Compressible turbulence in astrophysical plasmas

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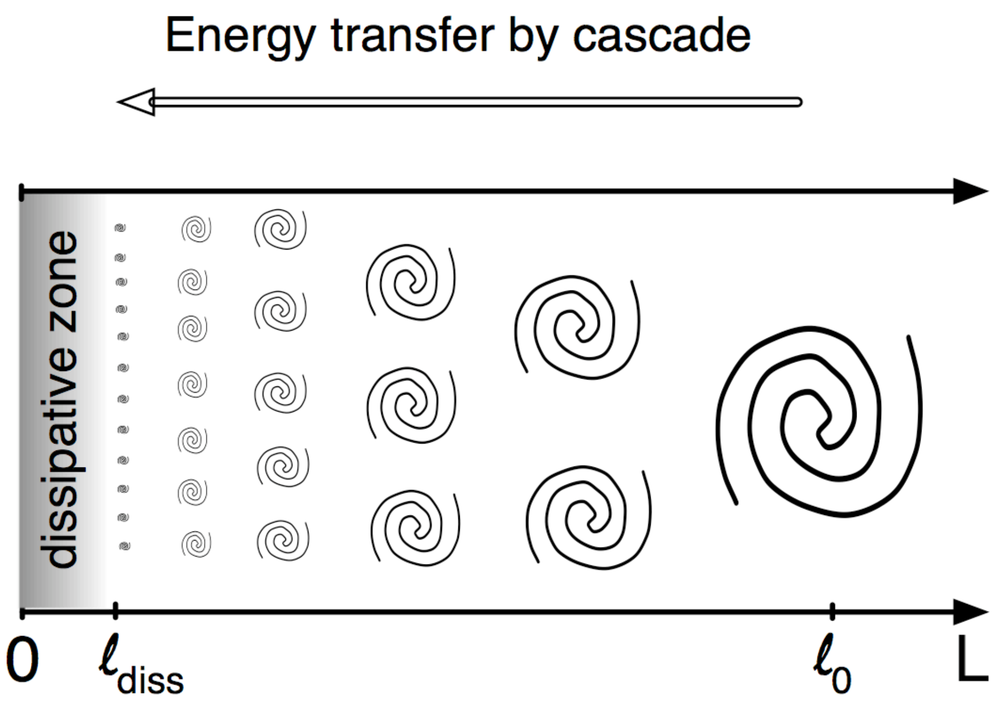
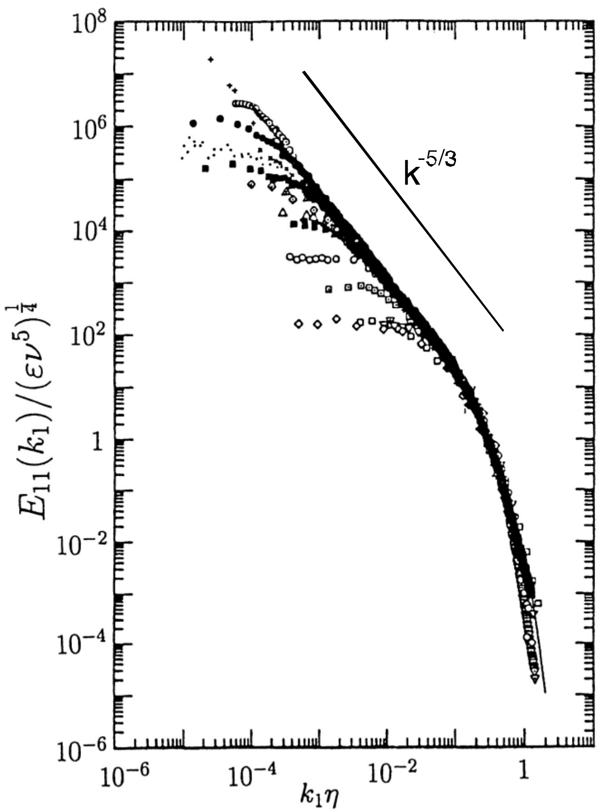
Incompressible to compressible



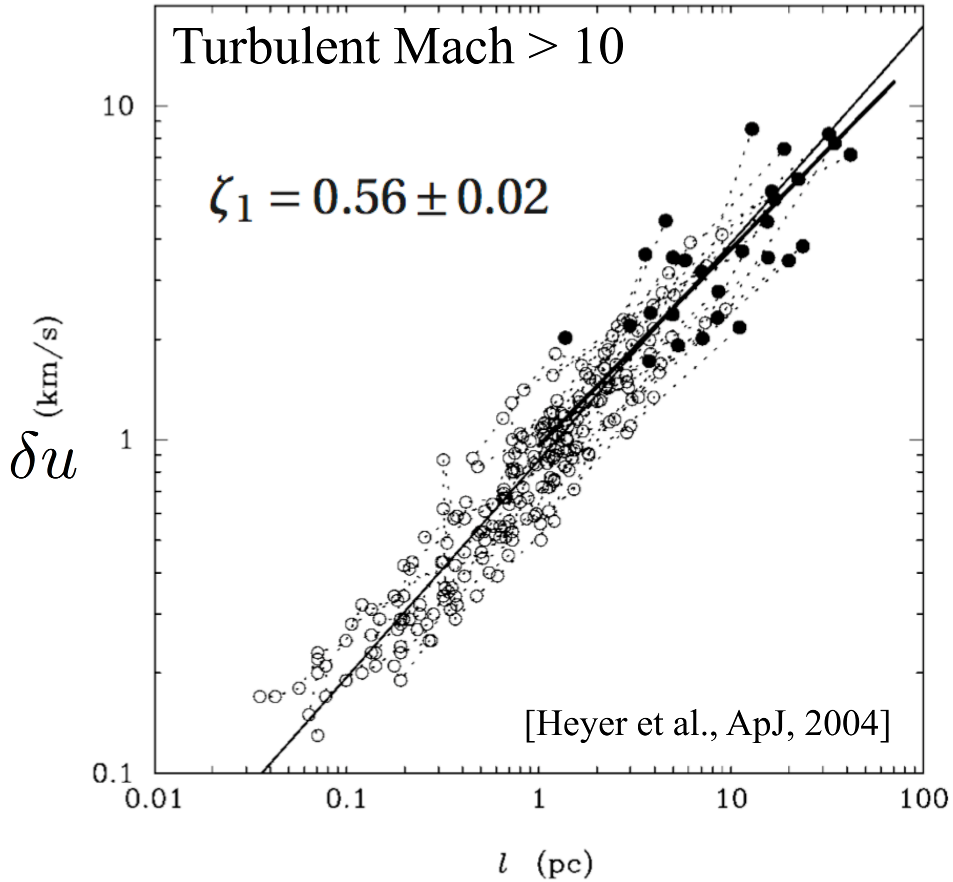


Exact relation: $$-\frac{4}{5}\varepsilon\ell = \langle(\delta u_\ell)^3\rangle\,,$$ [Kolmogorov, DAN, 1941]

Can we find universality in compressible fluids ??

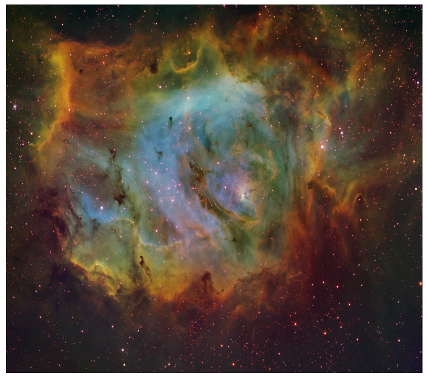
Compressible turbulence in astrophysics

Sample of 27 interstellar clouds



$1\ \mathrm{pc} = 3 \times 10^{16}\ \mathrm{m}$

Spectral line broadening *via* Doppler shifts gives (non-thermal) turbulent velocities



$\Longrightarrow E^u(k) \sim k^{-2.1}$

Few analytical works

- Acoustic wave turbulence : $E \sim k^{-3/2}$ [Zakharov & Sagdeev, Sov. Phys., 1971]
- Model of equation : $\frac{\partial E(k)}{\partial t} + \frac{\partial P(k)}{\partial k} = -D\frac{P(k)}{k}$ [Kadomtsev et al., Sov. Phys, 1973]

  ➔ Burgers as a benchmark

- Phenomenology: $\varepsilon \sim \rho u^3/\ell$ => $u \sim (\ell / \rho)^{1/3}$ [Fleck, ApJ, 1983]

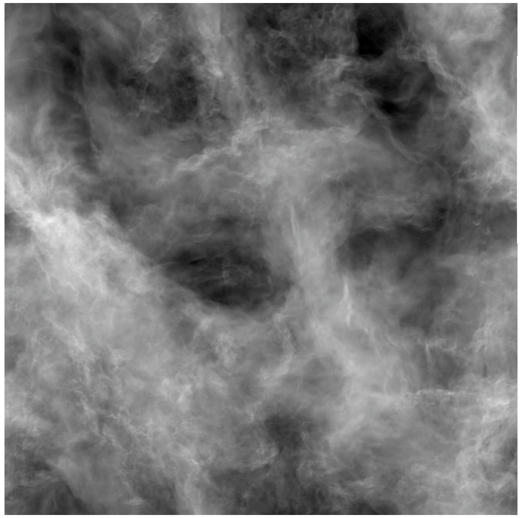
  ➔ lack of rigorous justification

- Numerical simulations at low turbulent Mach number

  [eg. Passot et al., A&A, 1988; Porter et al., PRL, 1992]

Can we derive an exact Kolmogorov law ?

Compressible isothermal HD turbulence



[Kritsuk et al., ApJ, 2007]

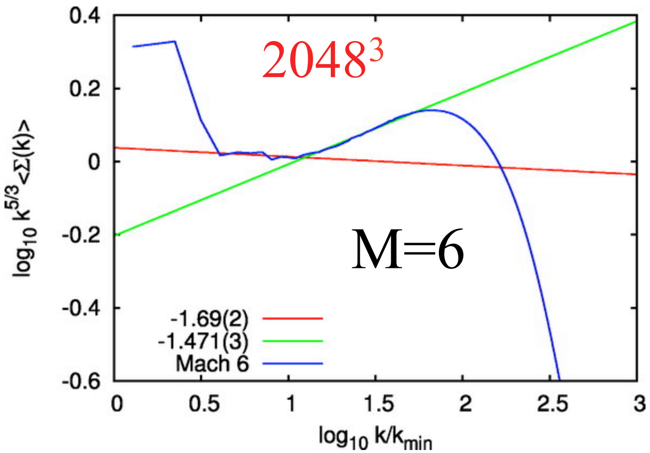


Fig. 19.—Time-averaged power spectrum of the density-weighted velocity $\boldsymbol{v} \equiv \rho^{1/3}\boldsymbol{u}$ compensated by $k^{5/3}$. The straight lines represent the least-squares fits to the data for $\log k/k_{\rm min} \in [0.5, 1.1]$ and $\log k/k_{\rm min} \in [1.2, 1.8]$. The inertial subrange slope is in excellent agreement with the model prediction. [*See the electronic edition of the Journal for a color version of this figure.*]

$$
\begin{aligned}
\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \mathbf{u}) &= 0\,, \\
\partial_t (\rho \mathbf{u}) + \boldsymbol{\nabla} \cdot (\rho \mathbf{u}\mathbf{u}) &= -\nabla P + \mu \Delta \mathbf{u} + \frac{\mu}{3} \nabla (\boldsymbol{\nabla} \cdot \mathbf{u}) + \mathbf{f}\,, \\
P &= C_s^2 \rho
\end{aligned}
$$

E^u does not scale as $k^{-5/3}$: it is steeper !

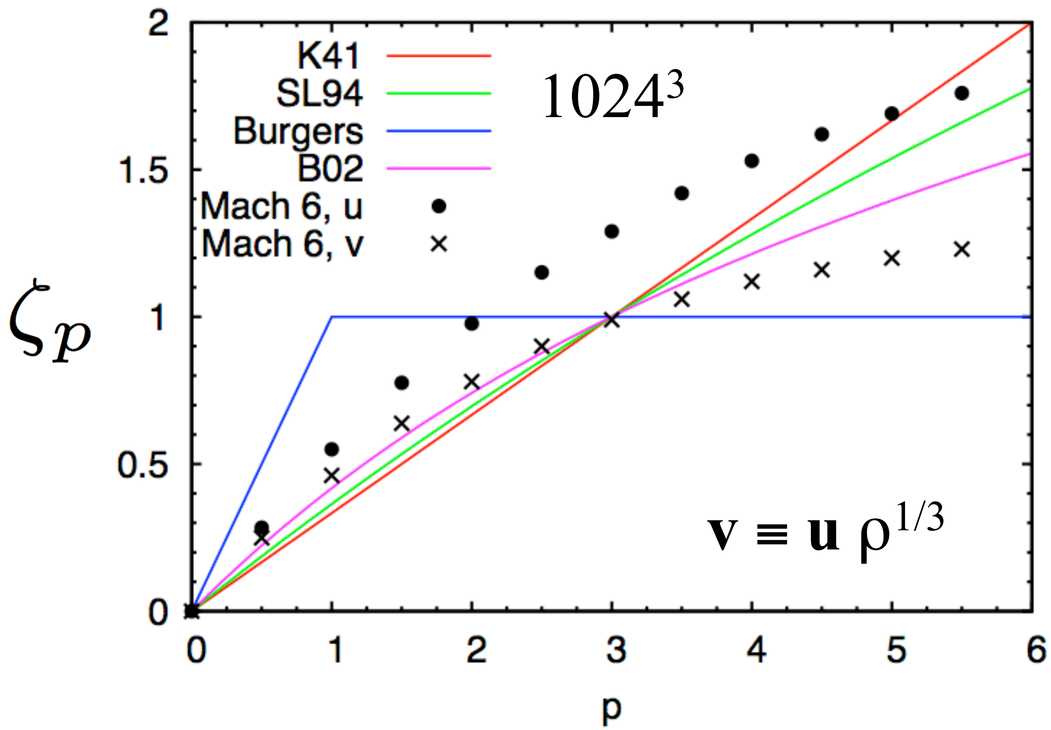
However if: $\mathbf{v} \equiv \mathbf{u}\, \rho^{1/3}$

then: $E^v \sim k^{-5/3}$

WHY ??

Compressible isothermal HD turbulence

$$S_p(\ell) \equiv \langle (\delta u_\ell)^p \rangle = C_p(\varepsilon \ell)^{\zeta_p}$$



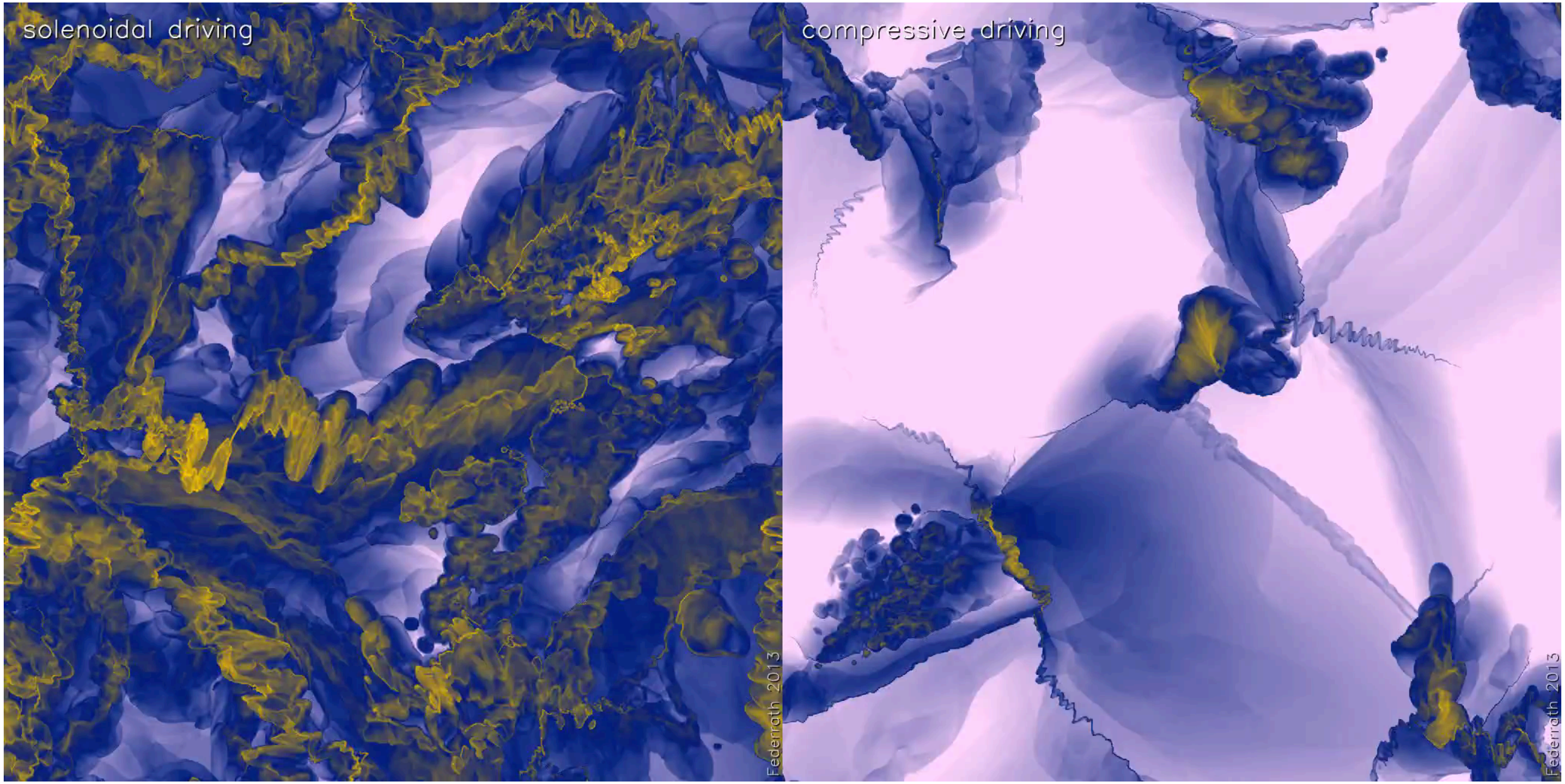
3D numerical simulations

[Kritsuk et al., ApJ, 2007]

A universal behavior seems to emerge from the density-weighted velocity: $\mathbf{v} \equiv \mathbf{u}\, \rho^{1/3}$

Role of the forcing

Density variation

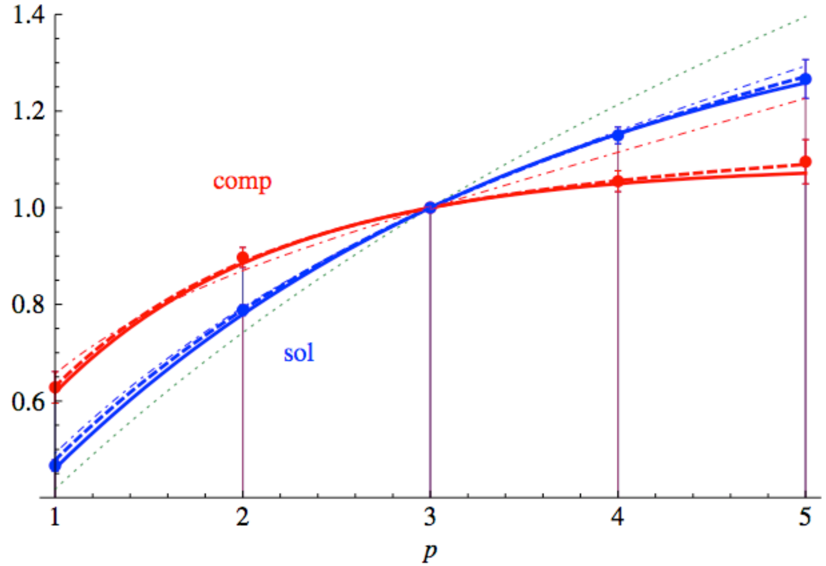


4096^3 Turbulent Mach = 17

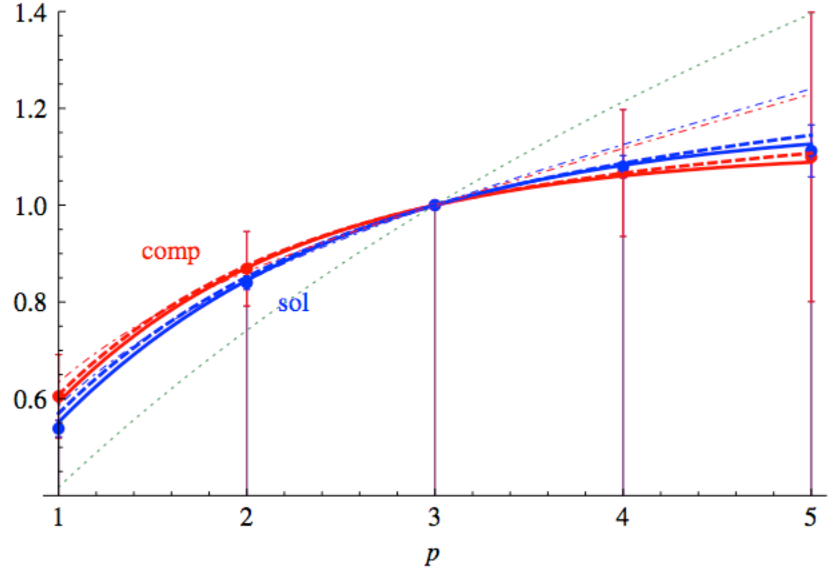
[Federrath, MNRAS, 2013]

Role of the forcing

$$\zeta_p/\zeta_3$$



$\langle \delta v_\ell^p \rangle$



$\langle \delta(\rho^{1/3} v)_\ell^p \rangle$

[Schmidt et al., PRL, 2008]

Exact relation for compressible turbulence

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0,$$
$$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla P + \mu \Delta \mathbf{u} + \frac{\mu}{3} \nabla(\nabla \cdot \mathbf{u}) + \mathbf{f},$$

$$\begin{cases} E = \rho u^2/2 + \rho e \\ e = C_s^2 \ln(\rho/\rho_0) \end{cases}$$

- Isothermal closure: $P = C_s^2 \rho$
- Homogeneity; stationarity for E: $\partial_t \langle E \rangle = -\mu \langle (\nabla \times \mathbf{u})^2 \rangle - \frac{4}{3} \mu \langle (\nabla \cdot \mathbf{u})^2 \rangle + F$
- Forcing localized at large-scale
- Dissipation negligible in the inertial range

} [Aluie, Phys. D, 2013]

- ε (the mean energy dissipation rate) becomes constant when Re → +∞
- Two-points correlation function: $\mathcal{R}(\mathbf{r}) \equiv \langle \rho \mathbf{u} \cdot \mathbf{u}'/2 + \rho e' \rangle \equiv \langle R \rangle$

Exact relation for compressible turbulence

$$-2\varepsilon = \nabla_{\mathbf{r}} \cdot \left\langle \left[\frac{\delta(\rho \mathbf{u}) \cdot \delta \mathbf{u}}{2} + \delta\rho\delta e - C_s^2 \bar{\delta}\rho \right] \delta \mathbf{u} + \bar{\delta} e \delta(\rho \mathbf{u}) \right\rangle + \langle (\nabla' \cdot \mathbf{u}')(R - E) \rangle + \langle (\nabla \cdot \mathbf{u})(\tilde{R} - E') \rangle$$

$\bar{\delta} f = (f' + f)/2$

$\delta f = f' - f$

[SG & Banerjee, PRL, 2011]

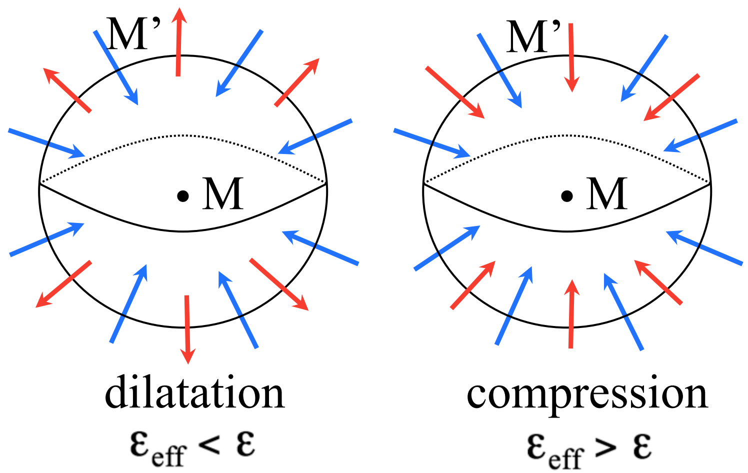
$\Updownarrow$

$$-2\varepsilon = \nabla.F + S$$

ISOTROPIC TURBULENCE :

$\varepsilon_{eff} = \varepsilon + S/2$

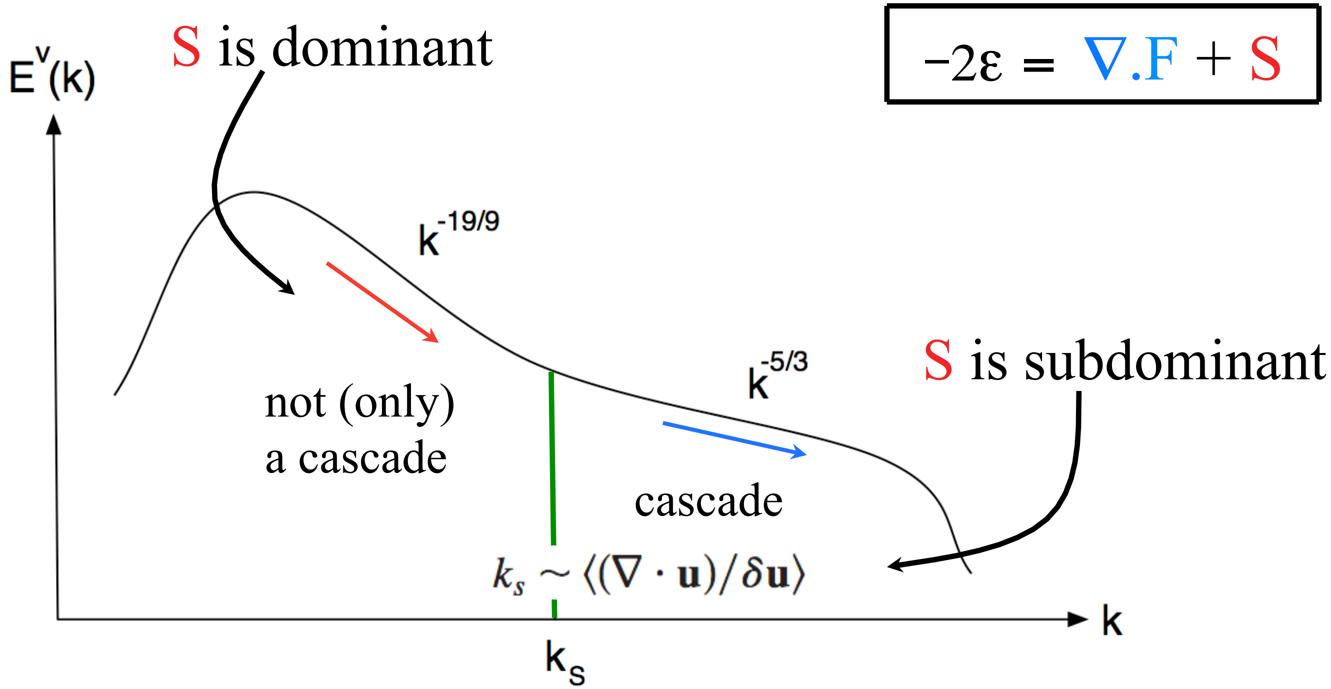
$-2\varepsilon_{eff} = \nabla.F$



Phenomenology

$v \equiv u\, \rho^{1/3}$

$-2\varepsilon = \nabla.F + S$

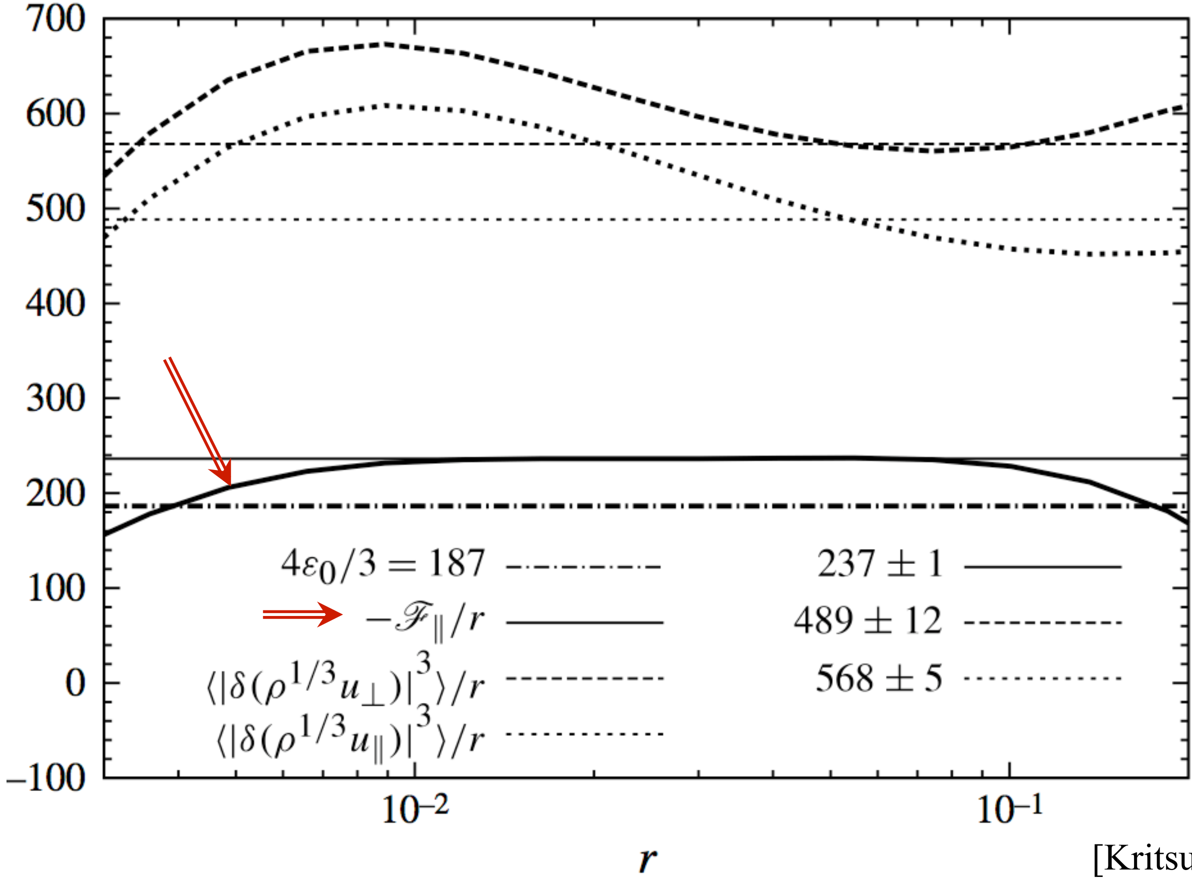


$$E^v(k) \sim \varepsilon_{\text{eff}}^{2/3} k^{-5/3}$$

Comparison with 3D simulations

$-2\varepsilon = \nabla . F + S$

$$F_{\parallel}(r) \approx \mathscr{F}_{\parallel}(r) \equiv \langle \delta(\rho \boldsymbol{u}) \cdot \delta \boldsymbol{u}\, \delta u_{\parallel} \rangle$$



Mach = 6

1024^3

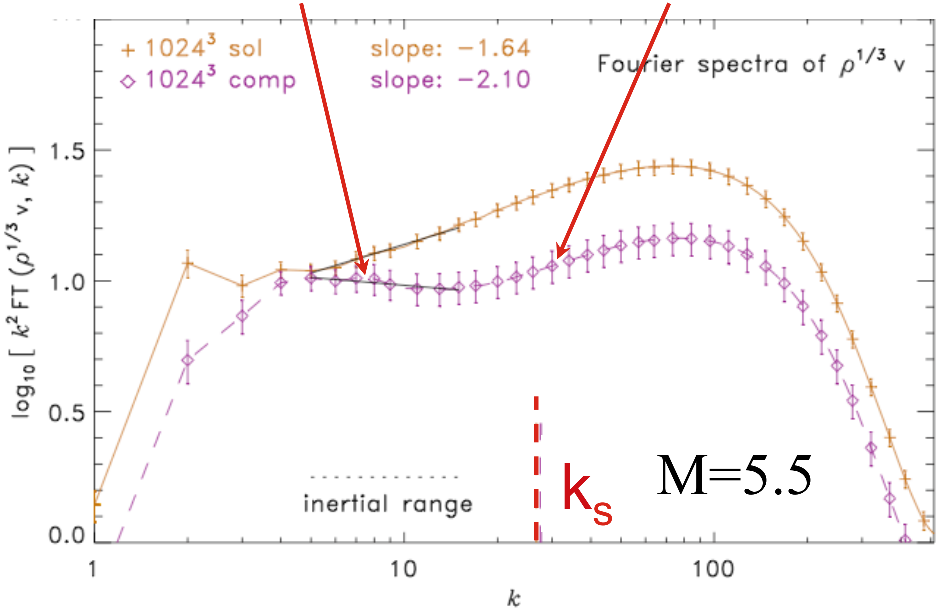
[Kritsuk et al., JFM Rapids, 2013]

The source term S is subdominant

Comparison with 3D simulations

$E^v \sim k^{-2.1}$ $E^v \sim k^{-5/3}$

$\mathbf{v} \equiv \mathbf{u}\,\rho^{1/3}$



[Federrath et al., A&A, 2010]

Sonic scale k_s may be defined as : $\int_{k_s}^{k_c} E(k)\mathrm{d}k \simeq \frac{1}{2}\langle c_s \rangle^2$

- The exact relation gives a universal definition for k_s
- DNS in agreement with the theory (-19/9 = -2.11)

Exact relation for isothermal compressible MHD turbulence

[Banerjee & SG, PRE, 2013]

$$
\begin{aligned}
-2\varepsilon = {} & \frac{1}{2}\boldsymbol{\nabla}_r \cdot \left\langle \left[\frac{1}{2}\delta(\rho\mathbf{z}^-)\cdot\delta\mathbf{z}^- + \delta\rho\delta e\right]\delta\mathbf{z}^+ + \left[\frac{1}{2}\delta(\rho\mathbf{z}^+)\cdot\delta\mathbf{z}^+ + \delta\rho\delta e\right]\delta\mathbf{z}^- + \bar{\delta}\left(e + \frac{v_A^2}{2}\right)\delta(\rho\mathbf{z}^- + \rho\mathbf{z}^+)\right\rangle \\
& - \frac{1}{8}\left\langle \frac{1}{\beta'}\boldsymbol{\nabla}'\cdot(\rho\mathbf{z}^+ e') + \frac{1}{\beta}\boldsymbol{\nabla}\cdot(\rho'\mathbf{z}'^+ e) + \frac{1}{\beta'}\boldsymbol{\nabla}'\cdot(\rho\mathbf{z}^- e') + \frac{1}{\beta}\boldsymbol{\nabla}\cdot(\rho'\mathbf{z}'^- e)\right\rangle \\
& + \left\langle(\boldsymbol{\nabla}\cdot\mathbf{v})\left[R_E' - E' - \frac{\bar{\delta}\rho}{2}(\mathbf{v_A}'\cdot\mathbf{v_A}) + \frac{P_M' - P'}{2}\right]\right\rangle + \left\langle(\boldsymbol{\nabla}'\cdot\mathbf{v}')\left[R_E - E - \frac{\bar{\delta}\rho}{2}(\mathbf{v_A}\cdot\mathbf{v_A}') + \frac{P_M - P}{2}\right]\right\rangle \\
& + \langle(\boldsymbol{\nabla}\cdot\mathbf{v_A})[R_H - R_H' + H' - \bar{\delta}\rho(\mathbf{v}'\cdot\mathbf{v_A})]\rangle + \langle(\boldsymbol{\nabla}'\cdot\mathbf{v_A}')[R_H' - R_H + H - \bar{\delta}\rho(\mathbf{v}\cdot\mathbf{v_A}')]\rangle\,,
\end{aligned}
$$

Exact relation for isothermal compressible MHD turbulence

[Banerjee & SG, PRE, 2013]

$$-2\varepsilon = \frac{1}{2}\nabla_r \cdot \left\langle \left[\frac{1}{2}\delta(\rho \mathbf{z}^-)\cdot\delta\mathbf{z}^- + \delta\rho\delta e\right]\delta\mathbf{z}^+ + \left[\frac{1}{2}\delta(\rho \mathbf{z}^+)\cdot\delta\mathbf{z}^+ + \delta\rho\delta e\right]\delta\mathbf{z}^- + \bar{\delta}\left(e + \frac{v_A^2}{2}\right)\delta(\rho\mathbf{z}^- + \rho\mathbf{z}^+)\right\rangle$$

$$-\frac{1}{8}\left\langle \frac{1}{\beta'}\nabla'\cdot(\rho\mathbf{z}^+ e') + \frac{1}{\beta}\nabla\cdot(\rho'\mathbf{z}'^+ e) + \frac{1}{\beta'}\nabla'\cdot(\rho\mathbf{z}^- e') + \frac{1}{\beta}\nabla\cdot(\rho'\mathbf{z}'^- e)\right\rangle$$

$$+\left\langle(\nabla\cdot\mathbf{v})\left[R'_E - E' - \frac{\bar{\delta}\rho}{2}(\mathbf{v_A}'\cdot\mathbf{v_A}) + \frac{P'_M - P'}{2}\right]\right\rangle + \left\langle(\nabla'\cdot\mathbf{v}')\left[R_E - E - \frac{\bar{\delta}\rho}{2}(\mathbf{v_A}\cdot\mathbf{v_A}') + \frac{P_M - P}{2}\right]\right\rangle$$

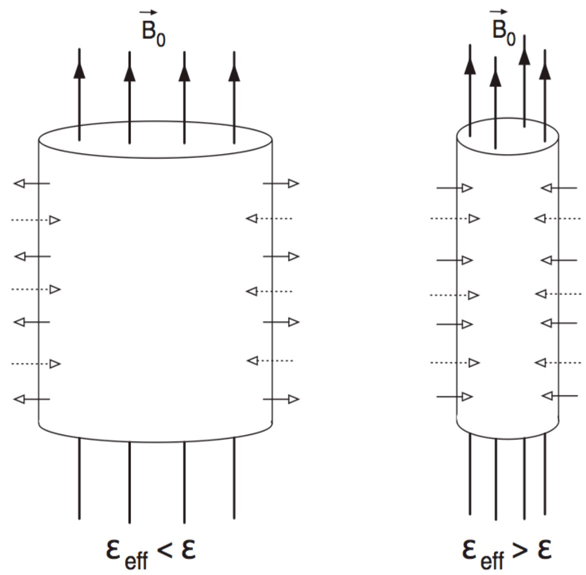
$$+\langle(\nabla\cdot\mathbf{v_A})[R_H - R'_H + H' - \bar{\delta}\rho(\mathbf{v}'\cdot\mathbf{v_A})]\rangle + \langle(\nabla'\cdot\mathbf{v_A}')[R'_H - R_H + H - \bar{\delta}\rho(\mathbf{v}\cdot\mathbf{v_A}')]\rangle\,,$$

Not easy to explain !

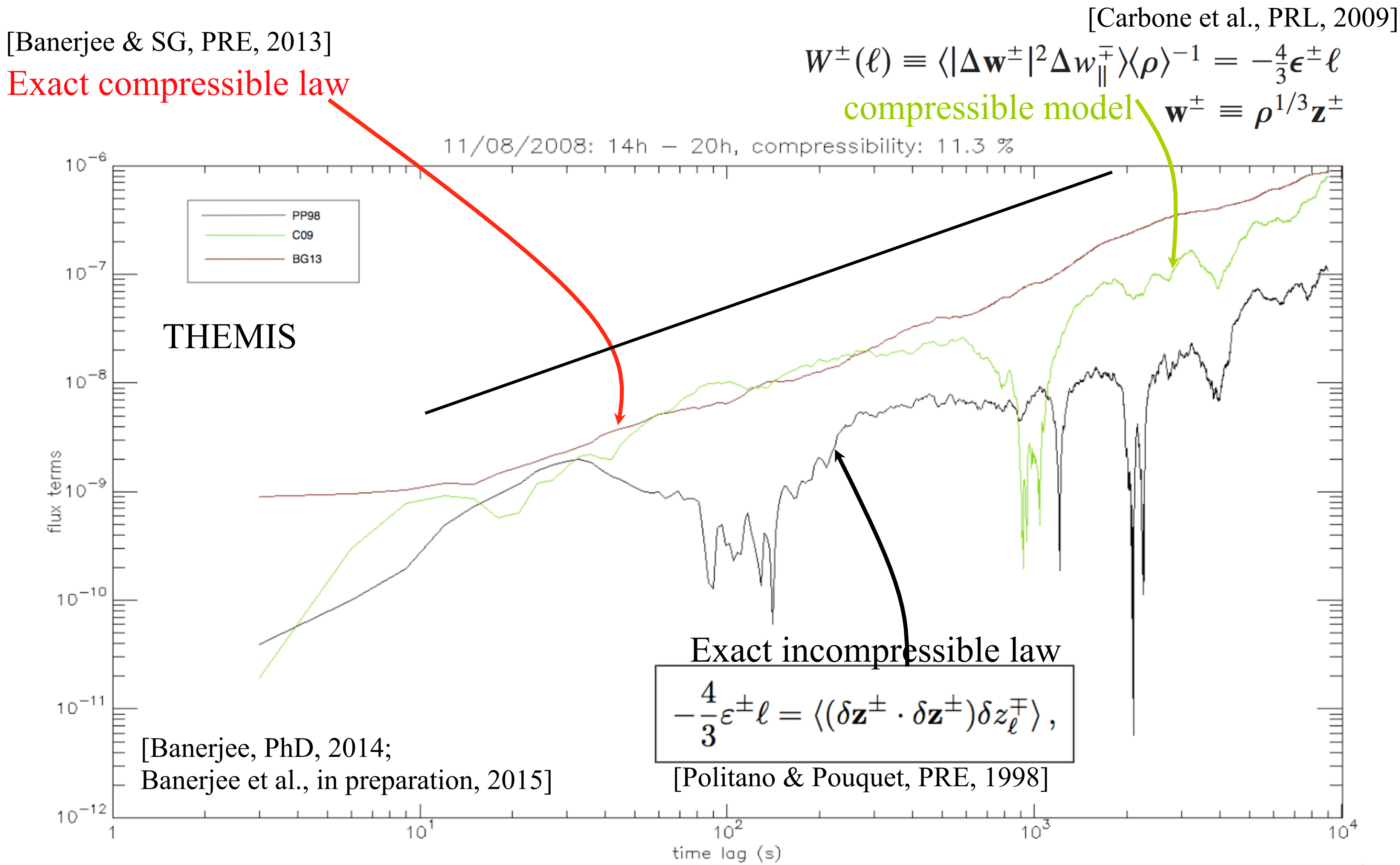
With a strong B_0 :

$$-4\varepsilon = B_0^2\nabla_{r_\perp}\cdot\left\langle\delta\left(\frac{1}{\sqrt{\rho}}\right)\delta(\sqrt{\rho})\delta\mathbf{v}_\perp\right\rangle$$

$$-\frac{B_0^2}{2}\left\langle(\nabla_\perp\cdot\mathbf{v}_\perp)\left(1+\sqrt{\frac{\rho}{\rho'}}\right) + (\nabla'_\perp\cdot\mathbf{v}'_\perp)\left(1+\sqrt{\frac{\rho'}{\rho}}\right)\right\rangle$$



Fast solar wind data

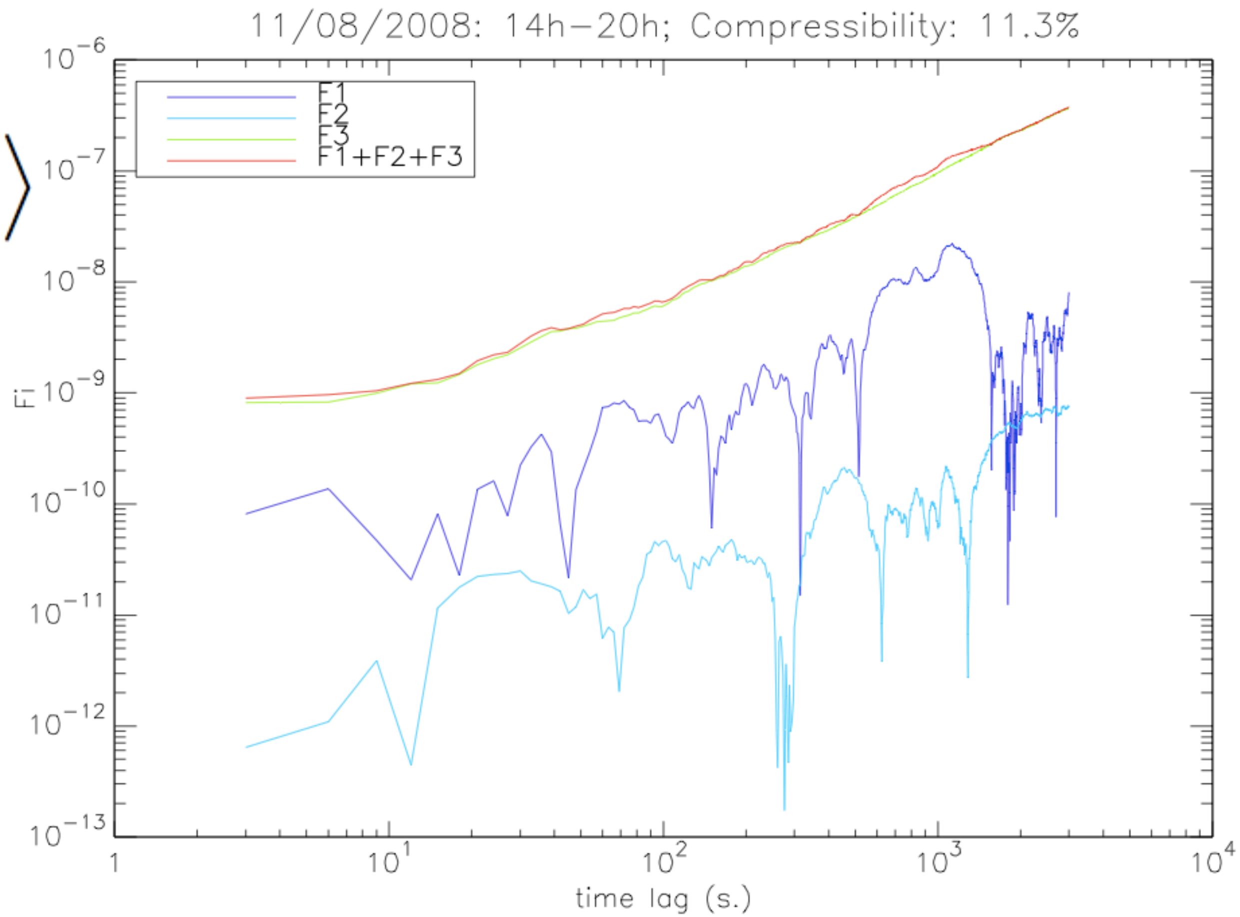


Fast solar wind data

$$F_1 = \left\langle \tfrac{1}{2}\left[\delta(\rho \mathbf{z}^-)\cdot\delta\mathbf{z}^-\right]\delta\mathbf{z}^+ + \tfrac{1}{2}\left[\delta(\rho\mathbf{z}^+)\cdot\delta\mathbf{z}^+\right]\delta\mathbf{z}^- \right\rangle$$

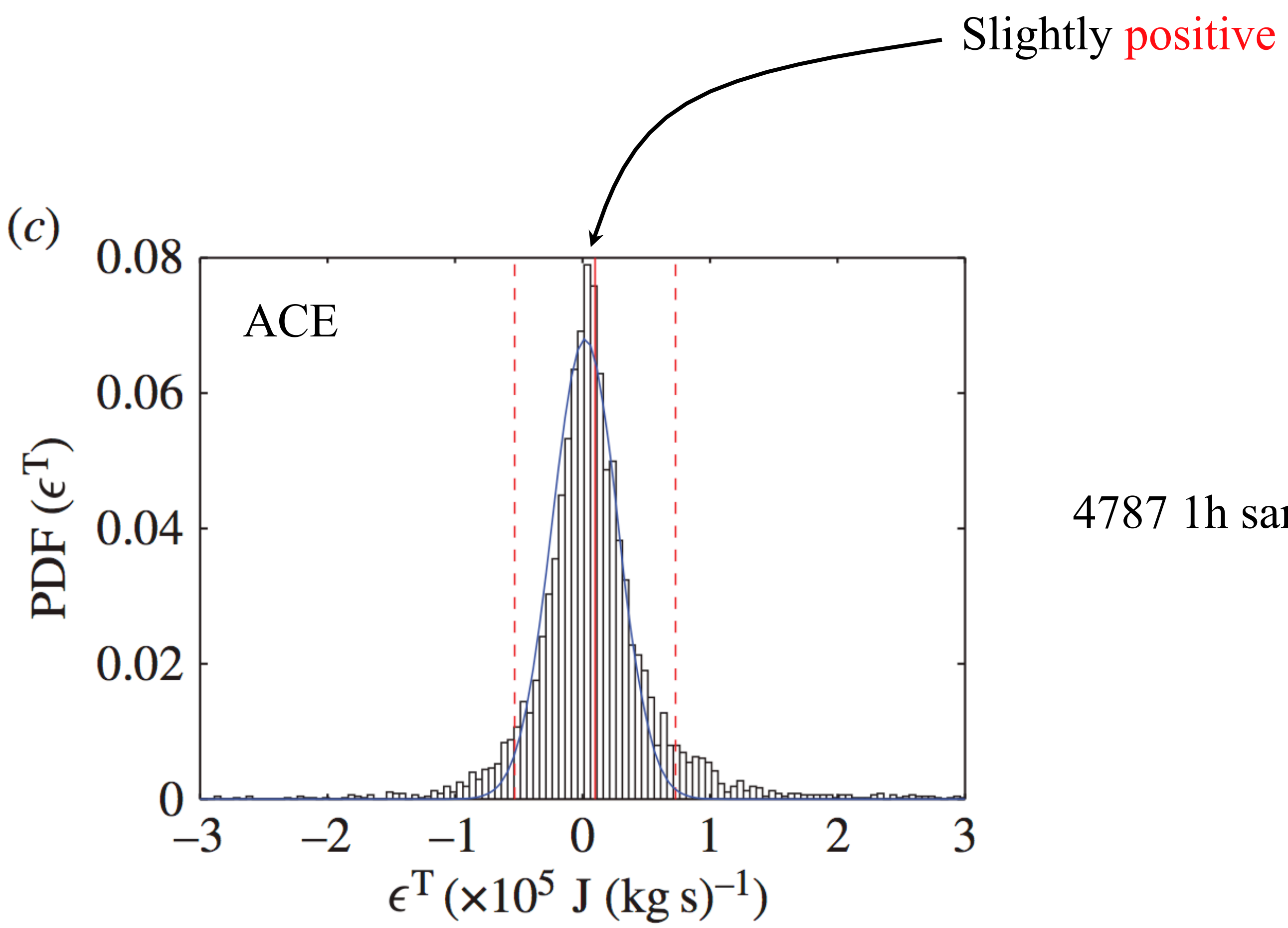
$$F_2 = \langle 2\delta\rho\delta e\delta\mathbf{v}\rangle$$

$$F_3 = \left\langle 2\bar{\delta}\left(e + \frac{v_A^2}{2}\right)\delta(\rho\mathbf{v})\right\rangle$$



[Banerjee et al., in preparation, 2015]

Statistical convergence



4787 1h samples

[Coburn et al., Phil. Trans. R. Soc. A, 2015]

Statistical convergence

[Taylor et al., PRE, 2003]