$$

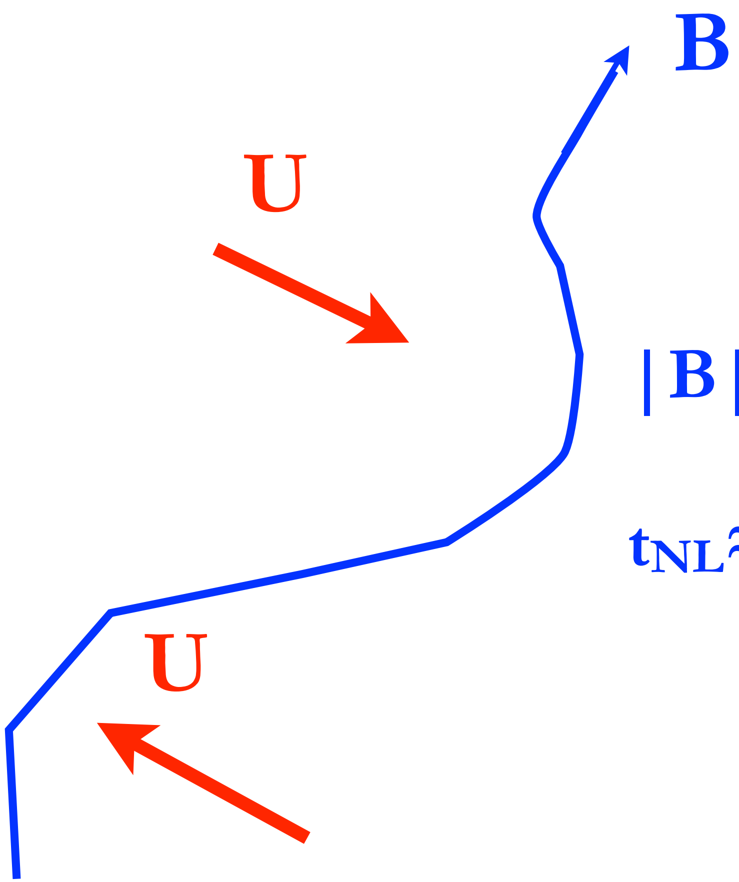
$$\partial(b^2 - u^2)/\partial t \approx -(b^2-u^2)/t_A$$

$b_\perp \neq 0,\ u_\perp = 0$ → $b_\perp{}^2 = u_\perp{}^2$



Scenario

2. Magnetic stretching → growth of B



$\mathbf{DB/Dt = (B.\nabla)u}$

$DB^2/Dt \approx B^2/t_{NL}$

⇓

$\mathbf{D(b^2-u^2)/Dt \approx (b^2+u^2)/t_{NL}}$

|B| grows due to stretching by ∇u

$\mathbf{t_{NL} \approx (\nabla u)^{-1} \approx (ku)^{-1}}$

Scenario

3. Alfvén / stretching balance

$$D(b^2-u^2)/Dt \approx -(b^2-u^2)/t_A + (b^2+u^2)/t_{NL}$$

Alfvén effect **stretching**

•Equilibrium: $\mathbf{b^2-u^2 \approx (t_A/t_{NL})\,(b^2+u^2)}$ (1)

•Definitions: residual energy = b^2-u^2 ; total energy = b^2+u^2
Residual and total spectra: $b^2-u^2 \approx k\,E_{res}(k)$; $u^2+b^2 \approx kE_{tot}(k)$

•Equilibrium (1) becomes

$$\mathbf{E_{res}(k) \approx (t_A/t_{NL})\,E_{tot}(k)} \qquad (2)$$

Scenario

4. Alfvén-stretching balance (followed)

•Assume *zero global mean field (→ local mean field ≈ brms)*

Define time scales: $\mathbf{t_A} = 1/(kb_{rms})$, $\mathbf{t_{NL}} \approx 1/(k(u^2+b^2)^{1/2}$

$$\mathbf{E_{res}(k) \approx (t_A/t_{NL})\ E_{tot}(k)}$$

becomes

$$E_{res} \approx (k^{1/2}/b_{rms})\ E_{tot}^{3/2}$$

• Scaling laws: $E^{res} \propto k^{-mres}$, $E^{tot} \propto k^{-mtot}$

$$\mathbf{m_{res} = -1/2 + 3\ m_{tot}/2}$$

Solar wind case *is* solution:

$$\mathbf{m_{tot} = 5/3 \Rightarrow m_{res}} = -1/2 + 3\ (5/3)/2 = \mathbf{2}$$

Results

1. Incompressible MHD

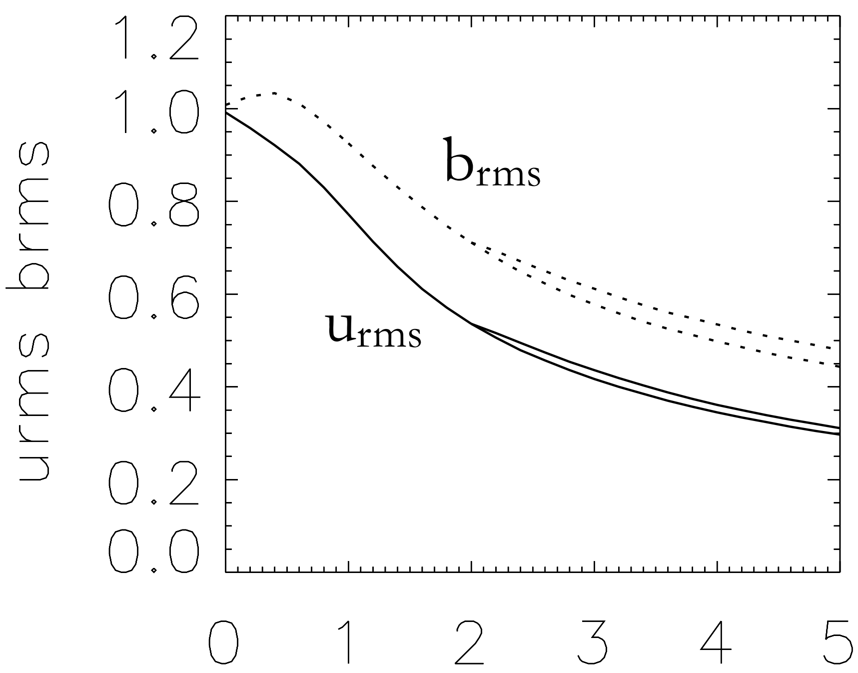
Initial conditions: Gaussian spectrum ($\Delta k=4$) $u_{rms}=b_{rms}=1$, $<u.b>=0$, divu=0, $<B>=0$

Two runs :

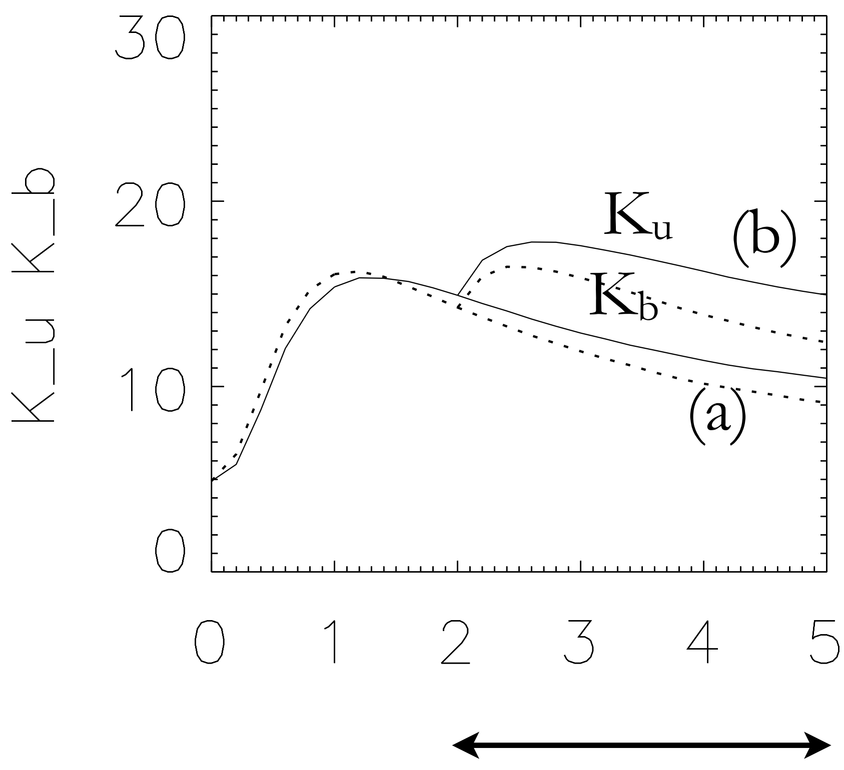
(a) constant viscosity

(b) viscosity/2 for $t \geq 2 \Rightarrow$ enlarged inertial range

b_{rms} and u_{rms}



Taylor wave numbers



period studied

Results

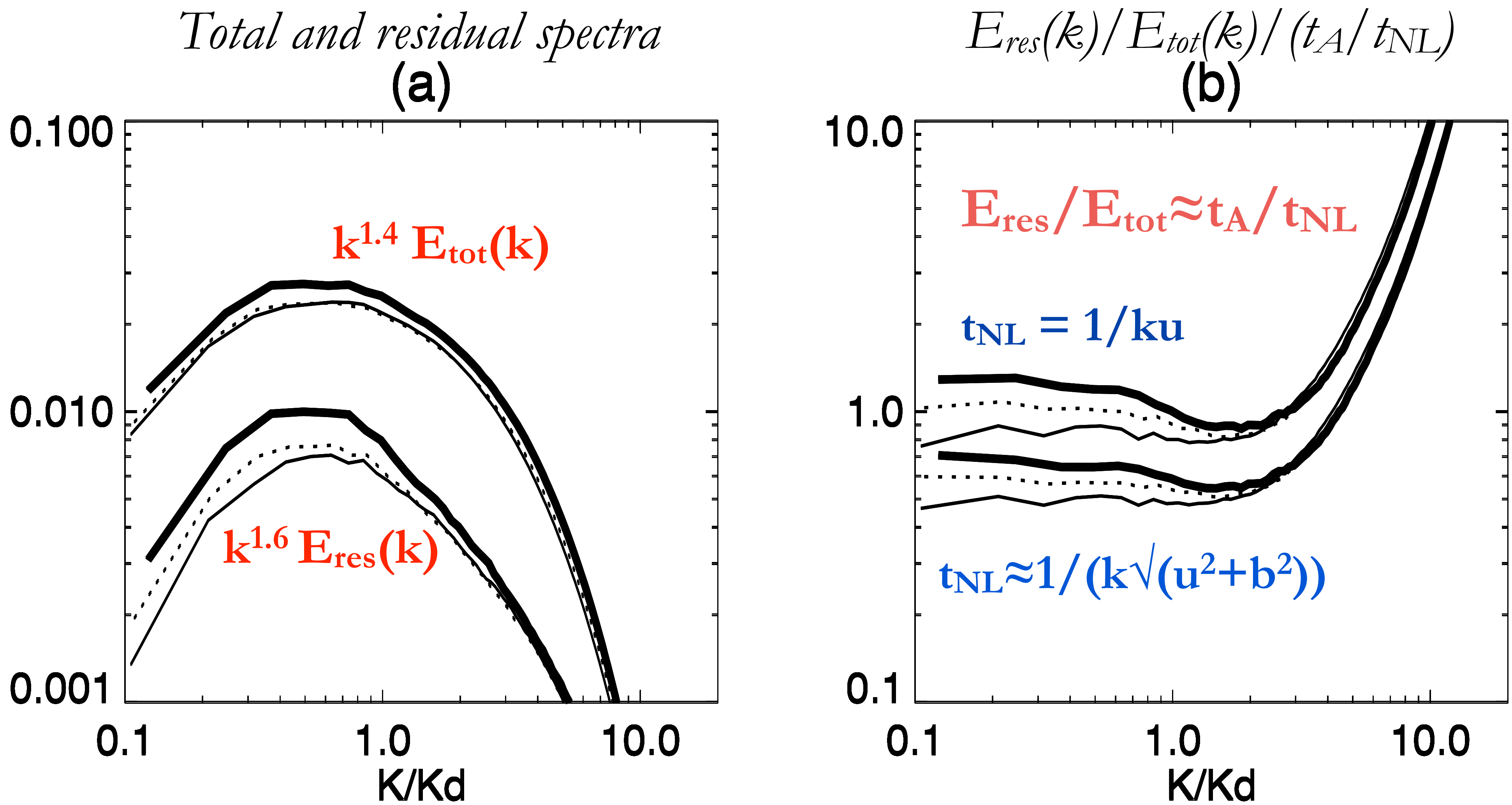
1. Incompressible MHD

Spectral averaging in 3 subsets of [2,5]; last interval [4,5] in *bold*

(a) Scaling: best fit **$m_T \approx 1.4$** $\Rightarrow$ **m_R** $= -1/2 + 3\, m_T/2 =$ **1.6 → correct slope**

→ stretch-Alfvén scenario correct

(b) Amplitudes: $E_{res}(k)/E_{tot}(k)/(t_A/t_{NL}) \approx 1$ with two definitions of t_{NL}



Results

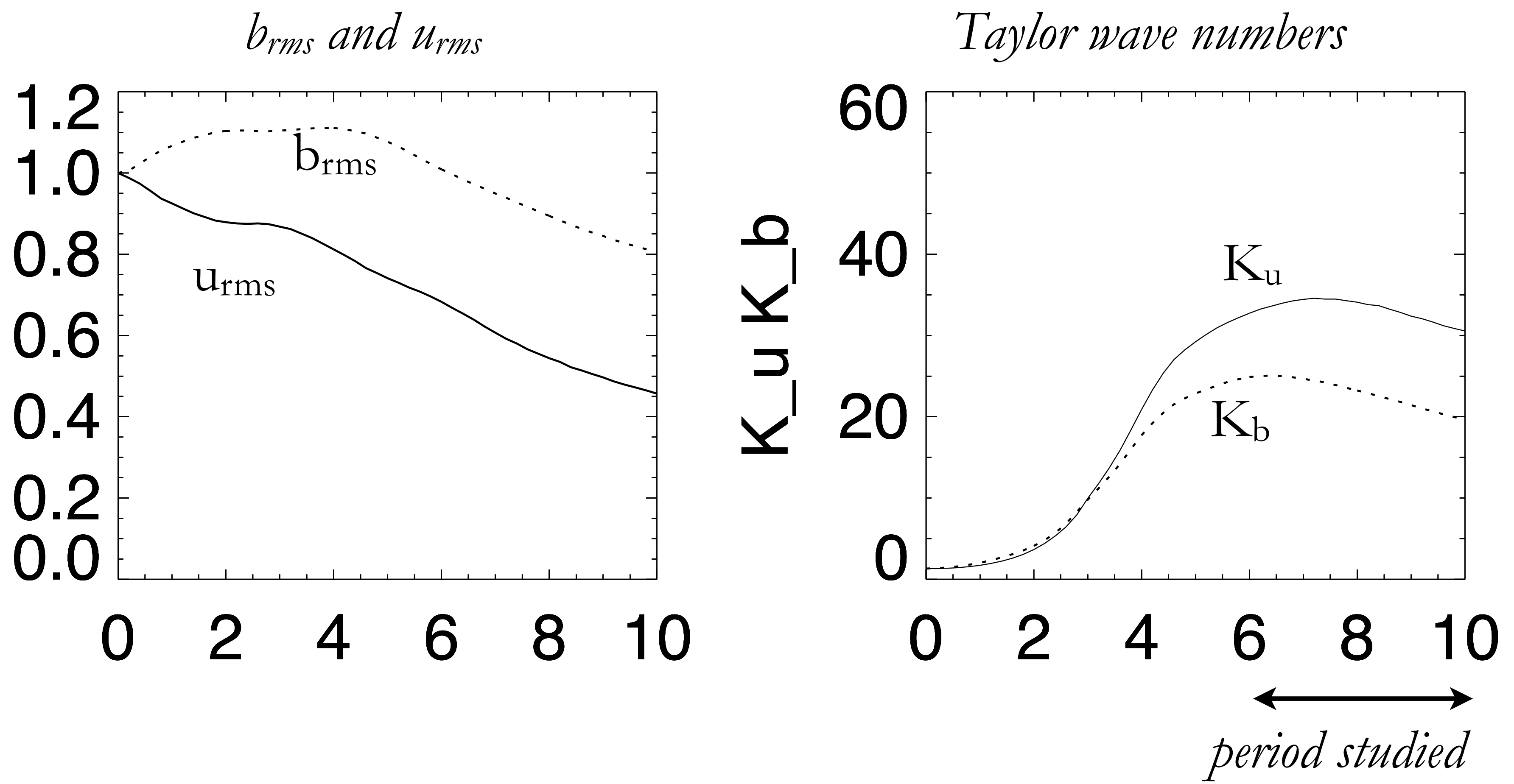
2. Compressible MHD

Initial conditions: *LS* spectrum (k≤2), $u_{rms}=b_{rms}=1$, <u.b>=0, divu=0, <B>=0

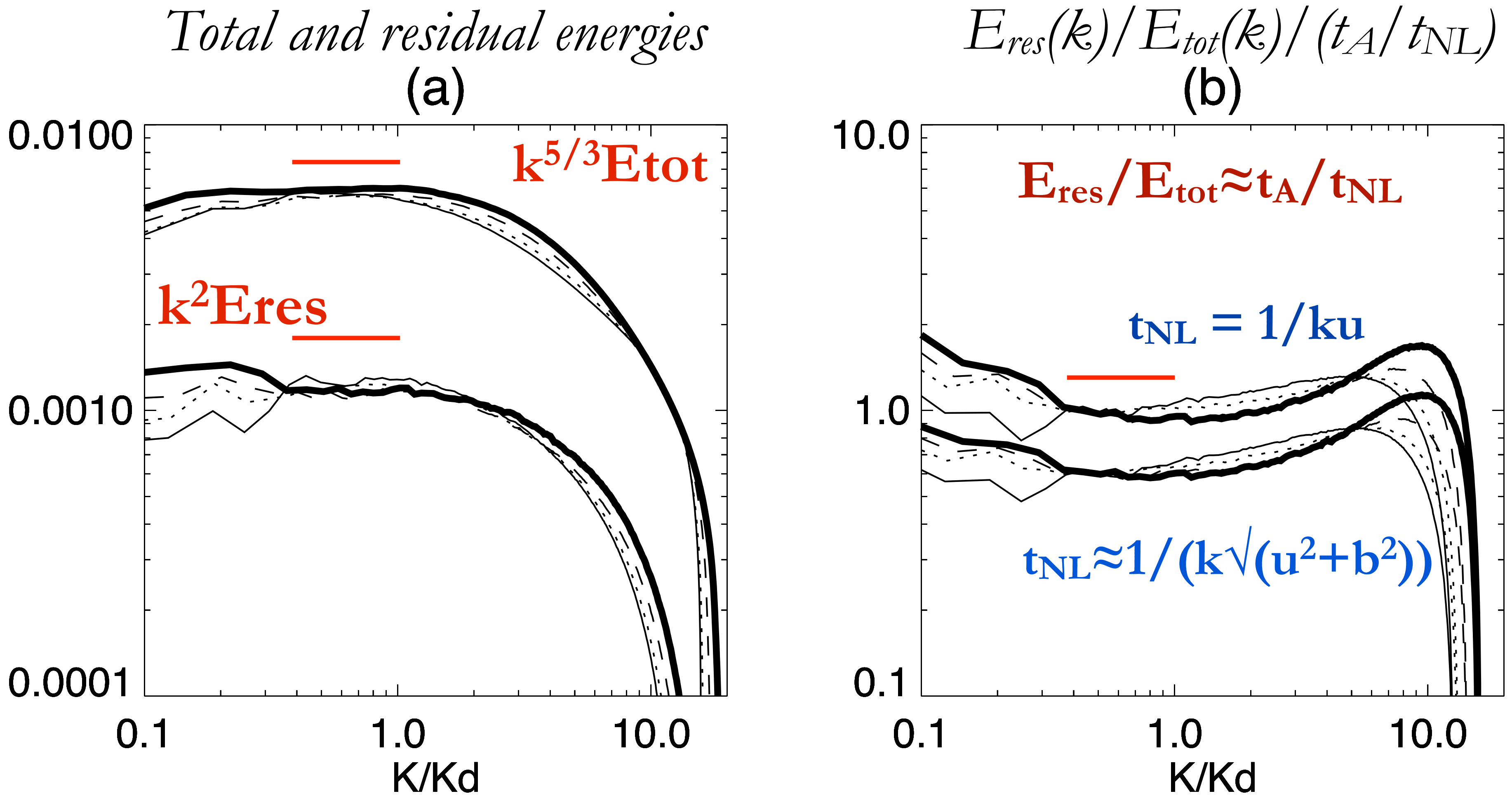
Two runs :

(a) constant viscosity

(b) viscosity/2 for t ≥2 ⇒ enlarged inertial range



2. Compressible MHD



Spectral averaging in 4 subsets of [6,10]; last interval [9,10] in bold

(a) Scaling: best fit $\mathbf{m_T \approx 5/3} \Rightarrow \mathbf{m_R} = -1/2 + 3\, m_T/2 = \mathbf{2} \rightarrow$ **correct slope**

$\rightarrow$ stretch-Alfvén scenario correct again

(b) Amplitude: $E_{res}(k)/E_{tot}(k)/(t_A/t_{NL}) \approx 1$ with two definitions of tNL

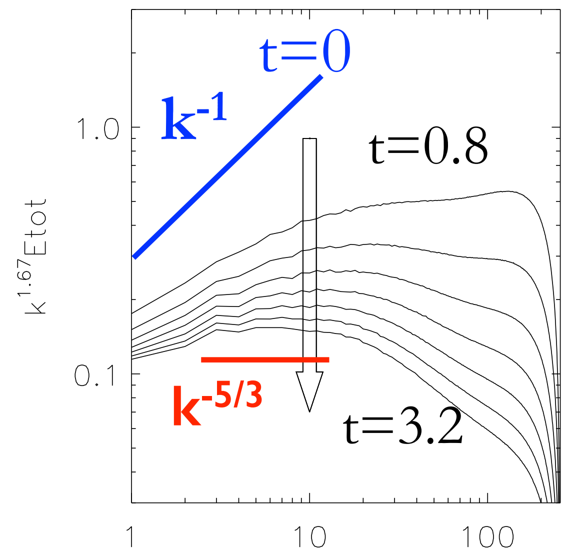
3. Comobile compressible MHD (*EBM*)

cf. Dong Verdini Grappin 2014

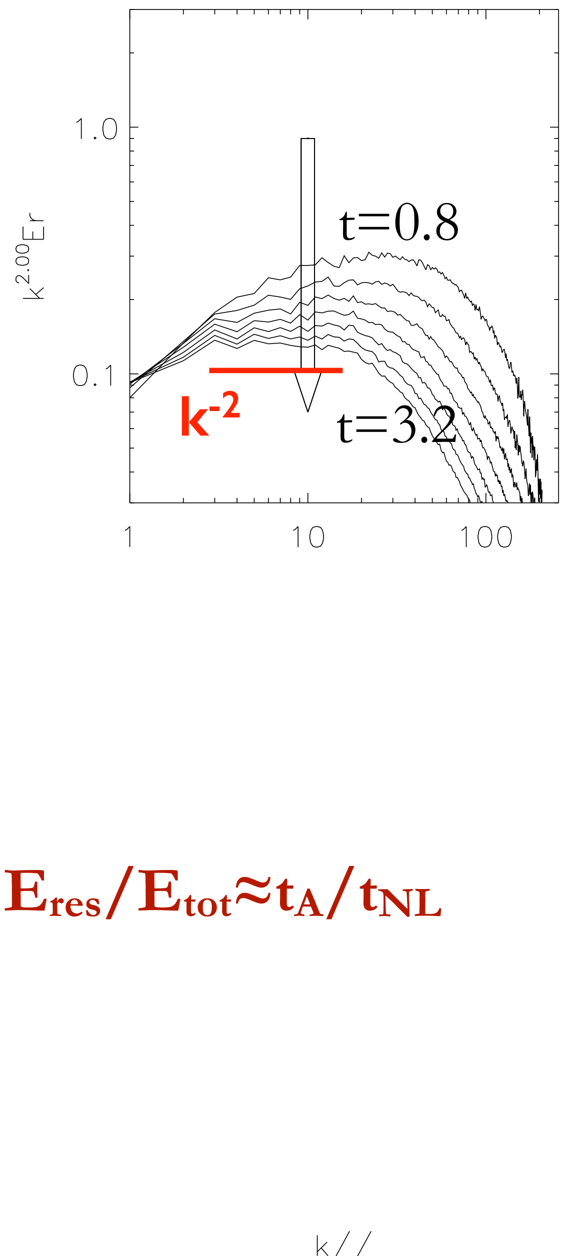
Initial conditions:
R=0.3 AU, aspect ratio 1
Isotropic $\mathbf{k^{-1}}$ spectrum,
$u_{rms}=b_{rms}$, <u.b>=0
expansion rate = $2/t_{NL}°$

Results:
$t=3.2\ t_{NL}°$, *final aspect ratio*= 7.4
(a)Scaling
best fit $m_T \approx 5/3$
$m_R = -1/2 + 3\ m_T/2 = 2$
correct slope → stretch-alfvén scenario correct
(b) Amplitude
$E_{res}(k)/E_{tot}(k)/(t_A/t_{NL}) \rightarrow 1$

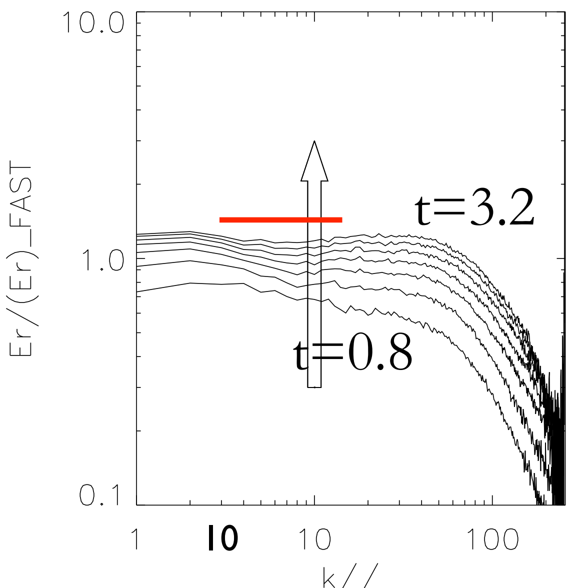
Total energy spectrum



Residual energy spectrum



$E_{res}(k)/E_{tot}(k)/(t_A/t_{NL})$



$E_{res}/E_{tot} \approx t_A/t_{NL}$

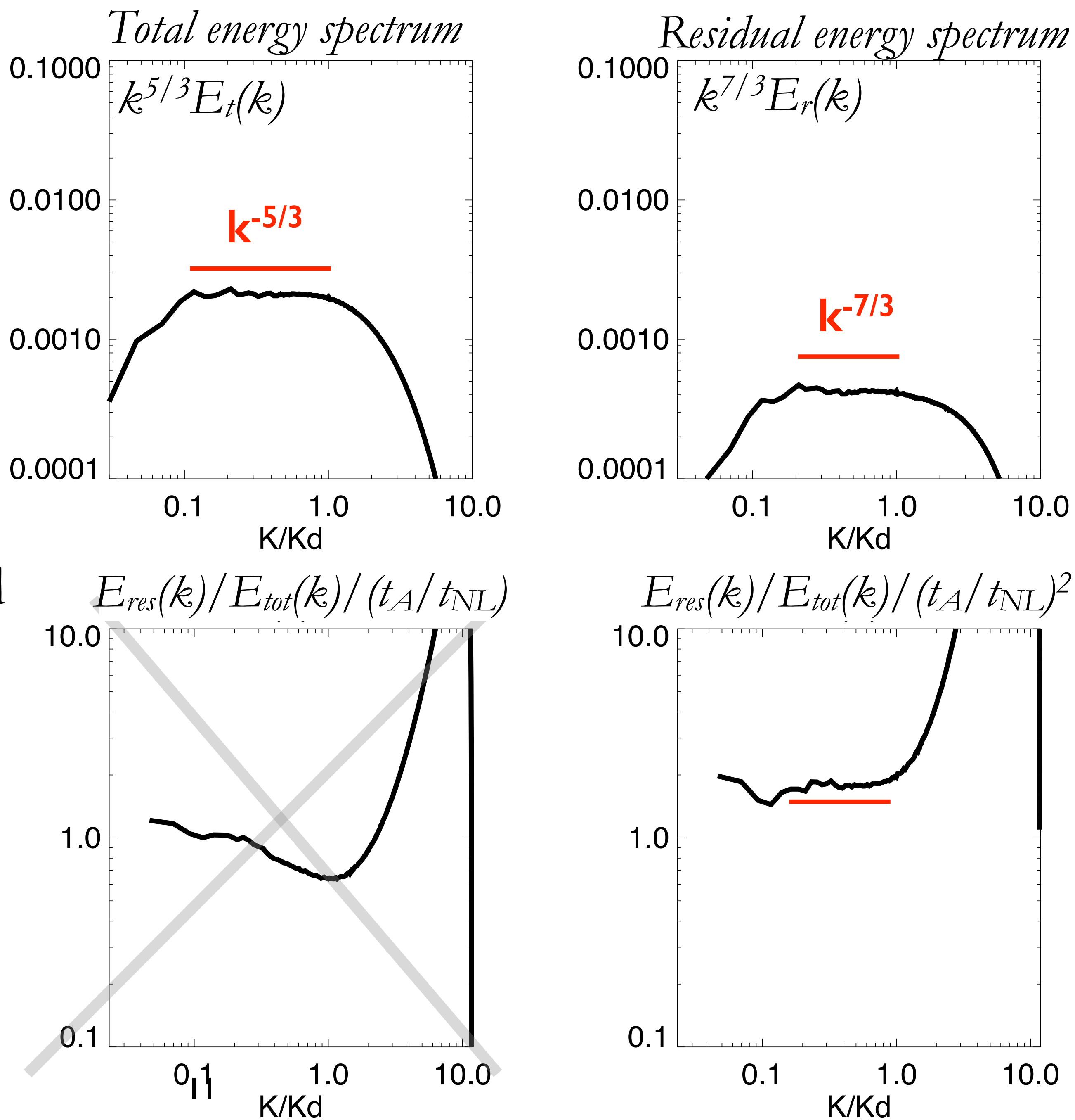
Results

4. Incompressible MHD, an alternative scenario

Müller Grappin 2005

Initial conditions: t=0
Gaussian $\Delta k=4$, <u.b>=0,
$u_{rms}=b_{rms}$

Results:
<u>Scaling</u>:
best fit $m_T=5/3$
• $m_R=-1/2+3/2m_T = 2$ *no* good fit !
• **$m_R=7/3$** *correct* fit
as predicted by SLOW version of stretching/Alfvén scenario
$E_{res}/E_{tot} \approx (t_A/t_{NL})^2$
→ $m_R=-1+2m_T = 7/3$



Summary

Fast versus slow scenario

General scenario: $E_{res}/E_{tot} = t_A/t^* = (t_A/t_{NL})^{\alpha}$

fast stretching: **(α=1) $t^* = t_{NL}$**

→ $m_R=-1/2+(3/2)m_T$

→ holds in incompressible, compressible and expanding (EBM) simulations shown here *and in solar wind*

slow stretching: **(α=2) $t^* = (t_{NL}/t_A)\ t_{NL}$** *(Grappin et al 1983, Müller Grappin 2005)*

→ $m_R=-1+2m_T$

→ holds in incompressible simulations with $B°=0$ and $B°\neq 0$