Hilbert-Huang and Morlet wavelet transformation

Sonny Lion (sonny.lion@obspm.fr)

LESIA, Observatoire de Paris

UPMC SORBONNE UNIVERSITÉS

l'Observatoire de Paris LESIA

Laboratoire d'Études Spatiales et d'Instrumentation en Astrophysique

The Hilbert-Huang Transform

The main objective of this talk is to serve as a guide for understanding, implementing and using the Hilbert-Huang transform.

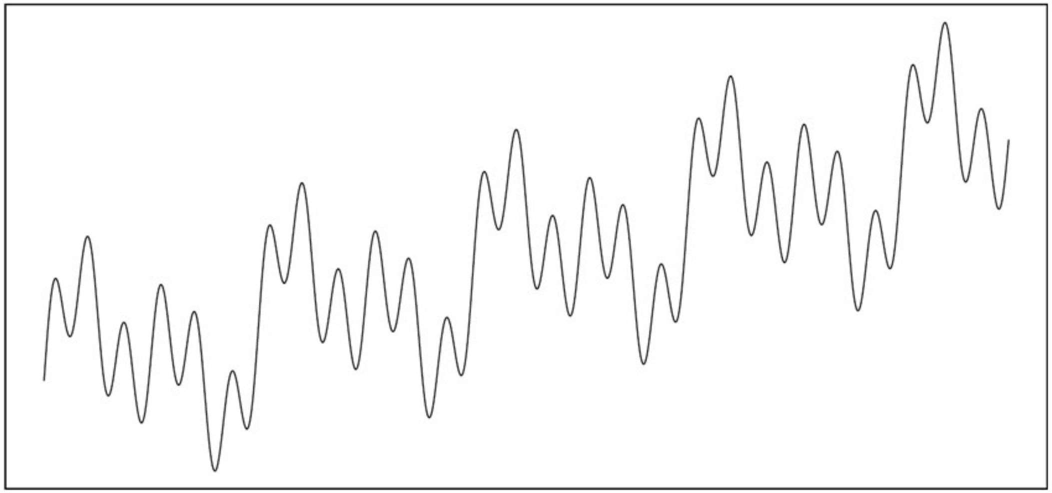
But why use HHT?

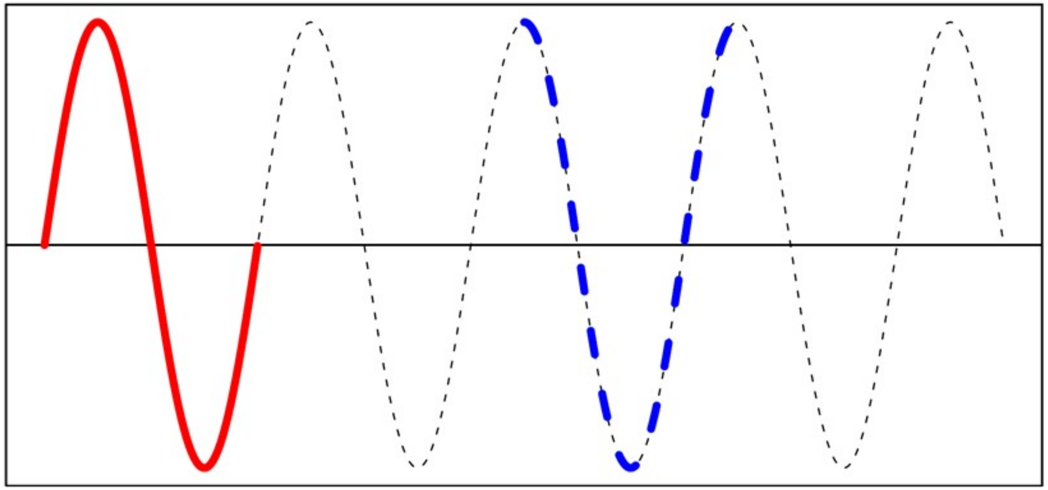
A comparative summary of Fourier, wavelet and HHT analyses is given in the following table:

	Fourier	Wavelet	Hilbert
Basis	*a priori*	*a priori*	adaptive
Frequency	convolution: global uncertainty	convolution: regional uncertainty	differentiation: local, certainty
Presentation	energy-frequency	energy-time-frequency	energy-time-frequency
Nonlinear	no	no	yes
Nonstationary	no	yes	yes
Feature Extraction	no	discrete: no; continuous: yes	yes
Theoretical base	theory complete	theory complete	empirical

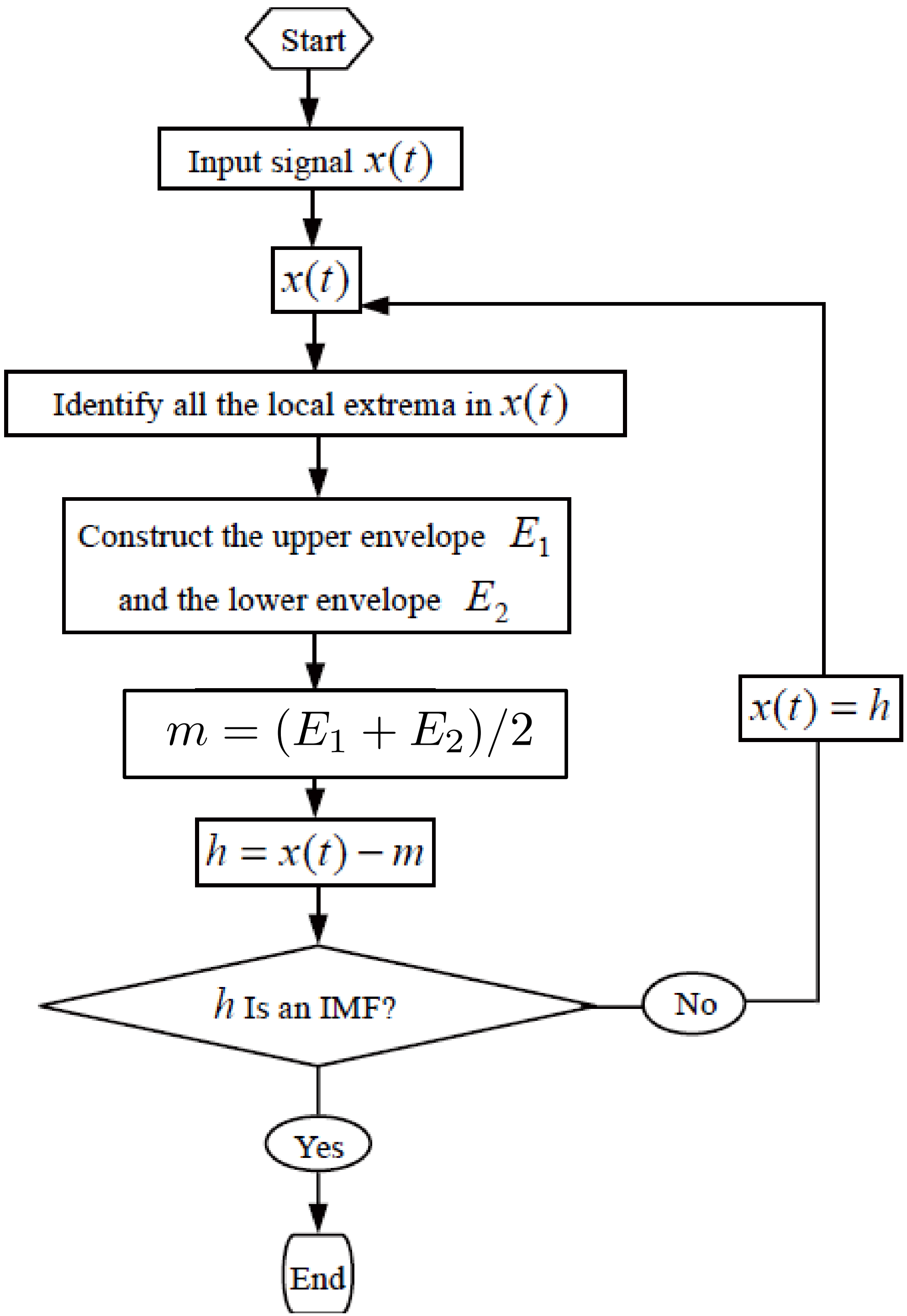
Intrinsic mode function

- The essential step extracting an IMF is to identify the intrinsic oscillation of the signal





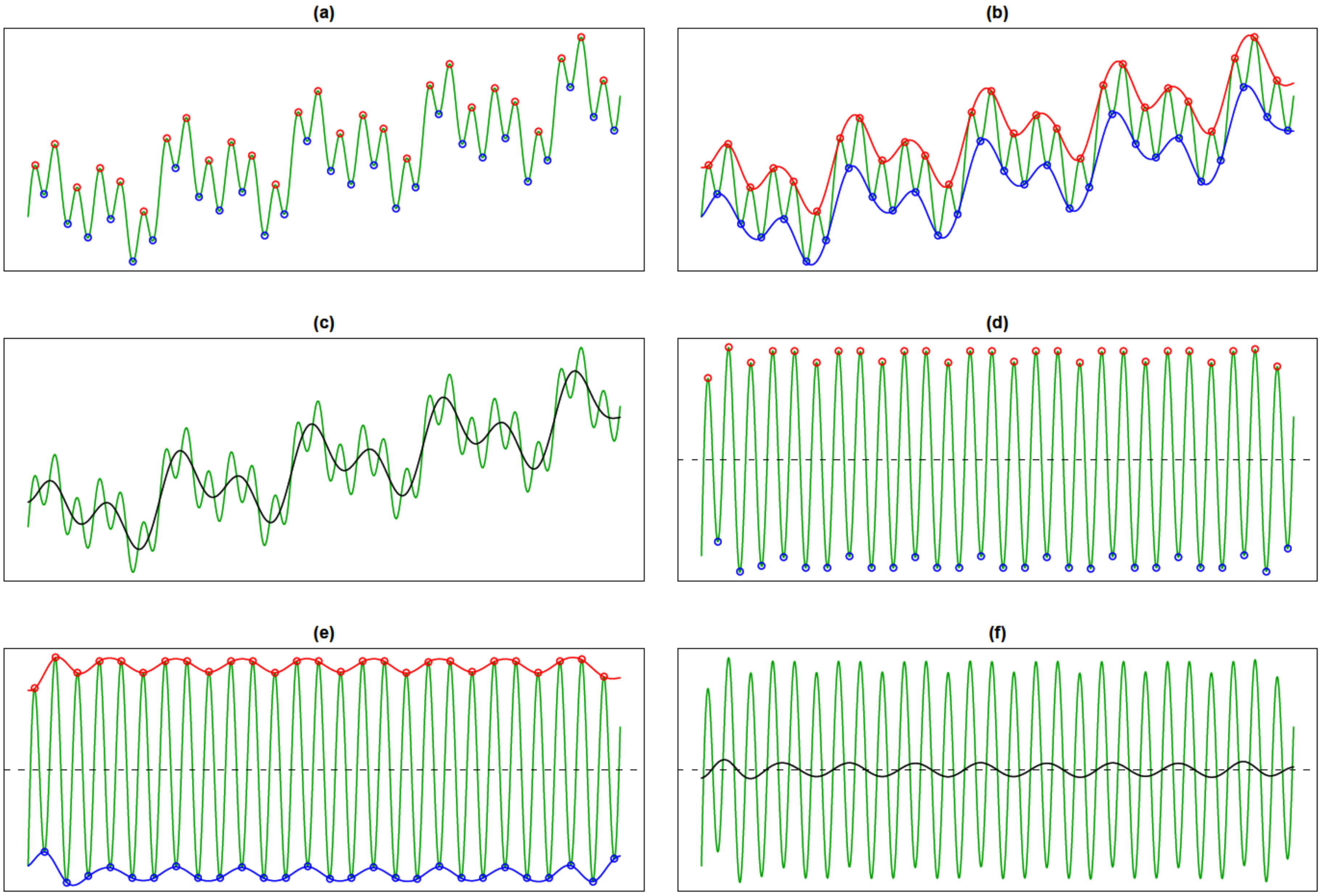
- An IMF can be defined as follows:
  - Its number of extrema and zero-crossings must be equal or differ at most by one
  - At any point, the mean value of its envelopes defined by the local maxima and the local minima should be zero.



Sifting process (part 1)

- Iterative process
- Envelopes are determined using interpolation between extrema
- Spline interpolation gives the best results

Sifting process (part 2)



Stopping criterion

- The purpose of the stopping criterion ideally is to end the sifting process when a proto-IMF verifies the two conditions of IMF but in reality we use approximate stopping criterion:

1. Based on the standard deviation (SD), computed from two consecutive sifting results

$$SD(h_{j(k-1)}, h_{jk}) = \sum_{t=0}^{T} \left[\frac{|(h_{j(k-1)}(t) - h_{jk}(t))|^2}{h^2_{j(k-1)}(t)} \right]$$

2. The IMF is chosen as the first proto-IMF of a series of S consecutive iterations which successfully verify the first IMF-requirement (the numbers of zero-crossings and extrema of the proto-IMF differ at most by one.)

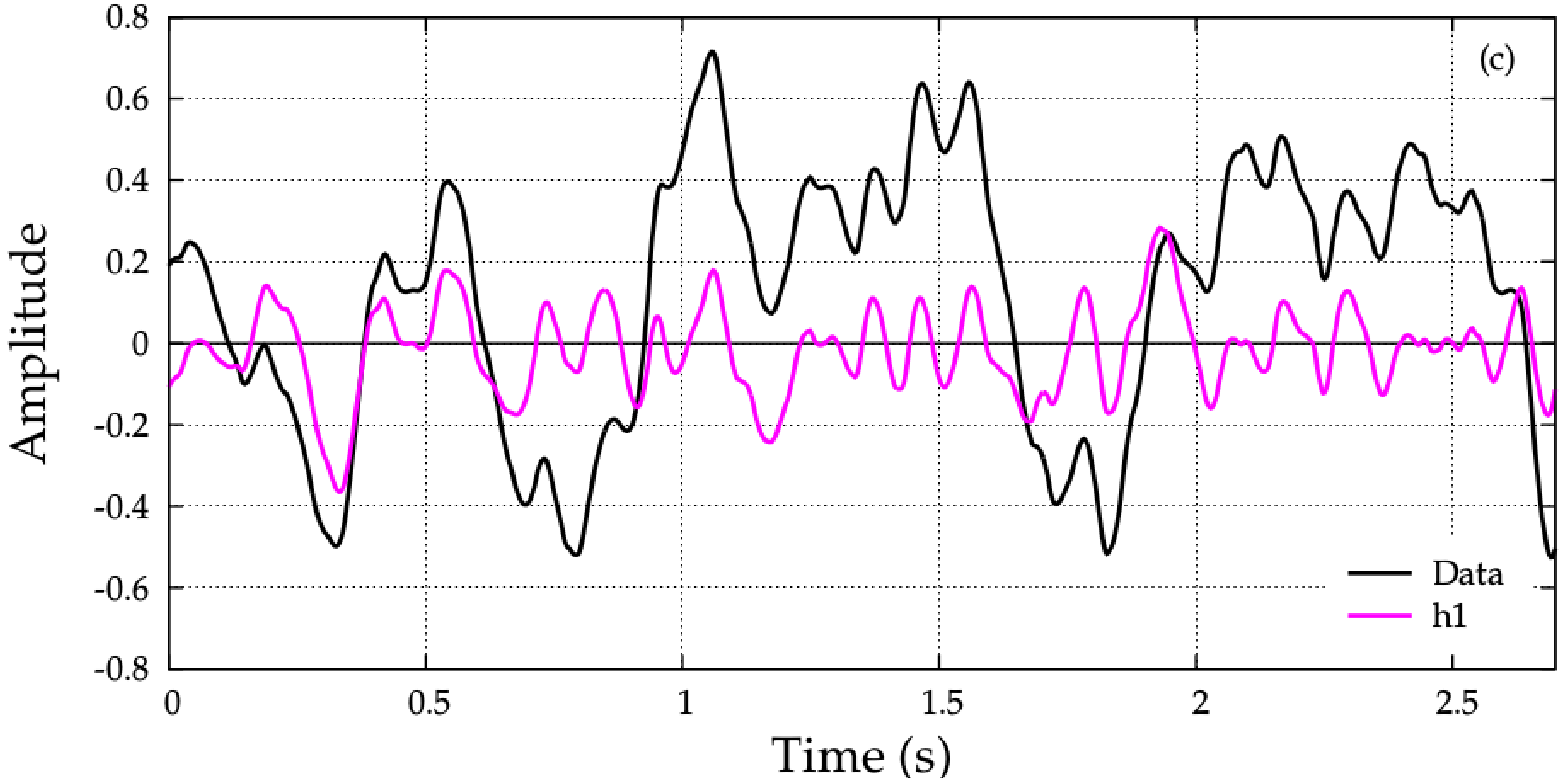
3. The sifting process is stopped after a predetermined number M of iterations, regardless of the two requirements

4. The most complete stop criterion, the sifting process is stopped if both the two following conditions are satisfied [Rilling et al. (2003)]:

   - the numbers of zero-crossings and extrema of the proto-IMF differ at most by one. (This is simply the first condition of IMF.)
   - the absolute value of the ratio of the mean of the proto-IMF to its mode amplitude is lower than a predetermined threshold for a fraction of the total signal size (condition on the amplitude of a portion of the signal)

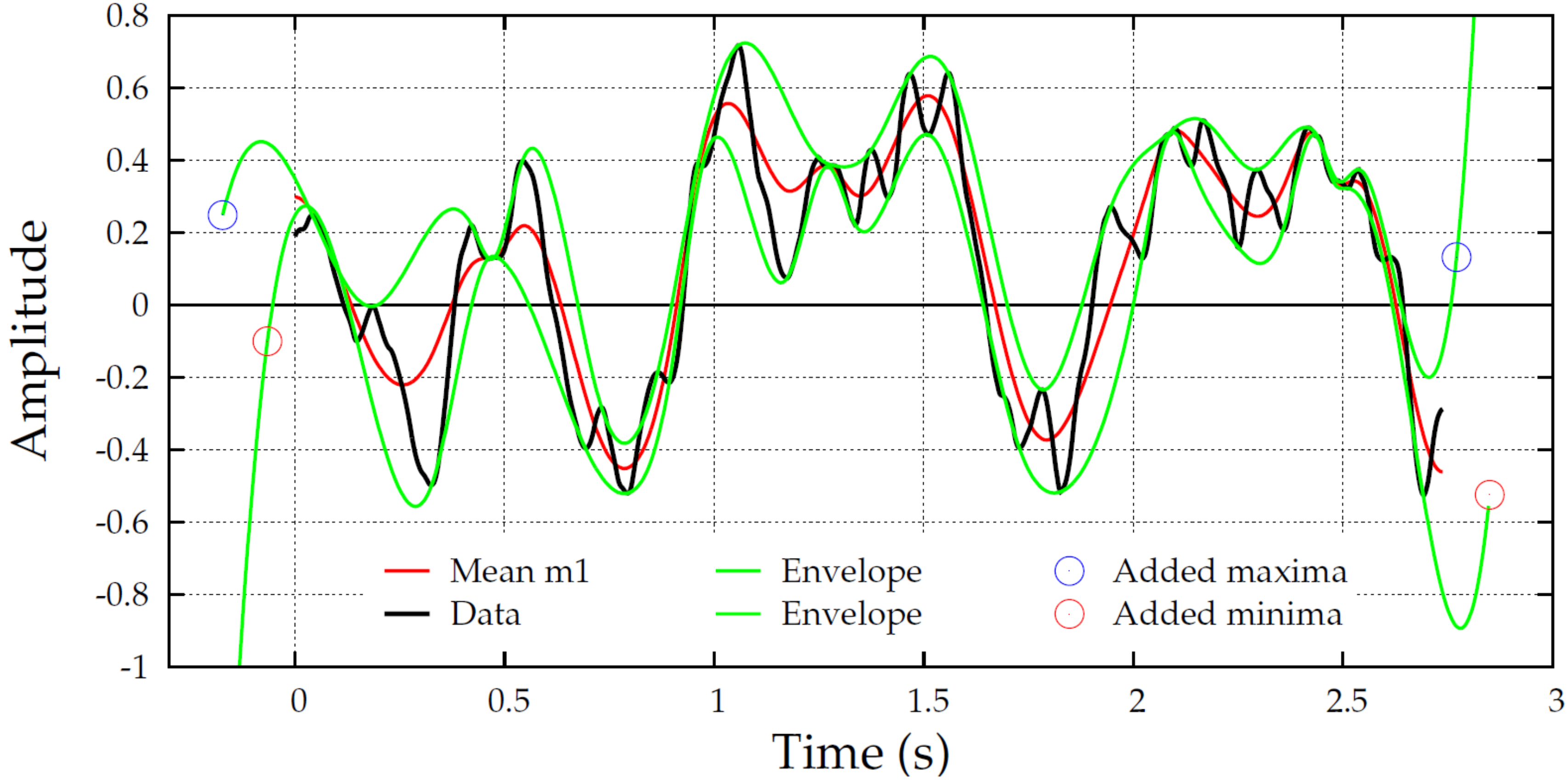
Boundary condition

- The side effects can lead to bad envelop fit, which causes incorrect signal decomposition

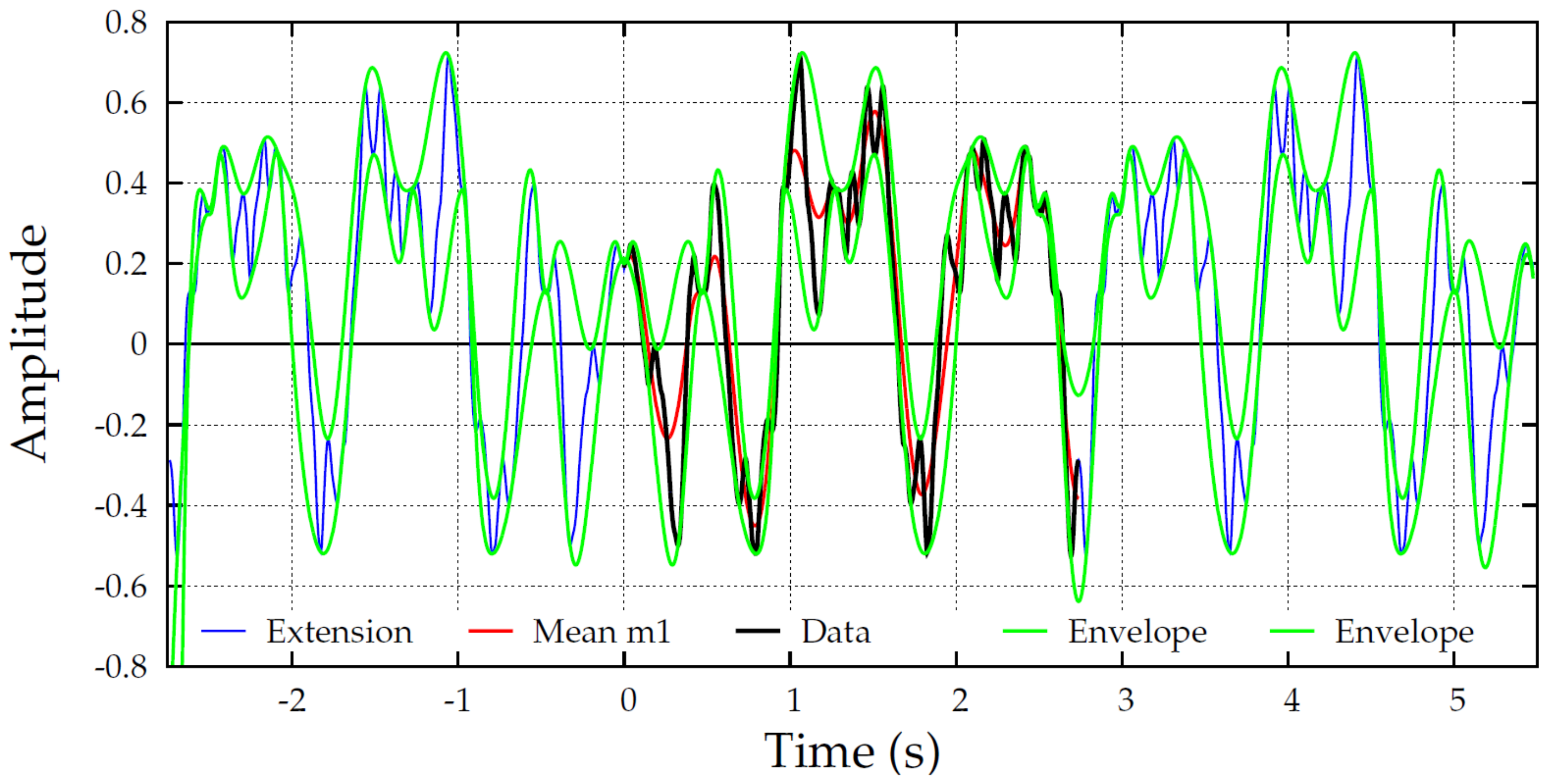


- Here we look at three options to prevent these effects:
  - Extrema extension
  - Mirror extension
  - Wave extension

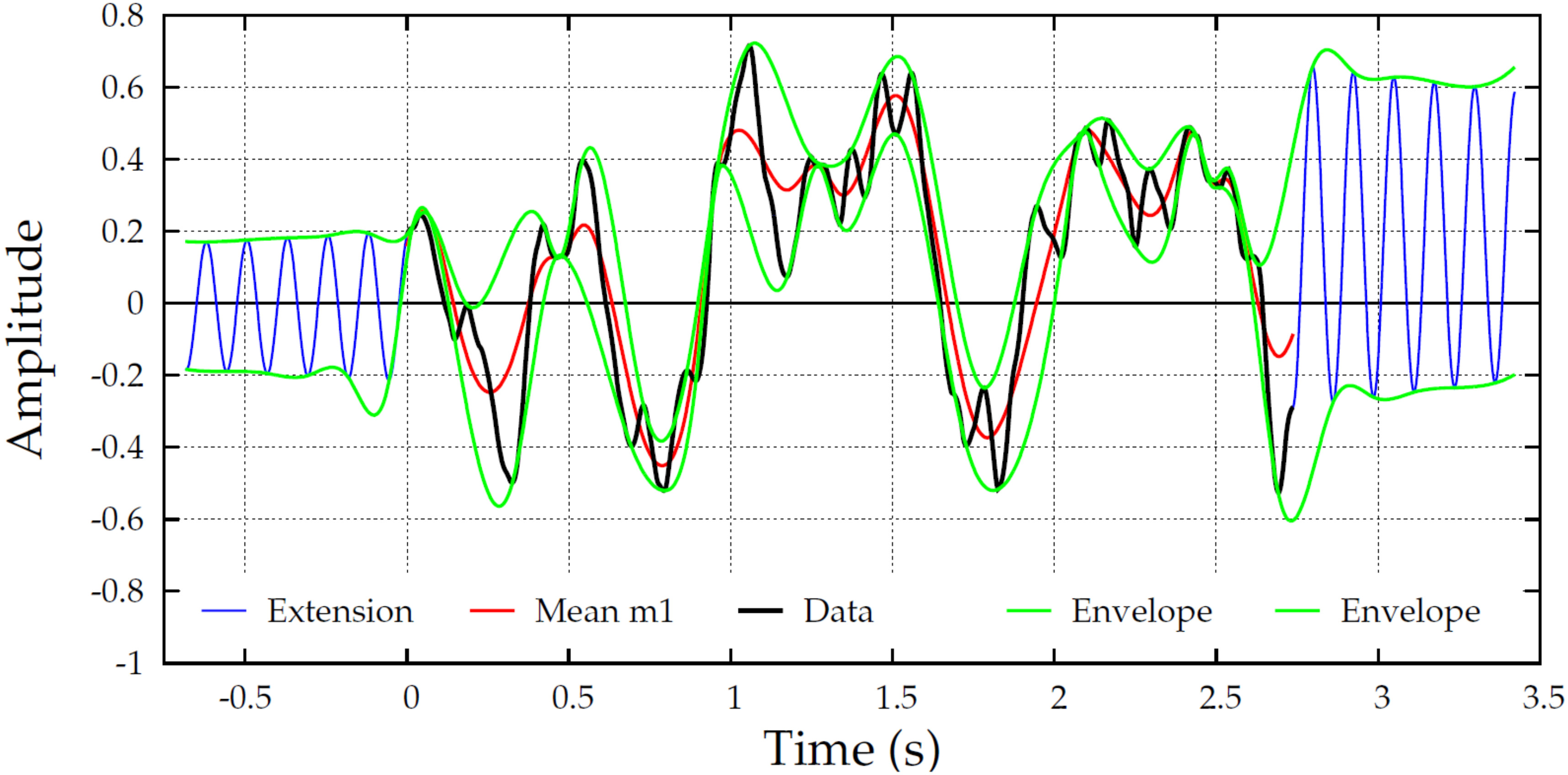
Extrema extension



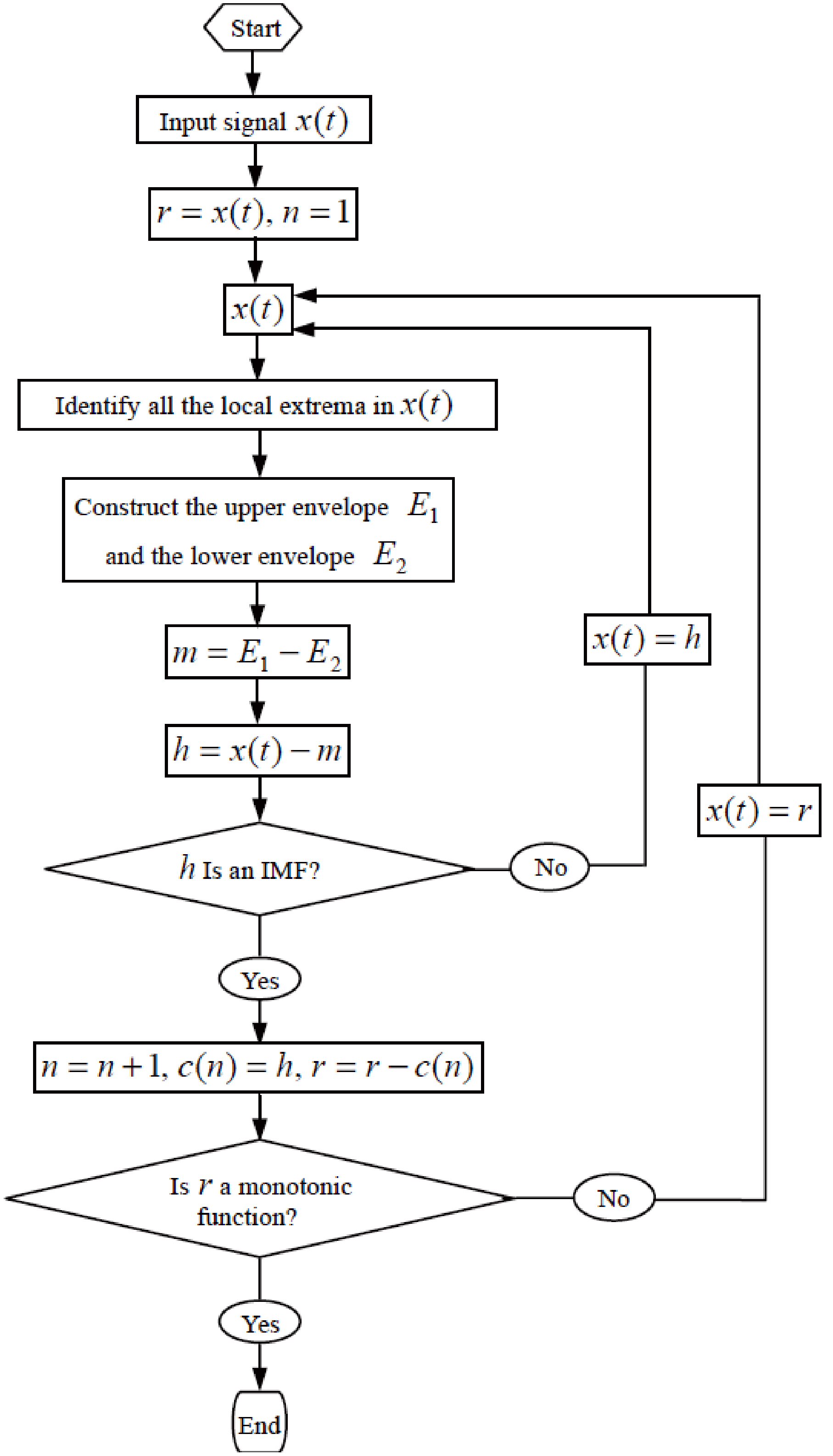
Mirror extension

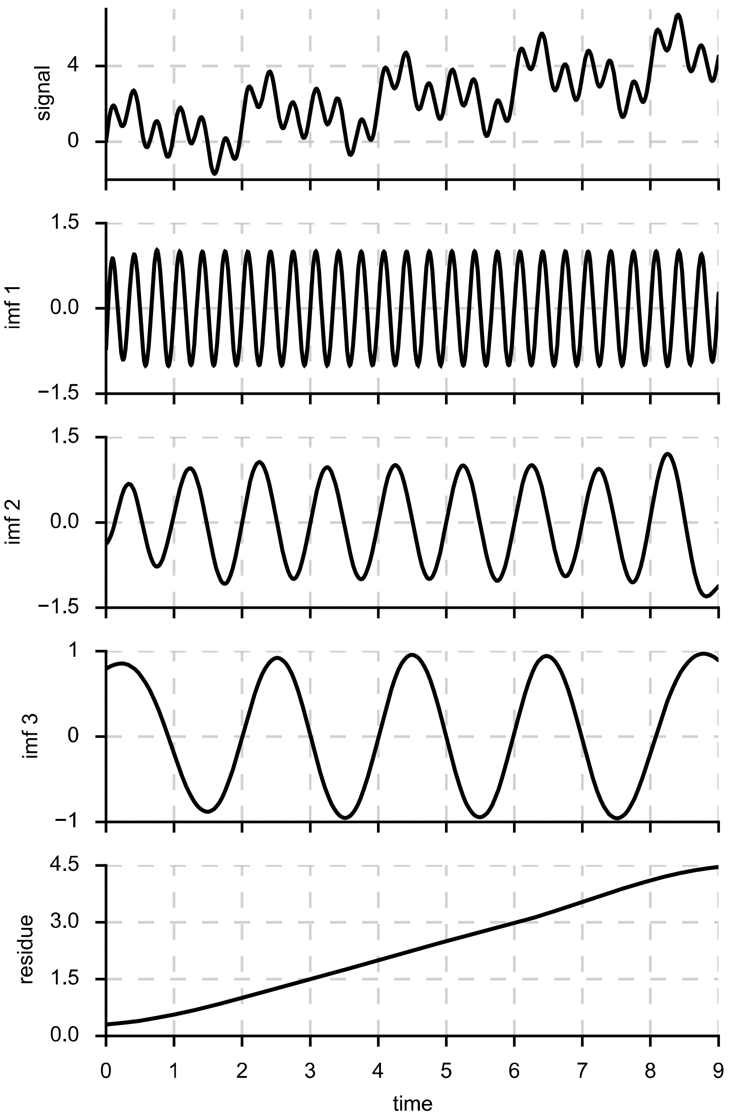


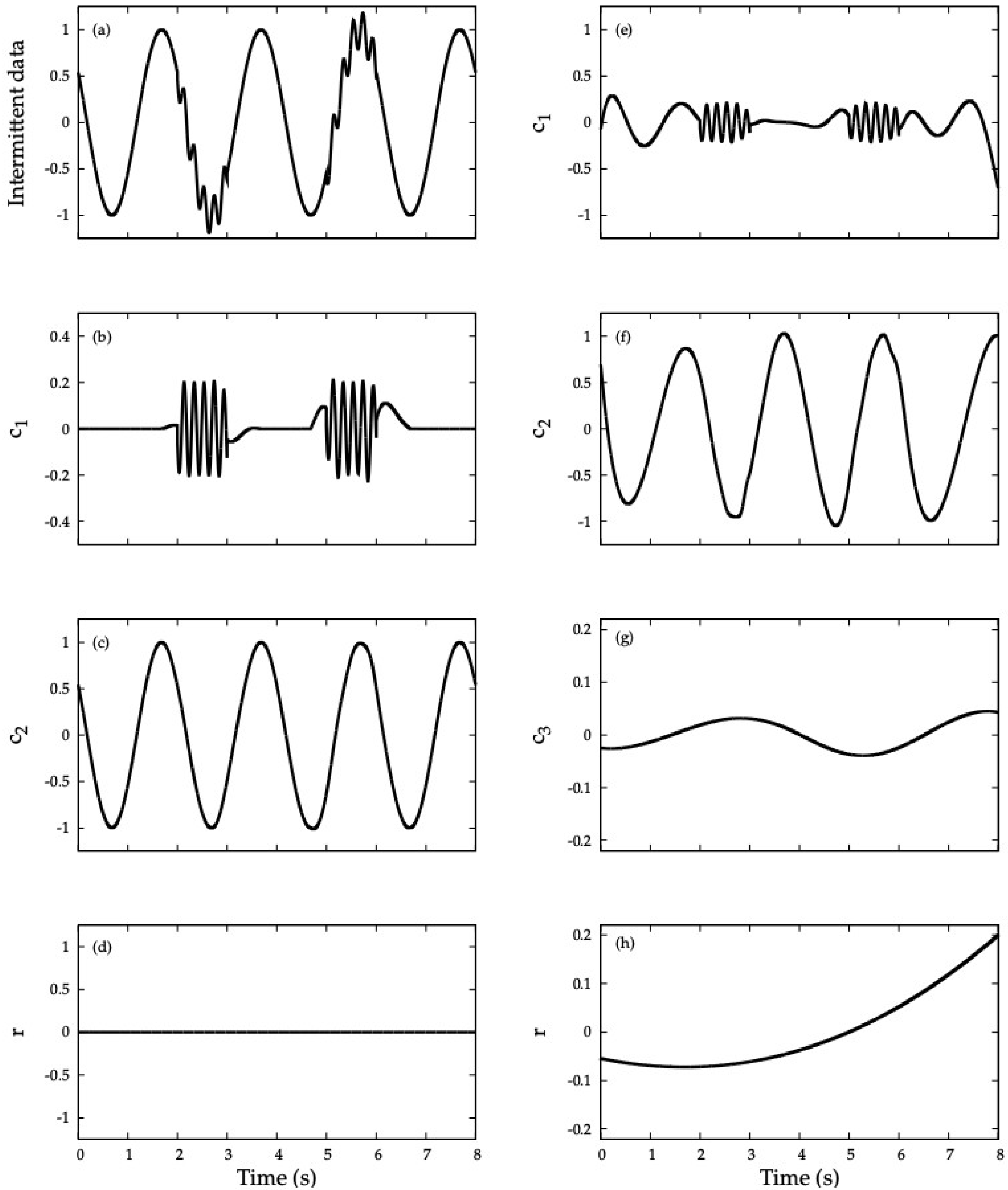
Wave extension



Empirical mode decomposition







Intermittence

- Intermittent high-frequency component can strongly affect the decomposition
- The intermittency test (Huang et al. 1999-2003)
  - We test the distance between two successive extrema.
  - If it is greater than a predetermined value n, then all the data between these two extrema must be discarded from the resulting IMF

Hilbert spectrum

- The analytic signal of x(t) is:

$$x(t) + i\mathcal{H}[x(t)] = a(t)e^{i\theta(t)}$$

- The imaginary part is denoted as:

$$y(t) = \mathcal{H}[x(t)]$$

- The amplitude is then given by:

$$a(t) = \sqrt{x^2(t) + y^2(t)}$$

- And the phase:

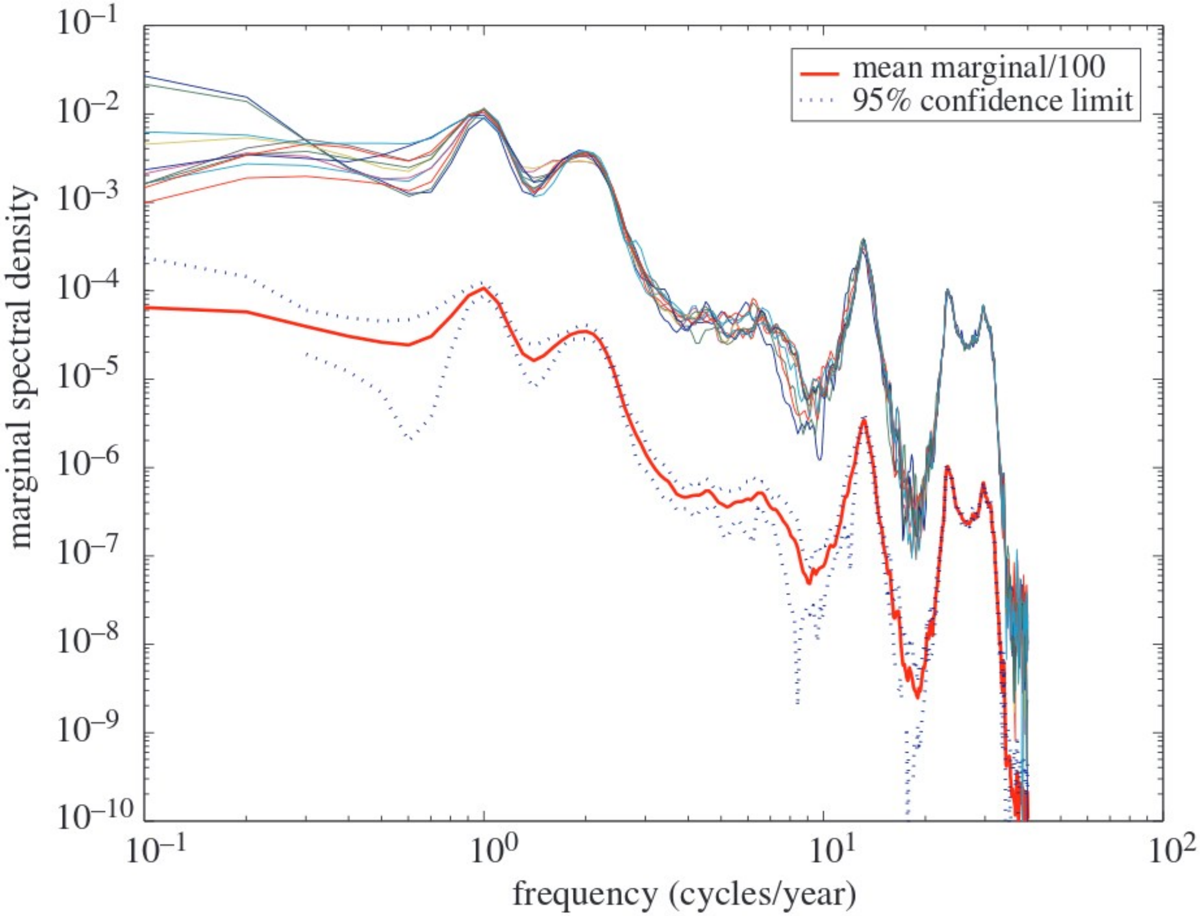
$$\theta(t) = 2\arctan\left(\frac{y(t)}{x(t) + a(t)}\right)$$

- Finally the instantaneous frequency is the first derivative of the phase:

$$\omega(t) = \frac{d\theta(t)}{dt} = \frac{x(t)y'(t) - y(t)x'(t)}{x(t)^2 + y(t)^2}$$

Confidence limit

- Based on the study of Huang et al. (2003)
- Aim: give a quantitative view of the results given by HHT
- Method:
  - Several sets of IMFs are calculated from the same data but with different control parameters
  - The resulting sets are assumed to have equal probability
  - Then, the algorithm calculates the ensemble mean of the sets of IMF and the standard deviation to give a confidence limit



How to improve Empirical Mode Decomposition? (EMD)

- The idea: Several sets of IMFs are calculated from the same data but with different realization of noise, and the resulting sets are assumed to have equal probability.

The Ensemble EMD (EEMD)

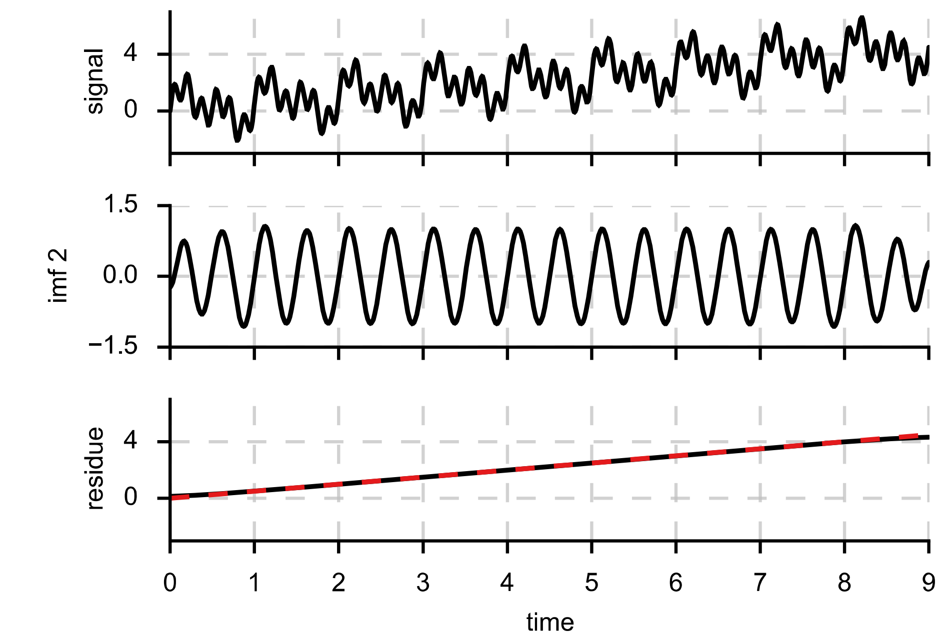
- EEMD defines the true IMF components as the mean of the corresponding IMFs obtained via EMD over an ensemble of trials, generated by adding different realizations of white noise of finite variance to the original signal
- The procedure can be summarize as follow:
  - Generate noisy time series s(t) = x(t) + white_noise(t)
  - Compute associated IMFs
  - Average all the IMFs

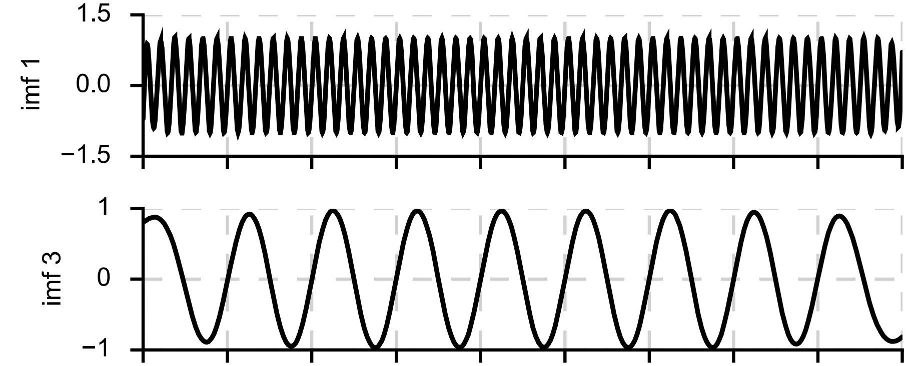
The Complete Ensemble EMD (CEEMD)

- Same kind of procedure BUT we use the mean imf to compute the residue at each step of the algorithm

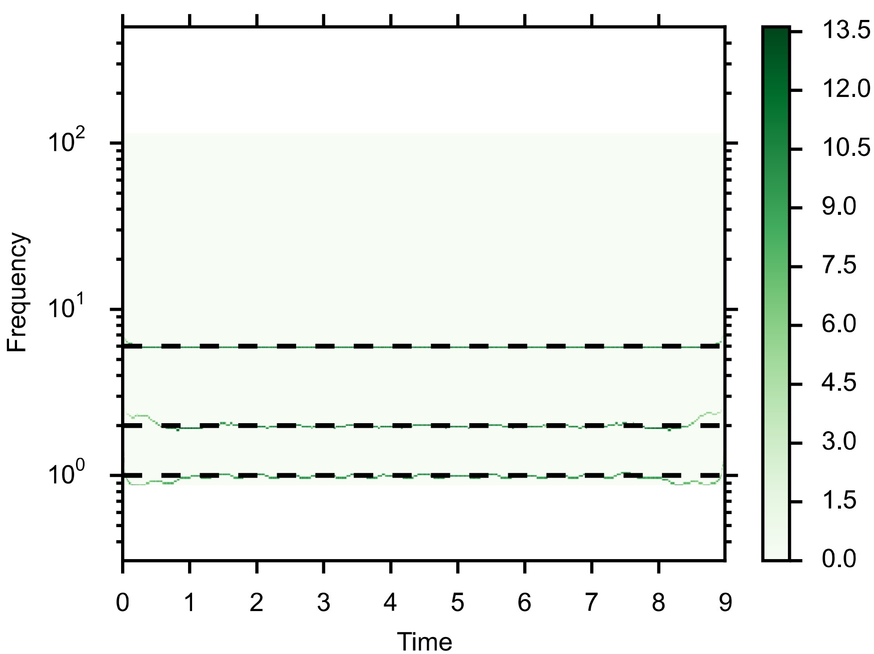
Comparison between HHT and Wavelet transform

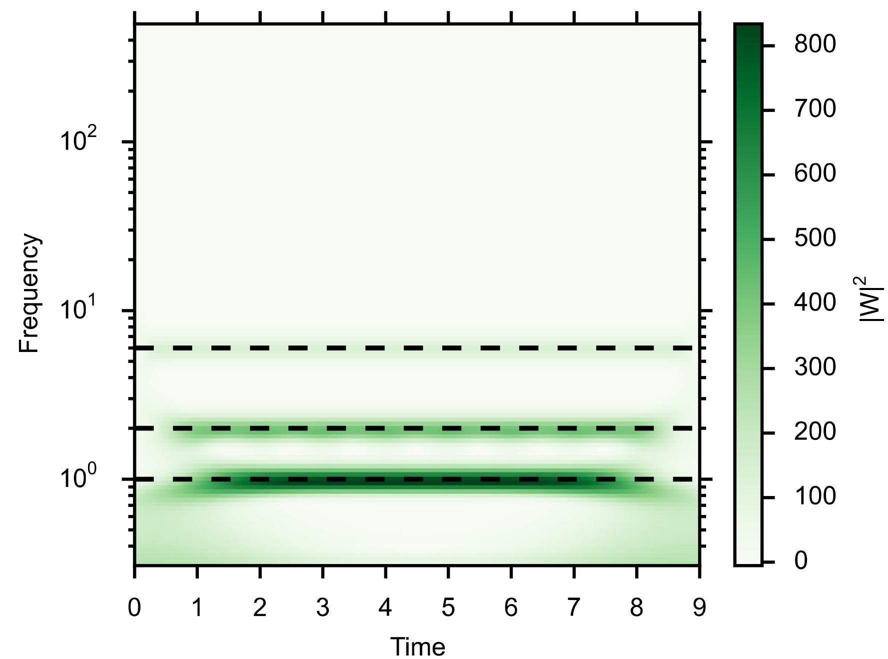
With a trend



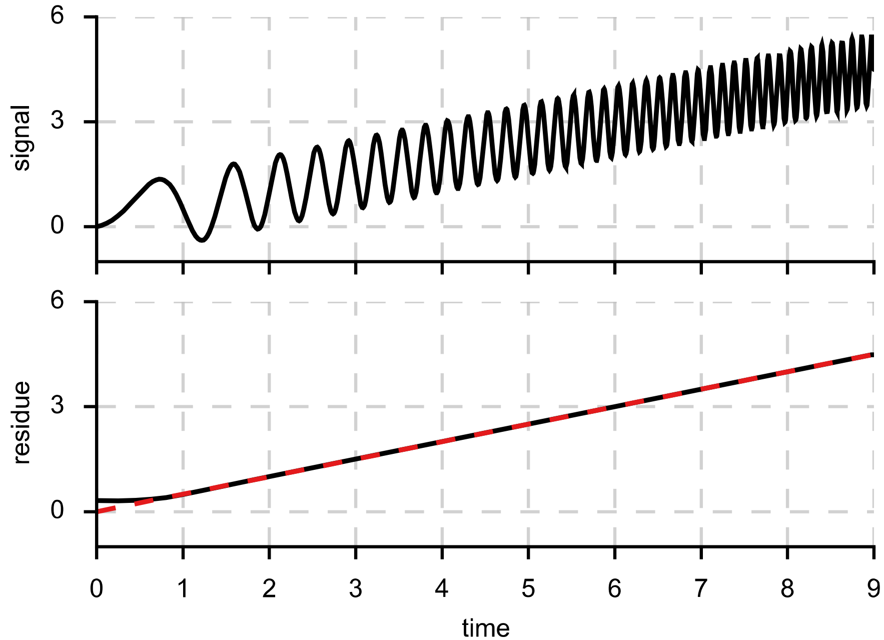


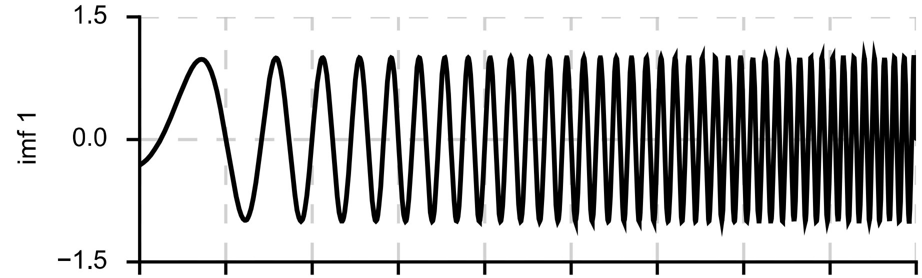
$$y(t)=0.5t+\sin(2\pi t) + \sin(4\pi t) + \sin(12\pi t)$$



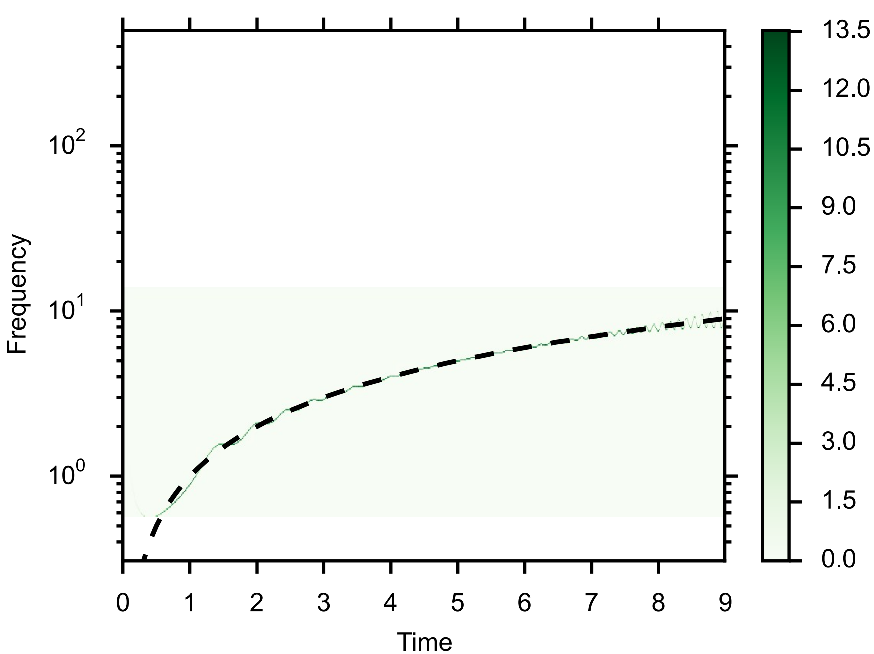


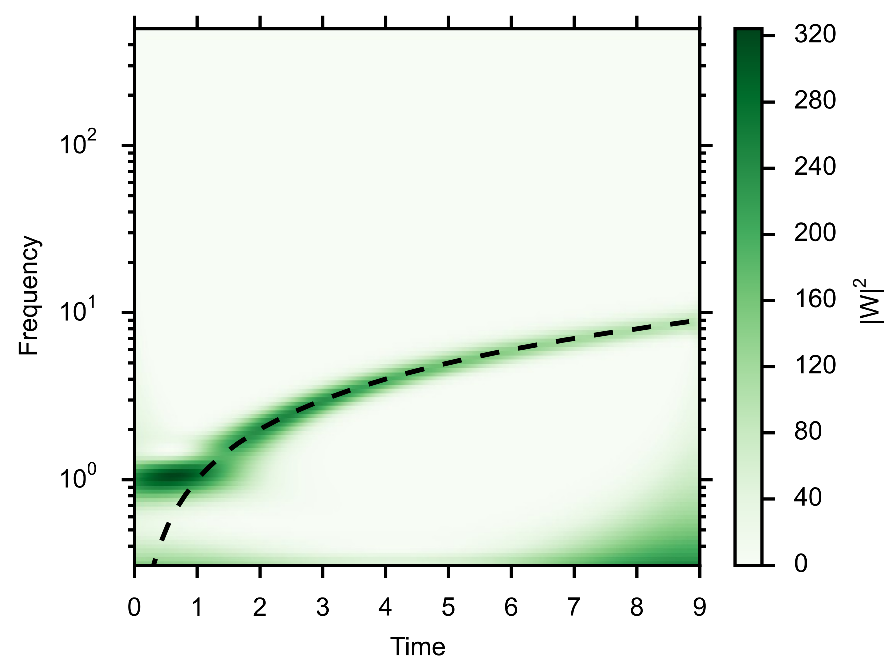
Changing frequency





$$y(t) = 0.5t + \sin(2\pi t^2)$$





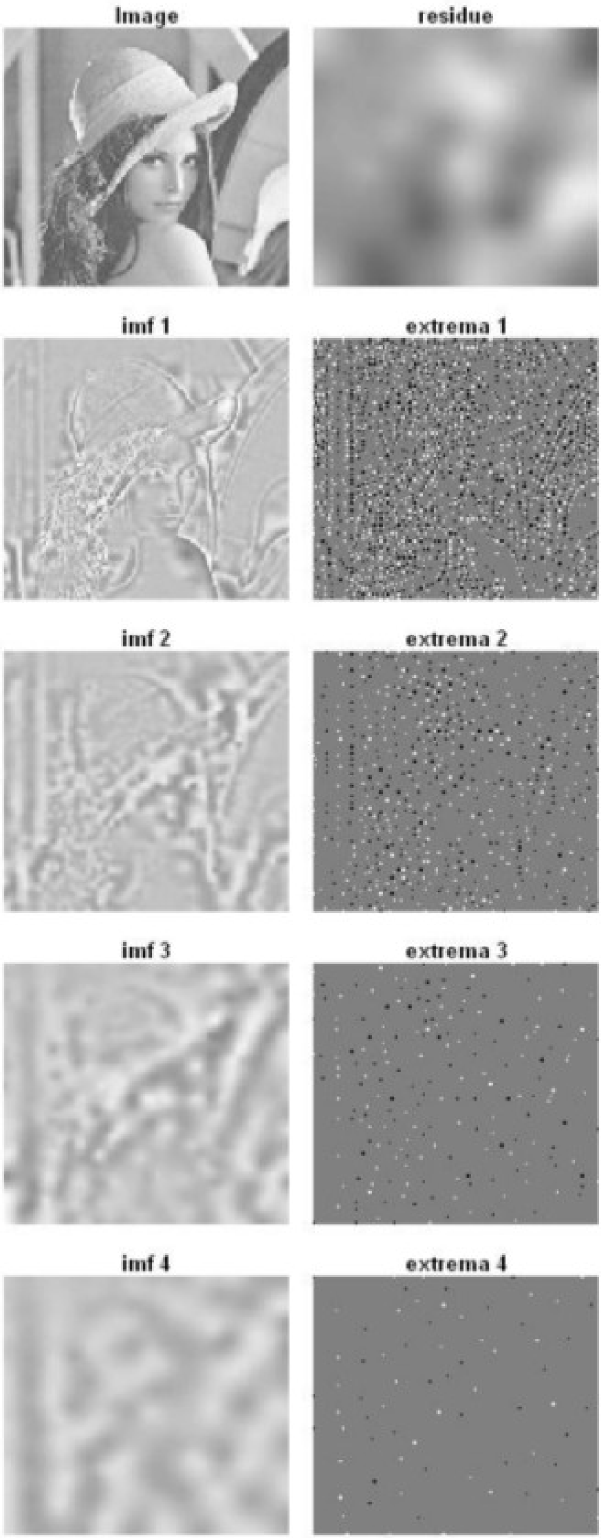


Image
residue
imf 1
extrema 1
imf 2
extrema 2
imf 3
extrema 3
imf 4
extrema 4

Also in 2D