Coexistence of weak and strong turbulence in incompressible Hall MHD

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Meudon Turbulence Workshop

ENERGY CASCADE AND DISSIPATION IN ASTROPHYSICAL TURBULENT PLASMAS

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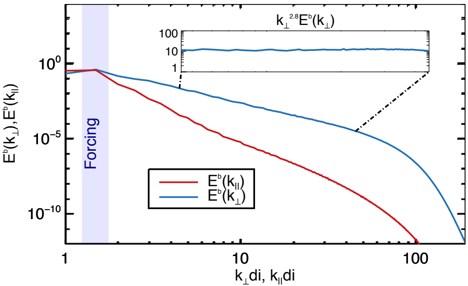
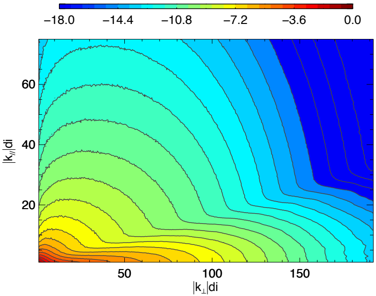
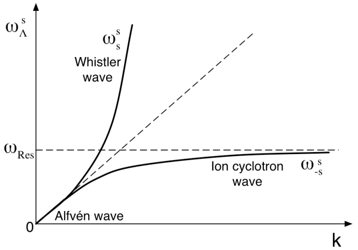
Numerical experiments of Hall MHD turbulence in the presence of a strong mean magnetic field

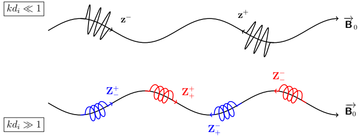
$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla P_* + \mathbf{b}\cdot\nabla\mathbf{b} + \mathbf{f}^u + \nu_h \nabla^6 \mathbf{u} \\ \dfrac{\partial \mathbf{b}}{\partial t} = \nabla\times[(\mathbf{u} - d_i \nabla\times\mathbf{b})\times\mathbf{b}] + \mathbf{f}^b + \eta_h \nabla^6 \mathbf{b} \end{cases}$$

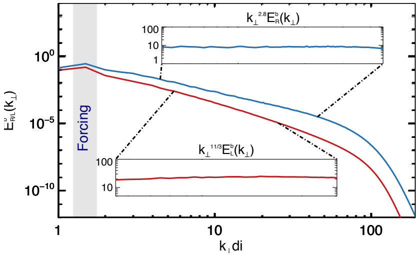


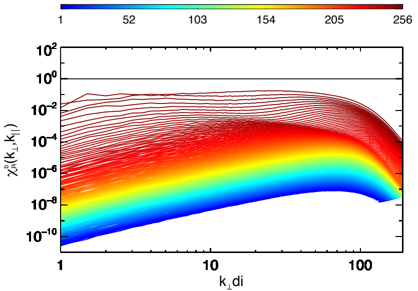


Which mode dominates the magnetic field fluctuations ?









Origin of anisotropy in whistler wave turbulence

In the long time statistical behavior most of the nonlinear terms will be destroyed by random phase mixing and only a few of them – called the resonance terms – will survive.

$$\begin{cases} skk_{\|} + s_p pp_{\|} + s_q qq_{\|} = 0 \\ \mathbf{k} + \mathbf{p} + \mathbf{q} = 0 \end{cases} \Rightarrow \frac{s_p p - sk}{q_{\|}} = \frac{s_q q - s_p p}{k_{\|}} = \frac{sk - s_q q}{p_{\|}}$$

The local interaction limit simplifies the resonance conditions as follows : $(s_p - s)k_{\|} \sim (s_q - s_p)q_{\|}$.
Exact kinetic equations for EMHD at the level of three-wave interactions :

$$\partial_t E(\mathbf{k}) = \frac{\pi\varepsilon}{8} \sum_{ss_p s_q} \int \left(\frac{s_q q - s_p p}{k_{\|}} \right)^2 \times \{complicated\ \ stuff\}$$

Only the interaction between two waves with opposite polarities will contribute significantly to the nonlinear dynamics.

It implies that either $q_{\|} \sim 0$ or $p_{\|} \sim 0$.

Does the anomalous spectrum can be explained by G.I.K phenomenology ?

Bicoherence Analysis

Bicoherence

$$C^2(\omega_k,\omega_l) = \frac{|\langle\langle \Psi(x,\omega_k)\Psi(x,\omega_l)\Psi^*(x,\omega_{k+l})\rangle\rangle|^2}{\langle\langle|\Psi(x,\omega_k)\Psi(x,\omega_l)|^2\rangle\rangle\,\langle\langle|\Psi^*(x,\omega_{k+l})|^2\rangle\rangle}$$

Schwartz's inequality $\Rightarrow C(\omega_k,\omega_l) \in [0,1]$

Interpretation of bicoherence : Bicoherence measures the proportion of the signal energy at any bifrequency (k,l) that is quadratically phase coupled to $k+l$. A large bicoherence means that the phase difference $\arg\Psi_k + \arg\Psi_l - \arg\Psi_{k+l}$ reaches a fixed value, even though each phase, when taken separately, may vary in a random way.

Examples of utilization :

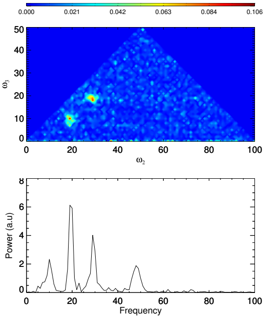
- Crack detection in aircraft, power generation plant, rail tracks, etc.
- Detection of "non-cooperative" aircraft target
- Analysis Electroencephalography
- Analysis of L-H transition in tokamak
- Analysis of gravity-capillary weak turbulence

Interpretation of bicoherence figures

$$\begin{cases} y(t) = \sum_{i=1}^{4} sin((\omega_i + \delta\omega_i)t + \phi_i) \\ \omega_1 + \omega_2 = \omega_3 \ ; \ \omega_1 - \omega_2 = \omega_4 \ ; \ \phi_{1,2} = random[-\pi, \pi] \end{cases}$$

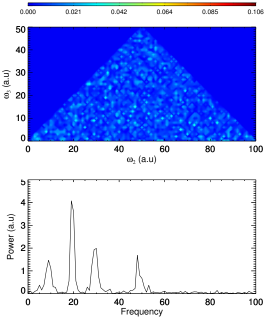
Coupled

$$\begin{cases} \phi_1 + \phi_2 = \phi_3 \\ \phi_1 - \phi_2 = \phi_4 \end{cases}$$

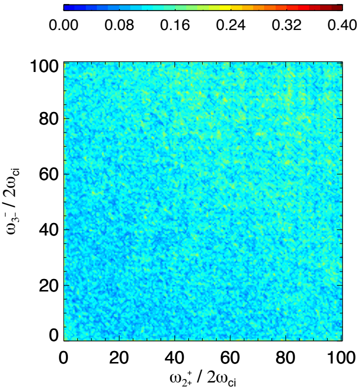


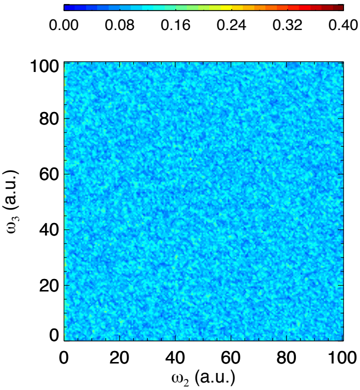
Uncoupled

$$\begin{cases} \phi_3 = random[-\pi, \pi] \\ \phi_4 = random[-\pi, \pi] \end{cases}$$

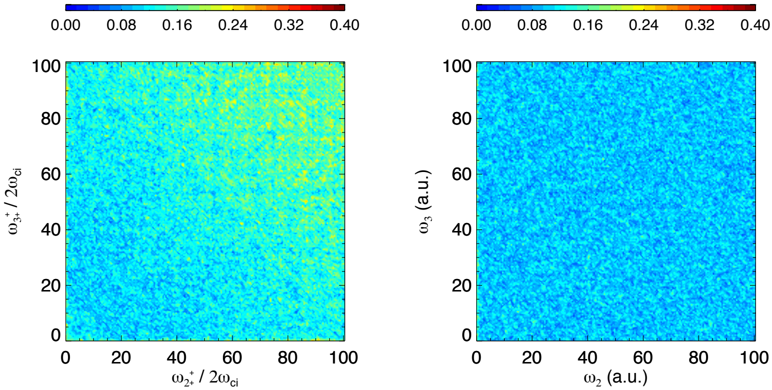


whistler↑+whistler↓ ⟹ whistler↑

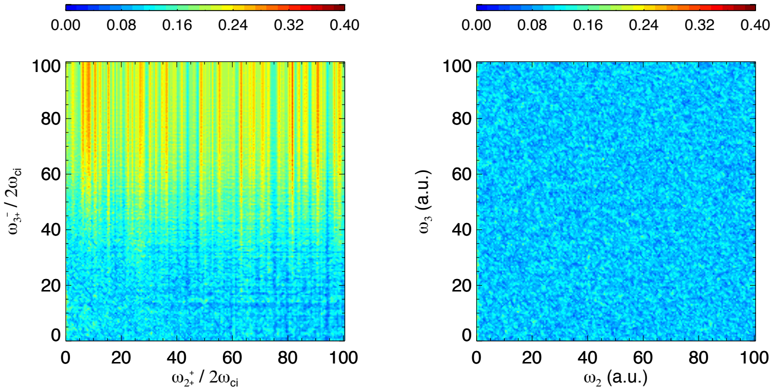




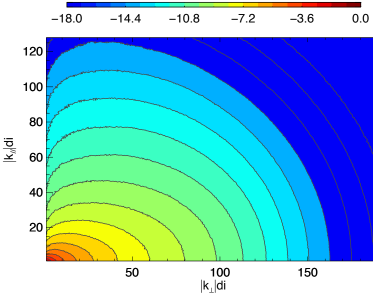
whistler$\uparrow$+whistler$\uparrow$ $\Longrightarrow$ whistler$\uparrow$

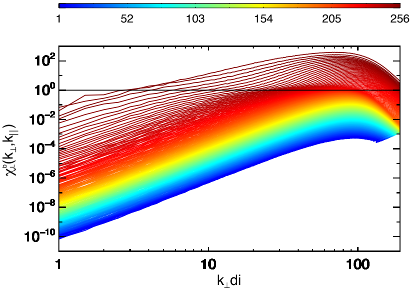


whistler↑+ion cyclotron↓ $\Longrightarrow$ whistler↑



Nature of the left handed fluctuations





$$\chi^R = \frac{\tau_w}{\tau_{nl}^R} \sim \frac{k_\perp \, \delta b_\perp^R}{k_\| \, b_0}, \quad \chi^L = \frac{\tau_{ic}}{\tau_{nl}^L} \sim \frac{di^2 k_\perp^3 \, \delta b_\perp^L}{k_\| \, b_0}$$

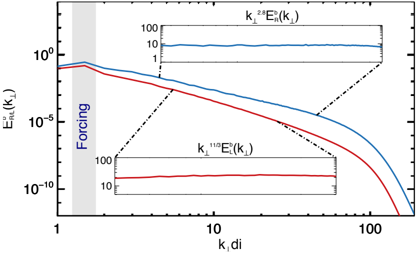
Left handed fluctuations are strongly nonlinear.

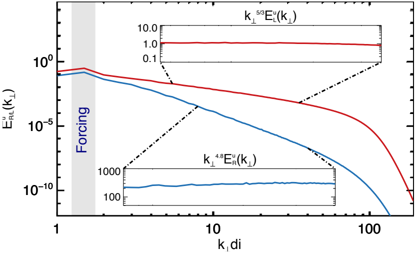
Nature of the left handed fluctuations

It is possible to show rigorously (heuristic explanation in Meyrand et al. PRL 2012) :

$$\begin{cases} E_L^u(\mathbf{k}) = k^2\, d_i^2\, E_L^b(\mathbf{k}), \\ E_R^b(\mathbf{k}) = k^2\, d_i^2\, E_R^u(\mathbf{k}). \end{cases}$$

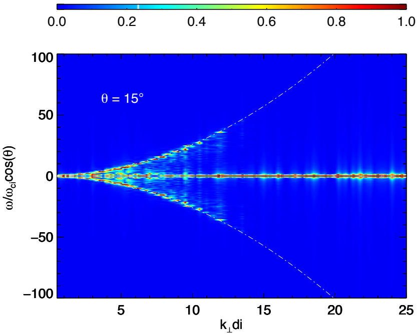
Left handed fluctuations are driven by the velocity field, right handed by the magnetic field.



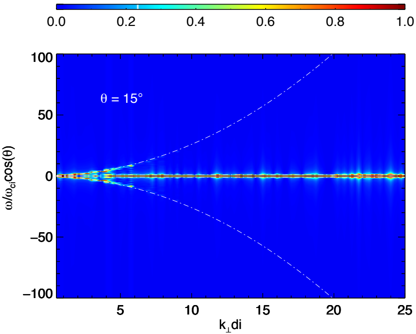


Wavenumber-frequency spectrum

Left handed fluctuations are driven by the velocity field, right handed by the magnetic field.



Wavenumber-frequency spectrum of the magnetic energy fluctuations



Wavenumber-frequency spectrum of the velocity energy fluctuations

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Weak magnetohydrodynamic turbulence and intermittency

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The overall home message

A sea of weakly interacting whistler waves can be vigorously influenced by a strongly turbulent velocity field background.